

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY
"KHARKIV POLYTECHNIC INSTITUTE"

GUIDELINES

**of calculation and graphical performing the task on the topic
"LINEAR ELECTRIC CIRCUITS OF HARMONIC CURRENT"**

for courses "Theoretical Basics of Electrical Engineering",
"Theory of Electrical and Magnetic Circuits",
"Theory of Electrical Circuits"

for students of specialties

141 "Electric Power, Electrical Engineering and Electromechanics",
151 "Automation and computer-integrated technologies",
171 "Electronics"

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GUIDELINES of calculation and graphical performing of the task on the topic "LINEAR ELECTRIC CIRCUITS OF HARMONIC CURRENT" for courses "Theoretical Basics of Electrical Engineering", "Theory of Electrical and Magnetic Circuits", "Theory of Electrical Circuits" for students of specialties 141 "Electric Power, Electrical Engineering and Electromechanics", 151 "Automation and computer-integrated technologies", 171 "Electronics" / comp. M. M. Rezynkina, O. H. Kiessaiev, O. L. Rezynkin, S. A. Lytvynenko – Kharkiv: NTU "KhPI", 2020. – 44 p.

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INTRODUCTION

These guidelines contain general theoretical basis on the topic of "Linear electric circuits of harmonic current", tasks for calculation and examples of AC circuits calculations.

Calculation and graphic work is devoted to the topic "Linear electrical circuits of harmonic current" for the students of the first semester of study of the disciplines: "Theoretical basics of electrical engineering", "Theory of electric and magnetic circuits", "Theory of electrical circuits".

Knowledge and skills of this topic are crucial for further successful study of students on the special disciplines of the electrical engineering. In practice, the method of phasors or complex amplitudes, which uses algebra of the complex numbers and allows application of all methods of DC calculation to the AC circuits, is widespread. The method of phasor diagrams construction is a necessary condition for understating of the theory of three-phase circuits. All these methods are applied in various special disciplines of the electrical engineering.

In the process of the task performing, students can use the literature listed at the end of these guidelines, which contains additional information on theoretical issues and examples of the mentioned problems solution. **It is explained how to choose your variant in p. 36.**

The methodical instructions are intended for students of the specialties 141 "Electric Power, Electrical Engineering and Electromechanics", 151 "Automation and computer-integrated technologies", 171 "Electronics" of full-time and distance forms of study.

1. SCOPE

These guidelines establish the requirements for performing calculation and graphical part of the task "Linear electrical circuits of harmonic current" on the courses "Theoretical basics of electrical engineering", "Theory of electrical and magnetic circuits", "Theory of electrical circuits".

The content of the calculation and graphical part of the task corresponds to the training program for students in electrical engineering specialties.

2. PERFORMANCE PURPOSE

As a result of this task performance, a student must:

- 1) know the basic laws and methods of linear AC circuits calculations;
- 2) be able to:
 - a) apply the laws and methods of electrical circuits calculation to solve specific tasks;
 - b) build phasor diagrams and timelines on the results of the calculations.

3. PERFORMANCE ORDER

The calculation and graphical part of the task are performed in the study of the theoretical material on this topic.

4. THE TASK REQUIREMENTS

4.1 Each section should contain:

- 1) the task (number of the variant, electrical schematic, chosen directions of the branch currents, levels of the assigned parameters);

2) the calculation part (calculated currents and voltage drops of each branch);

3) graphical part (phasor diagram and timelines);

4) conclusions (evaluation of the results with the help of power balance usage).

4.2 Task is performed on A4 (297x210) mm paper sheets. If necessary, A3 (297x420) mm format can be used.

Left, bottom and top margins should be at least 20 mm, at least 10 mm on the right.

4.3 The work pages are numbered in Arabic numerals in the upper right corner. Page numbering should be end-to-end for all work. There is no number on the title page, but it is included in the overall page numbering. The form of the title page is given in Attachment B.

4.4 Text of the work should be written on one side of a sheet in one of the following ways:

a) handwritten – clear, legible handwriting with letters and numbers not less than 2.5 mm high. Font density in the work should be the same;

b) printed – size 12 – 14 at 1.5 intervals.

4.5 Work must be stitched.

5. GENERAL PROVISIONS

5.1 Harmonic electrical quantities

The electromagnetic process in an electric circuit is called periodic if values of the currents and the voltages are repeated at regular intervals. The time after which instantaneous values of the quantities are repeated is called a period $[T]=s$.

$$f(t \pm kT) = f(t). \quad (5.1)$$

The frequency f is inversely proportional to a period $[f] = s^{-1} = \text{Hz}$:

$$f = \frac{1}{T}. \quad (5.2)$$

The most common and important type of the periodic currents and voltages is the sinusoidal (harmonic) currents and voltages.

If applied voltage is sinusoidal

$$u = U_m \sin(\omega t + \Psi_u), \quad (5.3)$$

then the current in the linear circuit is also sinusoidal

$$i = I_m \sin(\omega t + \Psi_i), \quad (5.4)$$

where u, i are instantaneous voltage and current values;

$\omega=2\pi f$ is the angular frequency;

Ψ_u, Ψ_i are initial phases of the voltage and the current;

U_m, I_m are the voltage and the current amplitudes.

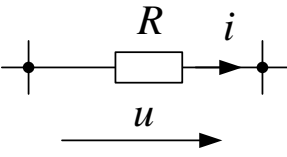
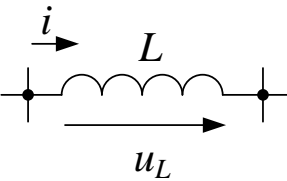
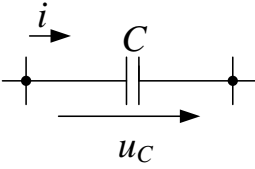
5.2 Parameters of passive circuit elements

Passive elements of AC electrical circuits are:

- resistive element with resistance R , in which electromagnetic energy is converted to heat (measured in Ohms – Ω);
- inductive element with inductance L , which characterizes the magnetic field energy $W_M = \frac{Li^2}{2}$ and the phenomenon of self-induction $e_L = -L \frac{di}{dt}$ (measured in Henry – H);
- capacitive element with a capacitance C , which characterizes the electric field energy $W_E = \frac{Cu^2}{2}$ (measured in Farads – F).

Table 5.1 presents the circuit passive elements and the voltage drop dependence on the elements parameters.

Table 5.1

Element	Designation in the circuit	Connection between the current and the voltage
Resistance R		$u_R = iR$
Inductance L		$u_L = L \frac{di}{dt}$
Capacitance C		$u_C = \frac{1}{C} \int idt$

5.3 Phasors (complex amplitudes) of the currents and the voltages

All basic laws and methods for DC electrical circuits calculation can be used to calculate linear AC circuits. To do this, the method of **phasors** (or complex amplitudes) is used. It uses phasors (complex amplitudes or complex RMS) of the sinusoidal currents and voltages. This allows the integration and differentiation operations (see table 5.1) to be replaced by the algebraic operations.

A rotating vector, called a “phasor” is a scaled line which length represents an AC quantity that has both magnitude (“peak amplitude” or RMS) and direction (“phase”) which is “frozen” at some point in time (as a rule $t=0$).

Phasor voltage and current with magnitude in peak (maximum) value are designated by underlining of the corresponding values and written as follows:

$$\underline{U}_m = U_m e^{j\Psi_u}; \quad (5.5)$$

$$\underline{I}_m = I_m e^{j\Psi_i}, \quad (5.6)$$

where U_m is voltage amplitude;
 Ψ_u is initial voltage phase;
 I_m is current amplitude;
 Ψ_i is initial current phase;
 $e=2.7182818$ is constant;
 $j = \sqrt{-1}$ is imaginary unit.

From phasors with magnitude in peak value, it is possible to transect to phasors with magnitude in RMS:

$$\underline{U} = \frac{\underline{U}_m}{\sqrt{2}} = U e^{j\Psi_u}; \quad (5.7)$$

$$\underline{I} = \frac{I_m}{\sqrt{2}} = I e^{j\psi_i}. \quad (5.8)$$

If a phasor current or voltage is given without other qualification, it is assumed to be in RMS. If a phasor current or voltage is given with “m” index, it is assumed to be in maximum (peak) values.

Ohm’s and Kirchhoff’s laws can be used for the phasor voltages and currents.

Thus, for calculation of the harmonic current circuits, you can use all the methods of calculation of the electrical circuits used for DC circuits.

5.4 Ohm’s and Kirchhoff’s laws with phasors in the complex form

5.4.1 Ohm’s law with phasors in the complex form

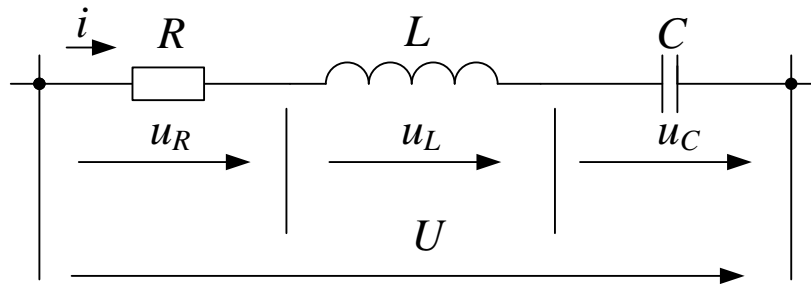


Figure 5.1 – Elementary branch of an AC electrical circuit

For the electrical circuit passive branch, the phasor current is equal to a ratio of the phasor voltage to the branch complex impedance:

$$\underline{I} = \frac{U}{\underline{Z}}, \quad (5.9)$$

where \underline{Z} is complex impedance of the branch.

The complex impedance is determined by the equation:

$$\underline{Z} = R + jX = Z e^{j\varphi}, \quad (5.10)$$

where R is active resistance of the branch;

$X = X_L - X_C = \omega L - \frac{1}{\omega C}$ is reactance of the branch;

$\varphi = \Psi_u - \Psi_i$ is phase shift angle between the voltage and the current.

Table 5.2 presents the passive elements complex impedances and dependence of phasor voltages on their parameters.

Table 5.3 presents the complex impedances of R - L and R - C elements connected in series.

Table 5.2

Element	Element complex impedance	Phasor of the voltage drop across the element
R	$\underline{Z} = R$	$\underline{U}_R = \underline{I}R$
L	$\underline{Z} = jX_L = j\omega L = \omega L e^{j90^\circ}$	$\underline{U}_L = \underline{I}j\omega L$
C	$\underline{Z} = jX_C = -j\frac{1}{\omega C} = \frac{1}{\omega C} e^{-j90^\circ}$	$\underline{U}_C = \underline{I}\left(-j\frac{1}{\omega C}\right)$

Table 5.3

Branch	Complex impedances of the branches
R - L	$\underline{Z} = R + jX_L = R + j\omega L = \sqrt{R^2 + (\omega L)^2} e^{j\arctg\frac{X_L}{R}}$
R - C	$\underline{Z} = R + jX_C = R - j\frac{1}{\omega C} = \sqrt{R^2 + \left(-\frac{1}{\omega C}\right)^2} e^{j\arctg\frac{X_C}{R}}$

5.4.2 Kirchhoff's laws with phasors in the complex form

Kirchhoff's current law in the complex form:

an algebraic sum of phasor currents of the electrical circuit branches that converge into a node is equal to zero:

$$\sum_{k=1}^n \underline{I}_k = 0. \quad (5.11)$$

Kirchhoff's voltage law in the complex form:

an algebraic sum of phasors of the voltage drops in any closed loop of an electrical circuit is equal to an algebraic sum of phasors EMFs in the loop:

$$\sum_{k=1}^m \underline{Z}_k \underline{I}_k = \sum_{k=1}^l \underline{E}_k. \quad (5.12)$$

The number of independent equations and the sign rules at writing Kirchhoff's equations are the same as for DC circuits.

5.5 Determination of the complex power, power balance of a circuit, wattmeter readings

The **complex power** (named **apparent power**) is determined by multiplying the phasor voltage by the complex conjugate of the phasor current:

$$\underline{S} = \underline{U} \underline{I}^* = U e^{j\Psi_u} \cdot I e^{-j\Psi_i} = S e^{j\varphi} = S \cos \varphi + j S \sin \varphi = P + jQ \quad (5.13)$$

where \underline{I}^* is complex conjugate of the phasor current;

P is active power;

Q is reactive power.

Two complex conjugate numbers differ only by a sign before imaginary unit j ($\underline{I}_m = I_m e^{j\Psi_i}$; $\underline{I}_m^* = I_m e^{-j\Psi_i}$).

Power balance for the harmonic current circuit:

$$\sum \underline{S}_{gen} = \sum \underline{S}_{cons}, \quad (5.14)$$

where \underline{S} is apparent power.

The power generated is an algebraic sum of all sources complex powers.

Complex power generated by an EMF source is:

$$\underline{S}_{gen} = \underline{E} \underline{I}^* = P_{gen} + jQ_{gen}, \quad (5.15)$$

where \underline{E} is phasor of the an source;

P_{gen} is generated active power;

Q_{gen} is generated reactive power;

\underline{I}^* is complex conjugate of phasor current in the branch to which a source is connected.

Consumed power is defined as a sum of the complex powers of each electrical circuit branch. The complex power consumed by the electrical circuit branches is determined by the equation:

$$\underline{S}_{cons} = \underline{U} \underline{I}^* = \underline{Z} I^2 = RI^2 + jXI^2 = P_{cons} + jQ_{cons}, \quad (5.16)$$

where \underline{U} is phasor voltage drop across the branch;

\underline{I}^* is complex conjugate of the phasor current of the branch.

The active power measurement is performed by a wattmeter. The wattmeter has two windings (fig. 5.2).

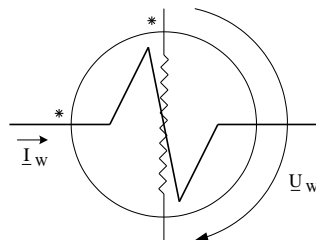


Figure 5.2

The winding through which the current flows has a small number of turns and a large cross-section of a wire with low resistance, is connected in

series to the test circuit (as an ammeter is usually connected). The current must flow into the beginning of the winding indicated by an asterisk in Fig. 5.2.

A voltage winding with a large number of turns of a wire with a small cross-section and high resistance is connected in parallel to the test section of the circuit (as a voltmeter is usually connected). The wattmeter voltage is measured between the terminal marked by an asterisk and the terminal with no mark (Fig. 5.2).

An electrical schematic of a wattmeter connection for measuring the active power consumed in an AC circuit is shown in fig. 5.3.

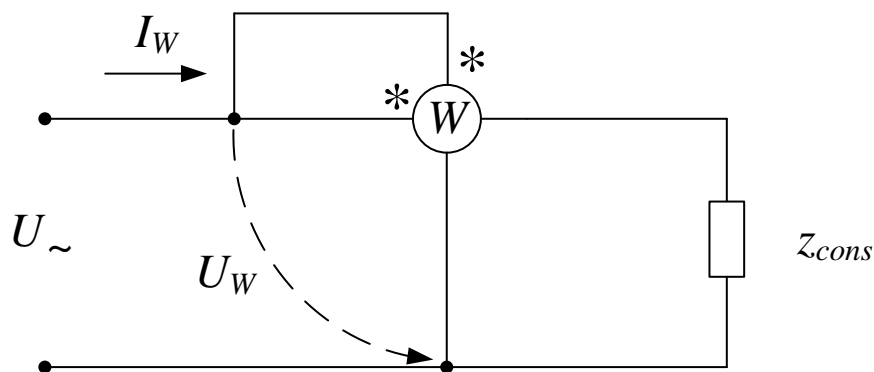


Figure 5.3

A wattmeter measures the active power of the circuit branch to which it is connected. Active power is defined as a real part of the complex power of this branch by the equation:

$$P_w = \text{Re}[\underline{U}_w \underline{I}_w^*] = U_w I_w \cos\varphi, \quad (5.17)$$

where \underline{U}_w is phasor of the voltage drop across the terminals of the wattmeter voltage measuring winding;

\underline{I}_w^* is complex conjugate of phasor current flowing through the wattmeter current measuring winding;

φ is phase shift angle between the voltage and the current of the wattmeter.

5.6 Complex numbers application for the electrical circuits calculations

It is known from the mathematics courses that a complex number can be presented in the rectangular and the exponential forms (table 5.4).

Table 5.4

Rectangular form	$\underline{A} = a + jb$	a is the complex number real part; b is the complex number imaginary part
Exponential form	$\underline{A} = Ae^{j\alpha}$	A is the complex number module; α is the complex number argument

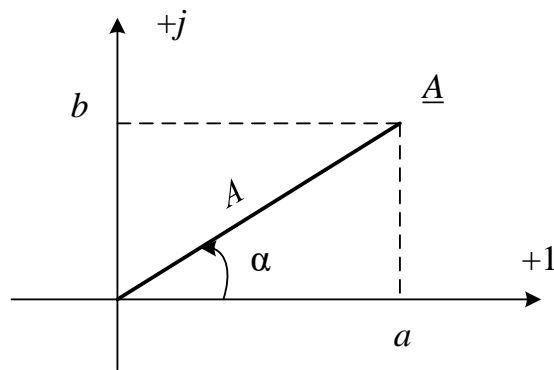


Figure 5.4

Transition from the exponential to the rectangular form is done by Euler's formula:

$$\underline{A} = Ae^{j\alpha} = A\cos\alpha + jA\sin\alpha = a + jb. \quad (5.18)$$

Transition from the rectangular to the exponential form is follows:

$$\underline{A} = a + jb = \sqrt{a^2 + b^2} e^{j\arctg\frac{b}{a}} = Ae^{j\alpha}. \quad (5.19)$$

It should be remembered that if a real part of a complex number is negative, then additional angle $\pm\pi$ must be summed to angle α as the tangents of two angles differing by $\pm\pi$ are the same.

Examples:

$$1. \underline{A}_1 = -5 + j6 = \sqrt{5^2 + 6^2} e^{j\left(\arctg\frac{6}{-5} \pm 180^\circ\right)} = 7,81e^{-j230,2^\circ} = 7,81e^{j129,8^\circ},$$

$$2. \underline{A}_2 = -4 - j3 = \sqrt{4^2 + 3^2} e^{j\left(\arctg\frac{-3}{-4} \pm 180^\circ\right)} = 5e^{j216,9^\circ} = 5e^{-j143,1^\circ}.$$

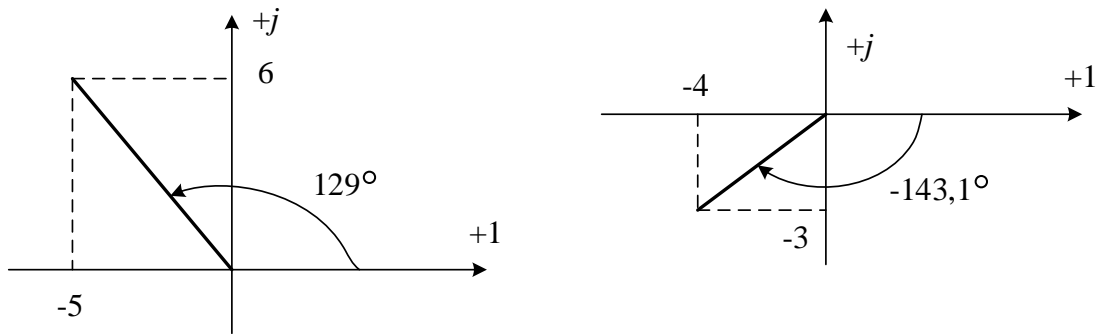


Figure 5.5

Operations with the complex numbers

1. **Addition and subtraction** of the complex numbers are performed in the rectangular form by adding (subtracting) separately a real and an imaginary parts:

$$\underline{A} = \underline{A}_1 \pm \underline{A}_2 = (a_1 + jb_1) \pm (a_2 + jb_2) = (a_1 \pm a_2) + j(b_1 \pm b_2) = a + jb \quad (5.20)$$

2. **Multiplication (division)** of the complex numbers is performed in the exponential form (but it is also possible in the rectangular form) by multiplication (division) of modules and addition (subtraction) of arguments:

$$\underline{A} = A_1 e^{j\alpha_1} \cdot A_2 e^{j\alpha_2} = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)} = A e^{j\alpha}; \quad (5.21)$$

$$\underline{A} = \frac{A_1 e^{j\alpha_1}}{A_2 e^{j\alpha_2}} = \frac{A_1}{A_2} e^{j(\alpha_1 - \alpha_2)} = A e^{j\alpha}. \quad (5.22)$$

3. Exponentiation is performed as follows:

$$\left(A e^{j\alpha}\right)^n = A^n e^{jn\alpha}. \quad (5.23)$$

6. THE TASK CONTENT

Calculation of the branched electrical circuit of harmonic current.

1) If **there is no** magnetic coupling between coils:

- a) determine currents and voltages in all branches;
- b) calculate active and reactive power balance, check its fulfillment and determine the wattmeter reading;
- c) draw a phasor diagram of currents and voltages;
- d) plot the instantaneous EMF values $e(t)$ and current in the branch with EMF $i(t)$ in one coordinate axis.

2) If **there is** a magnetic coupling between the coils:

- a) write a system of equations according to Kirchhoff's laws in the phasor form;
- b) calculate the currents of the branches and check correctness of the calculations by the method of decoupling of the inductive couplings, substituting the values of the currents found in the system of equations compiled by Kirchhoff's laws;
- c) determine the voltage drops across the branches;
- d) find the power balance for two branches containing inductively coupled coils.

Variants of the task are given in Attachment A.

7. EXAMPLES OF ELECTRICAL CIRCUITS CALCULATION

7.1 Calculation of the harmonic current circuit without the magnetic coupling between the coils

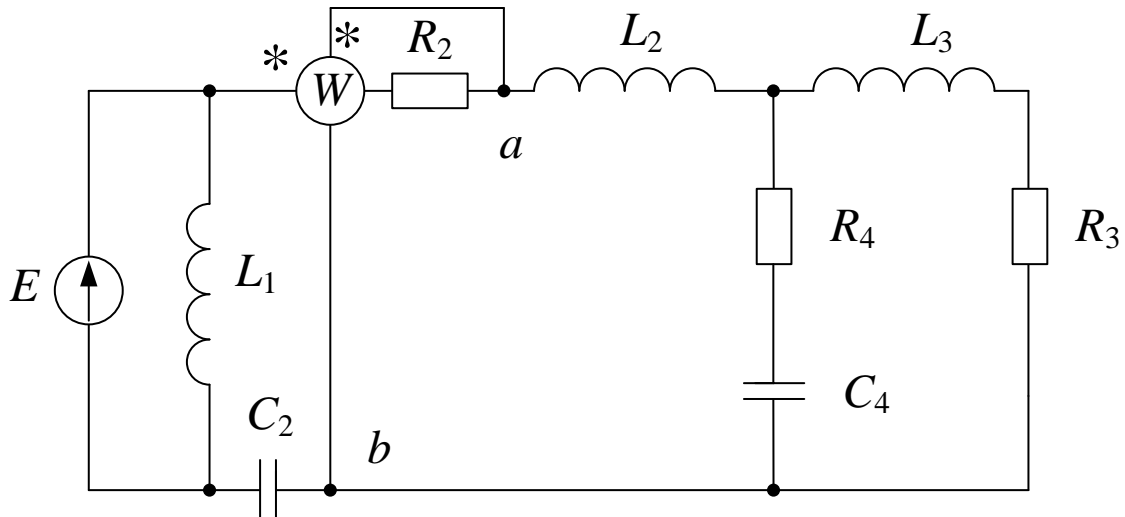


Figure 7.1

The electric circuit (fig. 7.1) parameters have the following values:

$E=120\text{ V}$	$L_1=8\text{ mH}$	$C_2=66,5\ \mu\text{F}$		$C_4=100\ \mu\text{F}$
$\Psi_E=30^\circ$		$L_2=5.59\text{ mH}$	$L_3=3,194\text{ mH}$	
$f=200\text{ Hz}$		$R_2=1\text{ Ohm}$	$R_3=3\text{ Ohm}$	$R_4=6\text{ Ohm}$

Important: the measuring devices (in this case, a wattmeter) should not be taken into account at the electrical circuit calculation.

For the circuit shown in fig. 7.2, determine the branches currents and voltages by the phasor method, check the power balance. Draw the phasor diagram and graphs of $e(t)$, $i(t)$ instant values.

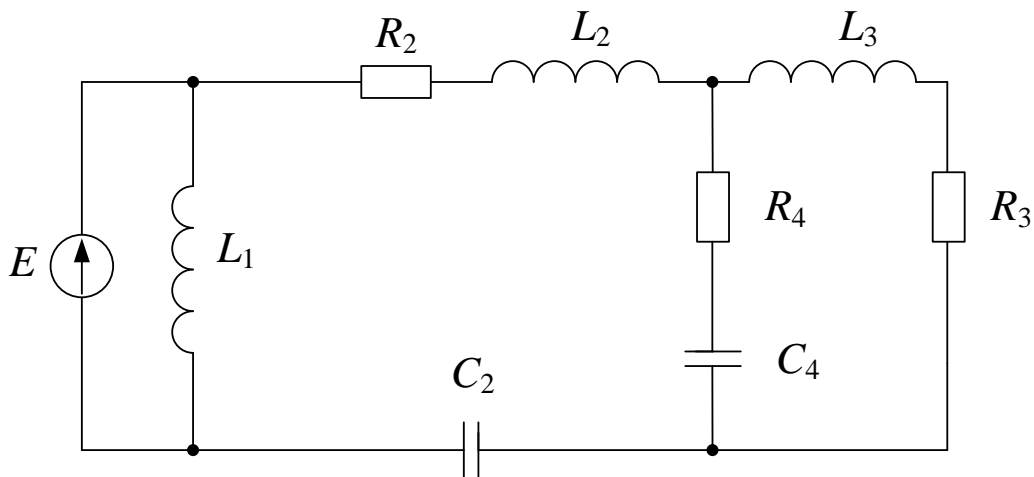


Figure 7.2

Solution:

1) Determination of the angular frequency:

$$\omega = 2\pi f = 2\pi \cdot 200 = 1257 \text{ s}^{-1}.$$

2) Branches reactances calculation (complex inductive and capacitive impedances):

$$X_L = \omega L; \quad X_C = -\frac{1}{\omega C}$$

$$X_{L1} = \omega L_1 = 1257 \cdot 8 \cdot 10^{-3} = 10,06 \text{ Ohm};$$

$$X_{L2} = \omega L_2 = 1257 \cdot 5,59 \cdot 10^{-3} = 7,03 \text{ Ohm};$$

$$X_{C2} = -\frac{1}{\omega C_2} = -\frac{1}{1257 \cdot 66,5 \times 10^{-6}} = -11,96 \text{ Ohm};$$

$$X_{L3} = \omega L_3 = 1257 \cdot 3,194 \cdot 10^{-3} = 4,01 \text{ Ohm};$$

$$X_{C4} = -\frac{1}{\omega C_4} = -\frac{1}{1257 \cdot 100 \cdot 10^{-6}} = -7,96 \text{ Ohm}.$$

3) Branches complex reactances calculation:

$$\underline{Z} = R + jX = \sqrt{R^2 + X^2} e^{j \arctg \frac{X}{R}} = Ze^{j\varphi}$$

Note: if the branch has only the reactance X ($R = 0$), then

$$\varphi = \operatorname{arctg} \frac{X}{R} = \operatorname{arctg} \frac{X}{0} = \operatorname{arctg} \pm \infty = \pm 90^\circ$$

$$\underline{Z}_1 = j\omega L_1 = j10,06 = 10,06 e^{j90^\circ} \text{ Ohm};$$

$$\begin{aligned} \underline{Z}_2 &= R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) = 1 + j(7,03 - 11,96) = 1 - j4,93 = \\ &= 5,03 e^{-j78,5^\circ} \text{ Ohm}; \end{aligned}$$

$$\underline{Z}_3 = R_3 + j\omega L_3 = 3 + j4,01 = 5,01 e^{j53,2^\circ} \text{ Ohm};$$

$$\underline{Z}_4 = R_4 - j\frac{1}{\omega C_4} = 6 - j7,96 = 9,97 e^{-j53^\circ} \text{ Ohm}.$$

As can be seen from fig. 7.3, this circuit is a mixed connection.

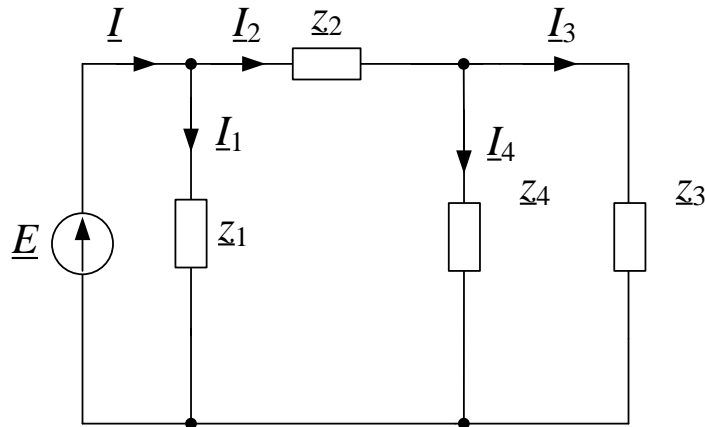


Figure 7.3

Branches \underline{Z}_3 and \underline{Z}_4 are connected in parallel. Their equivalent resistance is defined as follows:

$$\begin{aligned} \underline{Z}_{34} &= \frac{\underline{Z}_3 \cdot \underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = \frac{5,01 e^{j53,2^\circ} \cdot 9,97 e^{-j53^\circ}}{3 + j4,01 + 6 - j7,94} = \frac{43,95 e^{j0,2^\circ}}{9 - j3,95} = \frac{43,95 e^{j0,2^\circ}}{9,83 e^{-j23,7^\circ}} = 5,08 e^{j23,9^\circ} = \\ &= 4,64 + j2,06 \text{ Ohm}. \end{aligned}$$

\underline{Z}_2 and \underline{Z}_{34} are connected in series, as can be seen from fig. 7.4, so:

$$\underline{Z}_{234} = \underline{Z}_2 + \underline{Z}_{34} = 1 - j4,93 + 4,64 + j2,06 = 5,64 - j2,87 = 6,33 e^{-j27^\circ} \text{ Ohm}.$$

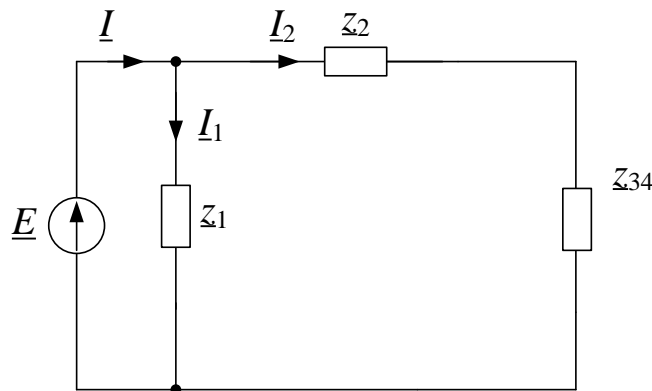


Figure 7.4

\underline{Z}_1 and \underline{Z}_{234} are connected in parallel, but it is not necessary to determine their equivalent resistance, as the voltage drop across these impedances is known (it is equal to the EMF voltage). Using Ohm's law in the phasor form, you can calculate the currents of these branches.

Phasor EMF:

$$\underline{E} = E e^{j\psi_E} = 120 e^{j30^\circ} \text{ V.}$$

3) Determination of the branches currents:

$$\underline{I}_1 = \frac{\underline{E}}{\underline{Z}_1} = \frac{120 e^{j30^\circ}}{10,06 e^{j90^\circ}} = 11,93 e^{-j60^\circ} = 5,96 - j10,33 \text{ A;}$$

$$\underline{I}_2 = \frac{\underline{E}}{\underline{Z}_{234}} = \frac{120 e^{j30^\circ}}{6,33 e^{-j27^\circ}} = 18,96 e^{j57^\circ} = 10,32 + j15,9 \text{ A.}$$

Current \underline{I}_3 is determined by the current divider rule:

$$\begin{aligned} \underline{I}_3 &= \underline{I}_2 \cdot \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = \\ &= 18,96 e^{j57^\circ} \cdot \frac{9,97 e^{-j53^\circ}}{3 + j4,01 + 6 - j7,96} = 19,23 e^{j27,7^\circ} = 17,03 + j8,94 \text{ A.} \end{aligned}$$

Currents \underline{I}_4 and \underline{I} can be determined by Kirchhoff's current law:

$$\underline{I}_4 = \underline{I}_2 - \underline{I}_3 = 10,32 + j15,9 - 17,03 - j8,94 = -6,71 + j6,96 = 9,67 e^{j134} \text{ A;}$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 5,96 - j10,33 + 10,32 + j15,9 = 16,28 + j5,57 = 17,21 e^{j18,9} \text{ A.}$$

4) Determination of the branch voltages:

$$\underline{U}_1 = \underline{E} = 120e^{j30^\circ} = 103,92 + j60 \text{ V};$$

$$\underline{U}_2 = \underline{Z}_2 \cdot \underline{I}_2 = 5,03e^{-j78,5^\circ} \cdot 18,96e^{j57^\circ} = 95,37e^{-j21,5^\circ} = 88,73 - j34,95 \text{ V};$$

$$\underline{U}_{34} = \underline{Z}_{34} \cdot \underline{I}_2 = 5,08e^{j23,9^\circ} \cdot 18,96e^{j57^\circ} = 96,32e^{j80,9^\circ} = 15,23 + j95,11 \text{ V}.$$

Correctness of the calculations is verified by Kirchhoff's voltage law:

$$\underline{U}_2 + \underline{U}_{34} = \underline{E};$$

$$88,73 - j34,95 + 15,23 + j95,11 = 103,96 + j60,16;$$

$$103,96 + j60,16 \approx 103,92 + j60.$$

Kirchhoff's voltage law is fulfilled, so the voltages of the branches are calculated correctly.

5) Checking the active and reactive power balance.

Apparent power - complex power generated by the EMF source:

$$\underline{S}_{gen} = \underline{E} \cdot \underline{I}^* = P_{gen} + jQ_{gen} = 120e^{j30^\circ} \cdot 17,21e^{-j18,9^\circ} = 2065e^{j11,1^\circ} = 2026 + j398 \text{ VA}.$$

Calculation of the complexes of power consumed in the branches:

$$\underline{S}_1 = \underline{U}_1 \cdot \underline{I}_1^* = 120e^{j30^\circ} \cdot 11,93e^{j60^\circ} = 1432e^{j90^\circ} = j1432 \text{ VA};$$

$$\underline{S}_2 = \underline{U}_2 \cdot \underline{I}_2^* = 95,37e^{-j21,5^\circ} \cdot 18,96e^{-j57^\circ} = 1808e^{-j78,5^\circ} = 361 - j1772 \text{ VA};$$

$$\underline{S}_3 = \underline{U}_3 \cdot \underline{I}_3^* = 96,32e^{j80,9^\circ} \cdot 19,23e^{-j27,7^\circ} = 1852e^{j53,2^\circ} = 1109 + j1483 \text{ VA};$$

$$\underline{S}_4 = \underline{U}_{34} \cdot \underline{I}_4^* = 96,32e^{j80,9^\circ} \cdot 9,67e^{-j134^\circ} = 931e^{-j53,1^\circ} = 559 - j745 \text{ VA}.$$

Power consumed by the whole circuit:

$$\begin{aligned} \underline{S}_{cons} &= P_{cons} + jQ_{cons} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3 + \underline{S}_4 = \\ &= j1432 + 361 - j1772 + 1109 + j1483 + 559 - j745 = 2029 + j398 \text{ VA}. \end{aligned}$$

The active power balance:

$$P_{gen} = P_{cons} ; 2026 \text{ W} \approx 2029 \text{ W}.$$

The reactive power balance:

$$Q_{gen} = Q_{cons} ; 398 \text{ VAr} = 398 \text{ VAr}.$$

Note: the error should not exceed 5 % in the active and the reactive power balance equations.

Thus, the balance of the active and the reactive power is satisfied, which confirms correctness of the electrical circuit currents and voltages calculation.

6) Determination of the wattmeter readings.

The active power of the circuit branch is determined by (5.17):

$$P_w = \text{Re} \left[\underline{U}_w \underline{I}_w^* \right].$$

Voltage \underline{U}_w is determined by Kirchhoff's voltage law for a loop that includes this voltage (fig. 7.5):

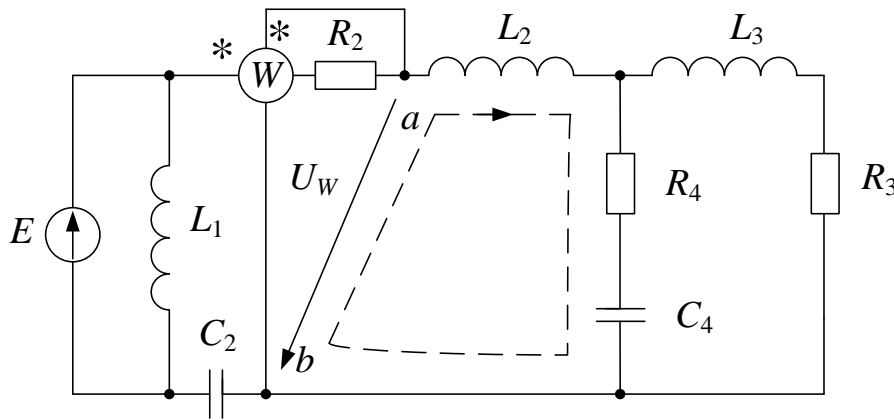


Figure 7.5

$$\underline{U}_{L2} + \underline{U}_{34} - \underline{U}_w = 0.$$

Thus

$$\begin{aligned} \underline{U}_w &= \underline{U}_{L2} + \underline{U}_{34} = \underline{I}_2 \cdot j\omega L + \underline{U}_{34} = (10,32 + j15,9) \cdot j7,03 + 15,23 + j95,11 = \\ &= j72,55 - 111,78 + 15,23 + j95,11 = -96,55 + j167,66 = \\ &= 193,36 e^{j119,9^\circ} \text{ V}. \end{aligned}$$

\underline{I}_w^* is the current flowing in the second branch:

$$\underline{I}_w^* = \underline{I}_2^* = 18,96e^{-j57^\circ} \text{ A.}$$

So, the wattmeter readings is:

$$P_w = \operatorname{Re}\left[193,36e^{j119,9^\circ} \cdot 18,96e^{-j57^\circ}\right] = \operatorname{Re}\left[3666,1e^{j62,9^\circ}\right] = 3666,1 \cdot \cos 62,9^\circ = 1670 \text{ W.}$$

7) The phasor diagram drawing.

The phasor diagram for the branched circle is drawing in accordance with Kirchhoff's laws:

$$\underline{I} = \underline{I}_1 + \underline{I}_2; \quad (7.1)$$

$$\underline{I}_2 = \underline{I}_3 + \underline{I}_4; \quad (7.2)$$

$$\underline{U}_1 = \underline{U}_{34} + \underline{U}_2. \quad (7.3)$$

To draw the phasor diagram, the RMS values of currents and voltages in the circuit branches are used, as well as the phase shift angles between them. RMS values of voltages, currents and phase shift angles have been calculated and are presented as a following table:

$U_1 = E = 120 \text{ V}$	$I_1 = 11,93 \text{ A}$	$\varphi_1 = 90^\circ$
$U_2 = 95,37 \text{ V}$	$I_2 = 18,96 \text{ A}$	$\varphi_2 = -78,5^\circ$
$U_{34} = 96,32 \text{ V}$	$I_3 = 19,23 \text{ A}$	$\varphi_3 = 53,2^\circ$
	$I_4 = 9,67 \text{ A}$	$\varphi_4 = -53^\circ$
	$I = 17,21 \text{ A}$	

Important:

- the phase shift angle φ is drawn from the current vector to the voltage vector in the phasor diagram.

If angle $\varphi > 0$ (inductive character of a branch), it is drawn **counter-clockwise**

If angle $\varphi < 0$ (capacitive character of a branch), it is drawn **clockwise**.

- A vector sum of two vectors is done by the parallelogram rule. If it is necessary to draw a vector sum of more vectors, each subsequent vector is

drawn from the end of the previous one. The sum vector connects the beginning of the first vector with the end of the last one.

- The scales of voltages m_U and of currents m_I should be selected.

The phasor diagram drawing for a branched circuit is started from the last branched section. As the third and fourth branches are connected in parallel, phasor voltage \underline{U}_{34} is drawn first and from it currents \underline{I}_4 and \underline{I}_3 are drawn by the phase shift angles φ_4 and φ_3 , as shown in fig. 7.6.

In drawing, phasor currents \underline{I}_4 and \underline{I}_3 are added geometrically, their sum is equal to phasor current \underline{I}_2 according to equation (7.2). According to equation (7.3), phasor voltage of the second branch \underline{U}_2 is added to phasor voltage \underline{U}_{34} . \underline{U}_2 direction is determined by turning relatively phasor current \underline{I}_2 by angle φ_2 . Obtaining phasor voltage \underline{U}_1 equal to EMF is done by adding geometrically phasors \underline{U}_{34} and \underline{U}_2 .

And finally, to draw \underline{I}_1 , according to (7.1), phasor current \underline{I}_1 should be added to phasor current \underline{I}_2 . Phasor current \underline{I}_1 is drawn lagged by angle φ_1 relatively phasor voltage \underline{U}_1 . A geometric sum of phasors \underline{I}_1 and \underline{I}_2 is equal to the source current I .

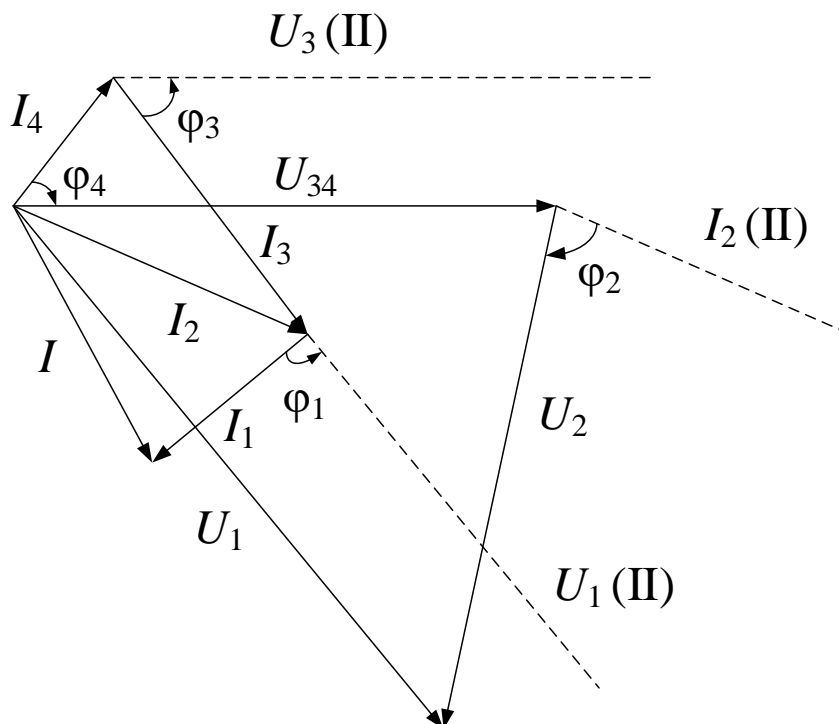


Figure 7.6

So, the phasor diagram is drawn in the following order:

$$\underline{U}_{34} \xrightarrow{-\varphi_4} \underline{I}_4 + \xrightarrow{\varphi_2} \underline{I}_2 \xrightarrow{-\varphi_3} \underline{I}_3 \xrightarrow{\varphi_2} \underline{U}_{34} \xrightarrow{\varphi_2} \underline{U}_{34} + \underline{U}_2 = \underline{U}_1 \xrightarrow{-\varphi_1} \rightarrow$$

$$\rightarrow \underline{I}_1 \rightarrow \underline{I}_1 + \underline{I}_2 = \underline{I}$$

8) Timelines drawing.

Instantaneous values of EMF $e(t)$ and current $i(t)$ in the source branch can be written by phasor EMF $\underline{E} = 120e^{j30^\circ}$ and calculated current $\underline{I} = 17,21e^{j18,9^\circ}$:

$$e(t) = 120\sqrt{2} \sin(\omega t + 30^\circ) = 170 \sin(\omega t + 30^\circ) \text{ V};$$

$$i(t) = 17,21\sqrt{2} \sin(\omega t + 18,9^\circ) = 24 \sin(\omega t + 18,9^\circ) \text{ A}.$$

Their graphs are shown in fig. 7.7.

The axis ωt can be scaled in radians or degrees (for example, 30° corresponds to $\pi/6$ radians and $18,9^\circ$ corresponds to $0,11\pi$ radians). A positive initial phase is laid on the horizontal axis on the left from the origin, a negative one is laid on the right. A positive half-wave of the sine wave starts from the right of this mark.

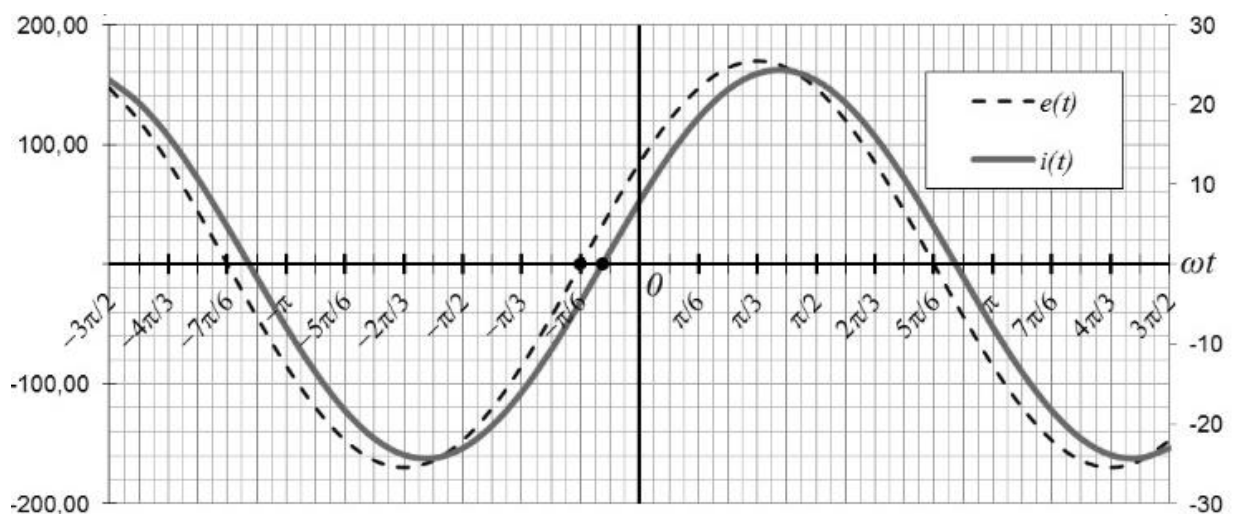


Figure 7.7

7.2 Calculation of the electrical circuit of harmonic current with the inductive coupling between the coils

If the alternating current in one of the coils causes the EMF in the other one, then these two coils have a magnetic coupling (they are inductively coupled) and the resulting EMF is called a mutual induction EMF. Presence of the mutual induction EMF must be taken into account at the electrical circuit calculation.

The voltage drop across a coil with inductance L_k depends on its own current i_k , which creates a self-inducing EMF $e_s = -L_k \frac{di_k}{dt}$ and on current i_m of an inductively coupled coil with inductance L_m that creates the EMF of mutual induction $e_m = \mp M \frac{di_m}{dt}$ in the coil with inductance L_k :

$$u_k = L_k \frac{di_k}{dt} \pm M \frac{di_m}{dt}. \quad (7.4)$$

where M is the mutual inductance of coils k and m .

If the *coils currents have the same direction relative to the same coils terminals*, the coils have the **aiding** connection. In this case, the self- and the mutual magnetic fluxes in each of them coincide in direction, amplifying the resulting flux and voltage, so in the expression for the voltage (7.4) the second term has a positive sign.

If the coils currents directions are opposite, the coils have the **opposite** connection. In this case, the self- and the mutual magnetic fluxes in each of them have opposite directions, and they weaken the resulting flux and voltage, so in the expression for voltage (7.4) the second term has a negative sign.

The expression for the phasor voltage is follows:

$$\underline{U}_k = j\omega L_k \underline{I}_k \pm j\omega M \underline{I}_m. \quad (7.5)$$

Kirchhoff's laws, the mesh current method, or a special technique called the **decoupling of inductive couplings method** are used for branching circuits calculations.

As an example, the schematic of the previous problem was used (see fig. 7.1). It includes the inductive coupling between the coils of the second and third branches, as shown in fig. 7.6.

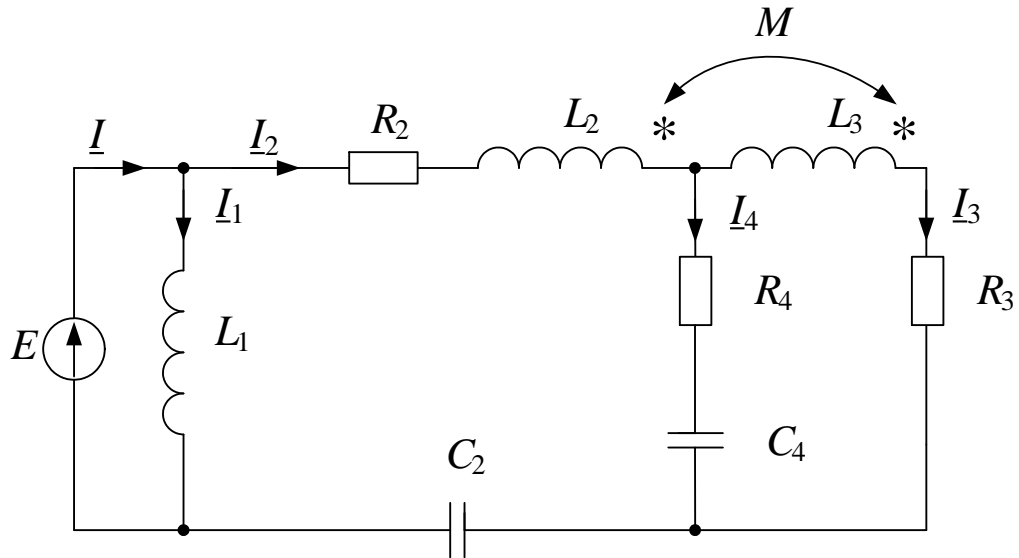


Figure 7.6

The values of the circuit parameters are:

$$E = 120 \text{ V}; \quad \Psi_E = 30^\circ; \quad \omega L_1 = 10,05 \text{ Ohm}; \quad R_2 = 1 \text{ Ohm}; \quad \omega L_2 = 7,02 \text{ Ohm};$$

$$\frac{1}{\omega C_2} = 11,97 \text{ Ohm}; \quad \omega L_3 = 4,01 \text{ Ohm}; \quad R_3 = 3 \text{ Ohm}; \quad R_4 = 6 \text{ Ohm};$$

$$\frac{1}{\omega C_4} = 7,96 \text{ Ohm}; \quad \omega M = 2 \text{ Ohm}.$$

- 1) *The system of equations in the phasor form according to Kirchhoff's laws.*

The directions of currents in the branches should be selected. The loops directions are selected clockwise.

$$\begin{cases} \underline{I} = \underline{I}_1 + \underline{I}_2; \\ \underline{I}_2 = \underline{I}_3 + \underline{I}_4; \\ j\omega L_1 \cdot \underline{I}_1 = \underline{E}; \\ (R_2 + j\omega L_2 - j\frac{1}{\omega C_2})\underline{I}_2 + j\omega M \underline{I}_3 + (R_4 - j\frac{1}{\omega C_4})\underline{I}_4 = \underline{E}; \\ (R_3 + j\omega L_3)\underline{I}_3 + j\omega M \underline{I}_2 - (R_4 - j\frac{1}{\omega C_4})\underline{I}_4 = 0. \end{cases} \quad (7.6)$$

Currents \underline{I}_2 and \underline{I}_3 have the same direction relative to the same terminals of the coils with inductance L_2 and L_3 , i.e. we have the aiding connection. In the case of different directions of currents \underline{I}_2 and \underline{I}_3 relative to the same coils terminals, it will be the opposite connection. So, terms $j\omega M \underline{I}_2$ and $j\omega M \underline{I}_3$ in equations (7.6) have a “plus” sign because directions of currents \underline{I}_2 and \underline{I}_3 coincide with the loops directions and there is the aiding connection.

2) Decoupling of inductive couplings.

This approach consists in replacing a circuit containing the inductive couplings by an equivalent circuit without such couplings.

The general rule of decoupling of inductive couplings is follows:

if two inductively coupled branches are connected to a node of a branched electrical circuit by the **same clamps**, then inductance “-M” should be additionally connected in these branches, and in the third branch free from the magnetic coupling (also connected to this node), inductance “+M” should be connected. If inductively coupled branches are connected to such a node by the **different clamps**, then inductance “+M” should be connected in these branches, and inductance “-M” should be connected in the third branch.

Note: Only currents can be calculated using the decoupling of inductive couplings method. Voltages are calculated by Kirchhoff’s voltage law according to the initial schematic (fig. 7.6).

Figure 7.7 shows examples of several different circuits with two inductively coupled coils and their decoupling.

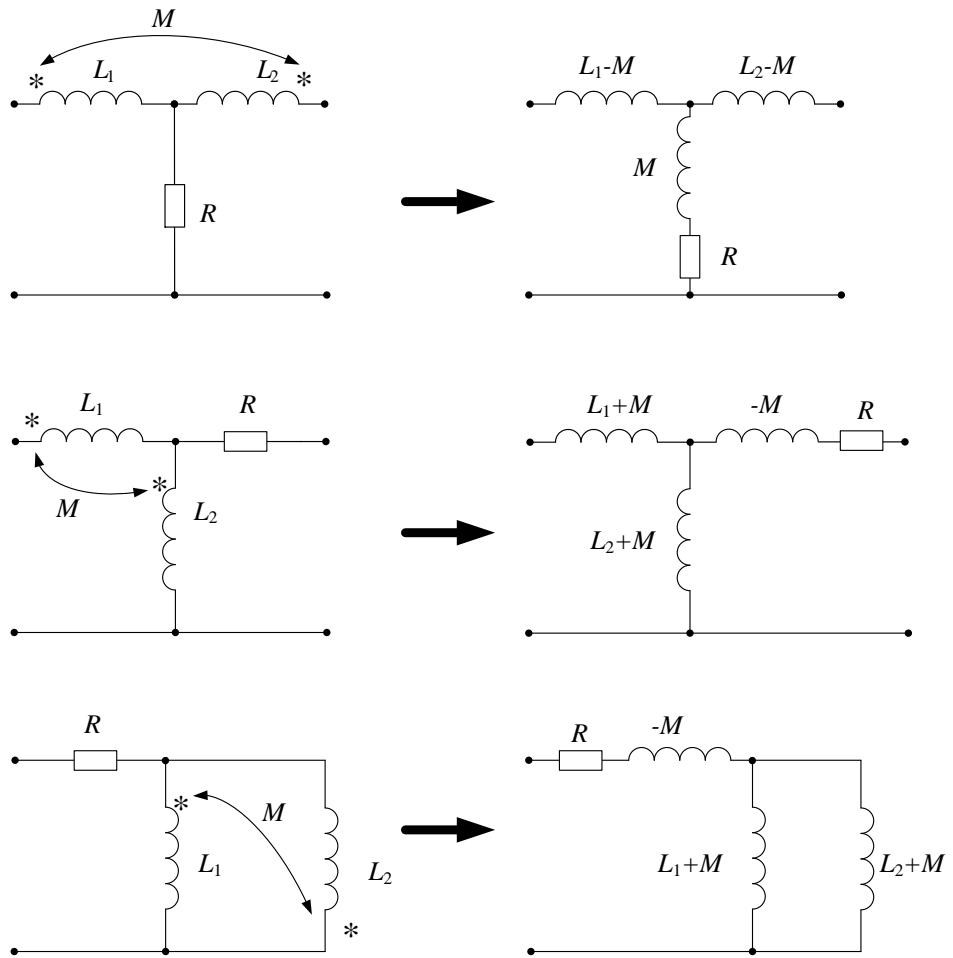


Figure 7.7

3) For calculations of the circuit currents (see fig. 7.6), the decoupling of inductive couplings method is used. By this method, we obtain the circuit shown in fig. 7.8.

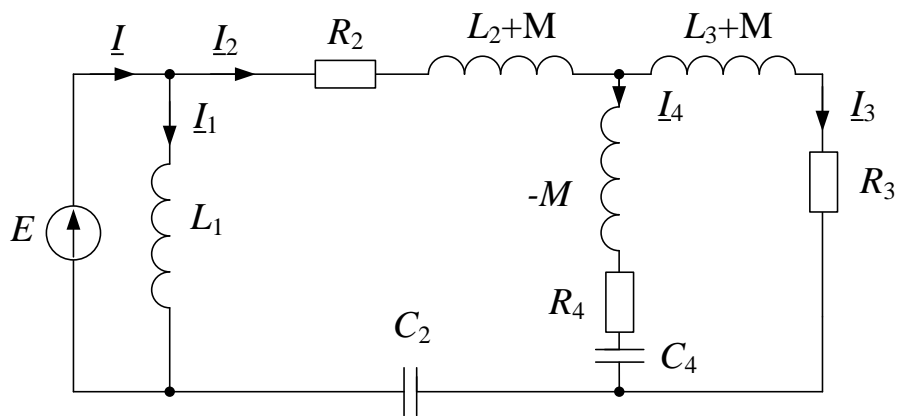


Figure 7.8

The resulting schematic is a mixed connection. In this schematic, the currents of the branches are the same as in the original schematic (see fig. 7.6), and the voltages are different.

4) Calculation of the branches impedances:

$$\underline{Z}_1 = j\omega L_1 = j10,06 = 10,06e^{j90^\circ} \text{ Ohm};$$

$$\begin{aligned} \underline{Z}_2 &= R_2 + j(\omega L_2 + \omega M - \frac{1}{\omega C_2}) = 1 + j(7,02 + 2 - 11,97) = 1 - j2,95 = \\ &= 3,11e^{-j71,5^\circ} \text{ Ohm}; \end{aligned}$$

$$\underline{Z}_3 = R_3 + j(\omega L_3 + \omega M) = 3 + j(4,01 + 2) = 3 + j6,01 = 6,72e^{j63,5^\circ} \text{ Ohm};$$

$$\underline{Z}_4 = R_4 + j(-\omega M - \frac{1}{\omega C_4}) = 6 + j(-2 - 7,96) = 6 - j9,96 = 11,63e^{-j58,9^\circ} \text{ Ohm};$$

$$\underline{Z}_{34} = \frac{\underline{Z}_3 \cdot \underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = \frac{6,72e^{j63,5^\circ} \cdot 11,63e^{-j58,9^\circ}}{3 + j6,01 + 6 - j9,96} = 7,95e^{j28,3^\circ} = 7 + j3,77 \text{ Ohm};$$

$$\underline{Z}_{234} = \underline{Z}_2 + \underline{Z}_{34} = 1 - j2,95 + 7 + j3,77 = 8 + j0,82 = 8,04e^{j5,9^\circ} \text{ Ohm}.$$

5) Calculation of the currents:

$$\underline{I}_1 = \frac{\underline{E}}{\underline{Z}_1} = 11,94e^{-j60^\circ} = 5,97 - j10,34 \text{ A};$$

$$\underline{I}_2 = \frac{\underline{E}}{\underline{Z}_{234}} = \frac{120e^{j30^\circ}}{8,04e^{j5,9^\circ}} = 14,93e^{j24,1^\circ} = 13,63 + j6,1 \text{ A};$$

$$\begin{aligned} \underline{I}_3 &= \underline{I}_2 \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} = 14,93e^{j24,1^\circ} \frac{11,63e^{-j58,9^\circ}}{3 + j6,01 + 6 - j9,96} = 17,66e^{-11,1^\circ} = \\ &= 17,33 - j3,4 \text{ A}; \end{aligned}$$

$$\underline{I}_4 = \underline{I}_2 - \underline{I}_3 = 13,63 + j6,1 - 17,33 + j3,4 = -3,7 + j9,5 = 10,2e^{111,3^\circ} \text{ A};$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 5,97 - j10,34 + 13,63 + j6,1 = 19,6 - j4,24 = 20,05e^{-j12,2^\circ} \text{ A}.$$

Correctness of the current calculations is checked by substituting the currents found in the system in the equations written by Kirchhoff's laws (7.4).

6) Calculation of voltages of the circuit branches.

The decoupling by the inductive couplings method (see fig. 7.8) **is used only to calculate currents**. To determine the voltage drops across the branches, the initial schematic is used (see fig. 7.6).

$$\underline{U}_1 = \underline{E} = 120e^{j30^\circ} = 103,92 + j60 \text{ V};$$

$$\begin{aligned} \underline{U}_2 &= (R_2 + j\omega L_2 - j\frac{1}{\omega C_2})\underline{I}_2 + j\omega M \underline{I}_3 = (1 + j7,02 - j11,97)(13,63 + j6,1) + \\ &+ j2(17,33 - j3,4) = 50,63 - j26,71 = 57,24e^{-j27,8^\circ} \text{ V}; \end{aligned}$$

$$\begin{aligned} \underline{U}_3 &= (R_3 + j\omega L_3)\underline{I}_3 + j\omega M \underline{I}_2 = (3 + j4,01)(17,33 - j3,4) + j2(13,63 + j6,1) = \\ &= 53,42 + j86,55 = 101,71e^{58,3^\circ} \text{ V}; \end{aligned}$$

$$\underline{U}_4 = (R_4 - j\frac{1}{\omega C_4})\underline{I}_4 = \underline{U}_3.$$

7) Power balance for the second and third branches.

Calculation of powers incoming in the second and third branches from the source:

$$\underline{S}_{2incom} = \underline{U}_2 \underline{I}_2^* = 57,24e^{-j27,8^\circ} \cdot 14,93e^{-j24,1^\circ} = 854,6e^{-j51,9^\circ} = 527 - j672 \text{ VA};$$

$$\underline{S}_{3incom} = \underline{U}_3 \underline{I}_3^* = 101,71e^{j58,3^\circ} \cdot 17,66e^{j11,1^\circ} = 1796,2e^{j69,4^\circ} = 632 + j1681 \text{ VA}.$$

Calculation of the powers consumed by the branches elements:

$$\underline{S}_{2cons} = R_2 I_2^2 + j(\omega L_2 - \frac{1}{\omega C_2}) I_2^2 = 1 \cdot 14,93^2 + j(7,02 - 11,97)14,93^2 =$$

$$= 223 - j1103 \text{ VA};$$

$$\underline{S}_{3cons} = R_3 I_3^2 + j\omega L_3 I_3^2 = 3 \cdot 17,66^2 + j4,01 \cdot 17,66^2 = 936 + j1251 \text{ VA}.$$

Calculations of the powers transferred by the magnetic fluxes due to a phenomenon of the mutual induction:

$$\begin{aligned}\underline{S}_{2M} &= j\omega M \underline{I}_3^* \underline{I}_2 = 2e^{j90^\circ} \cdot 17,66e^{-j11.1^\circ} \cdot 14,93e^{-j24.1^\circ} = 527,3e^{j54.8^\circ} = \\ &= 304 + j431 \text{ VA};\end{aligned}$$

$$\begin{aligned}\underline{S}_{3M} &= j\omega M \underline{I}_2^* \underline{I}_3 = 2e^{j90^\circ} \cdot 14,93e^{j24.1^\circ} \cdot 17,66e^{j11.1^\circ} = 527,3e^{j125.2^\circ} = \\ &= -304 + j431 \text{ VA}.\end{aligned}$$

The power incoming in the branches from the source is equal to a sum of the power consumed by the elements of that branch and the power transferred by the magnetic flux:

$$\underline{S}_{2\text{cons}} + \underline{S}_{2M} = 223 - j1103 + 304 + j431 = 527 - j672 = \underline{S}_{2\text{const}};$$

$$\underline{S}_{3\text{cons}} + \underline{S}_{3M} = 936 + j1251 - 304 + j431 = 632 + j1682 = \underline{S}_{3\text{const}}.$$

Powers \underline{S}_{2M} and \underline{S}_{3M} can also be presented as generated by the sources in the second and third branches, the EMFs of which is equal to the EMF of the mutual induction ($\underline{E}_{2M} = -j\omega M \underline{I}_3$; $\underline{E}_{3M} = -j\omega M \underline{I}_2$):

$$\begin{aligned}\underline{S}_{2M} &= -j\omega M \underline{I}_3^* \underline{I}_2 = -2e^{j90^\circ} \cdot 17,66e^{-j11.1^\circ} \cdot 14,93e^{-j24.1^\circ} = 527,3e^{-j125.2^\circ} = \\ &= -304 - j431 \text{ VA};\end{aligned}$$

$$\begin{aligned}\underline{S}_{3M} &= -j\omega M \underline{I}_2^* \underline{I}_3 = -2e^{j90^\circ} \cdot 14,93e^{j24.1^\circ} \cdot 17,66e^{j11.1^\circ} = 527,3e^{-j54.8^\circ} = \\ &= 304 - j431 \text{ VA}.\end{aligned}$$

Then the power balance is:

$$\underline{S}_{2\text{cons}} = \underline{S}_{2\text{const}} + \underline{S}_{2M} = 527 - j672 - 304 - j431 = 223 - j1103 \text{ VA};$$

$$\underline{S}_{3\text{cons}} = \underline{S}_{3\text{const}} + \underline{S}_{3M} = 632 + j1681 + 304 - j431 = 936 + j1251 \text{ VA}.$$

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ATTACHMENT A.

Guidelines for task variant choosing.

The number of the schematic for calculation and the number of the numerical data of the corresponding variant are shown in Table A.1 of the Attachments (see p. 37-38).

The variant number corresponds to the student's number in the journal of the student's academic group. It corresponds to the line number in Table A.1. If there is only one group in the lecture course, then the group index is «I». If there are several groups in the lecture course, then index «I» corresponds to the group which group number is the smallest, for the group with the next number it is «II» and so on. This is the column number in Table A.1.

In each cell of Table A.1 (with a row number corresponding to the student's variant number and a column number corresponding to the index of the group), the number in the circle corresponds to the circuit number (see Figure A.1, p. 39–42), the number in the square corresponds to the variant number of the parameters data (see Table A.2, pages 43–44).

Variants of the tasks of the AC circuits calculations

Table A.1 – Variants of the tasks of the AC circuits calculations.

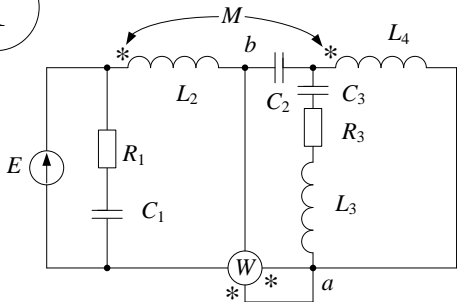
Variant	Group index											
	I		II		III		IV		V		VI	
1	25	19	19	7	13	25	7	13	1	1	25	7
2	2	26	8	8	14	8	20	14	26	26	2	14
3	27	21	21	9	15	27	9	15	3	3	21	15
4	10	10	16	22	22	10	4	4	4	22	16	16
5	23	5	17	11	5	23	11	17	5	5	17	23
6	24	18	12	6	18	12	6	24	12	12	18	24
7	7	7	13	19	19	1	25	13	1	7	13	7
8	26	14	20	8	14	2	8	26	2	20	14	20
9	15	21	9	9	21	3	27	15	3	27	21	9
10	22	16	16	4	10	22	4	4	10	16	22	22
11	17	17	5	11	23	23	17	5	11	11	5	17
12	6	18	12	12	18	24	6	12	24	18	12	6
13	19	25	13	13	19	19	25	1	19	13	1	25
14	2	14	8	20	14	26	20	2	26	8	14	14
15	27	9	21	27	15	15	21	21	27	3	9	15
16	16	22	16	4	10	10	22	4	22	22	4	16
17	11	5	23	17	5	5	17	11	23	23	5	11

Continuation of Table A.1.

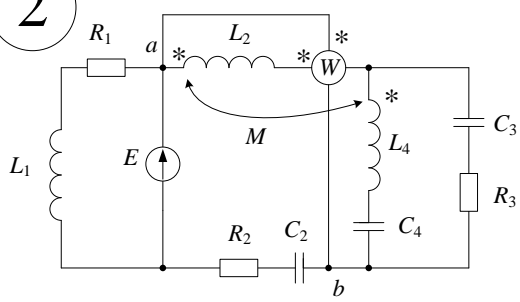
Variant	Group index					
	I	II	III	IV	V	VI
18	24 24	12 18	24 12	18 6	24 6	18 24
19	13 1	7 19	1 7	25 25	1 25	19 13
20	26 2	20 20	14 20	8 14	2 8	26 8
21	21 15	15 3	9 21	3 9	27 27	15 21
22	4 22	16 10	10 4	22 22	4 16	10 22
23	11 17	5 23	17 17	11 23	5 11	23 11
24	6 6	12 24	24 6	18 18	12 6	24 12
25	1 13	7 25	13 7	7 1	1 19	25 25
26	2 2	20 26	8 2	14 14	26 20	8 26
27	3 15	9 27	15 9	3 21	9 3	27 3
28	30 29	28 28	29 28	30 30	29 30	28 30
29	28 30	30 28	28 29	29 29	30 29	29 28
30	29 30	28 30	28 28	29 30	30 30	30 29
31	7 1	1 19	1 25	13 7	19 13	25 7
32	8 32	2 2	14 14	26 26	20 32	20 8
33	9 33	21 15	3 9	21 21	21 33	15 3
34	16 22	22 4	16 4	4 16	16 10	22 16

Variants of AC circuits.

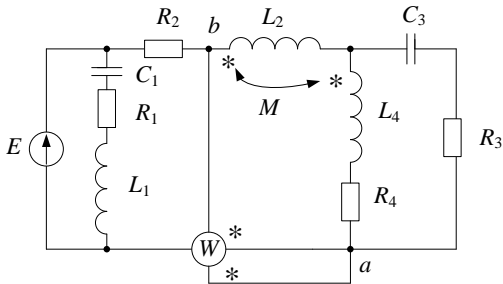
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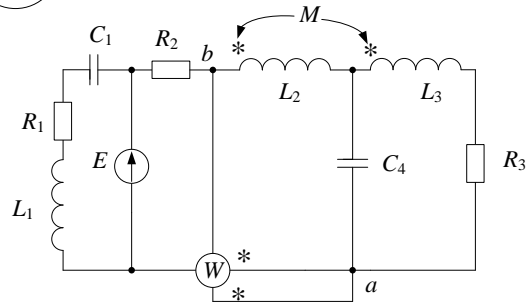
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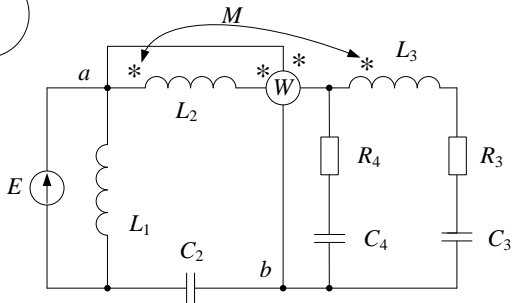
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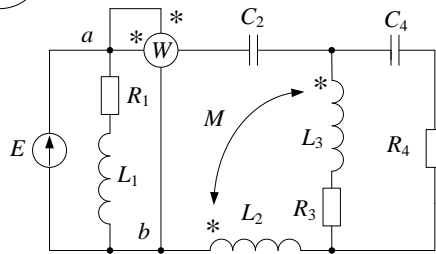
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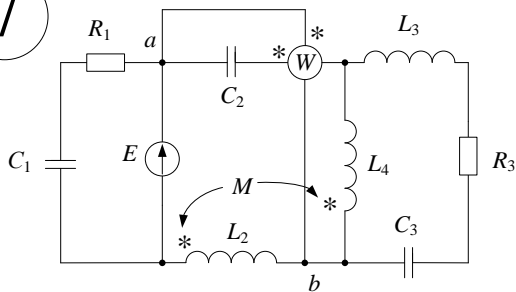
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6



7



8

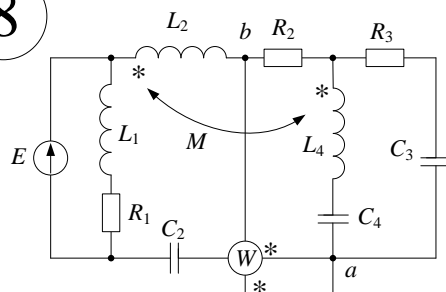


Figure A.1 (sheet 1)

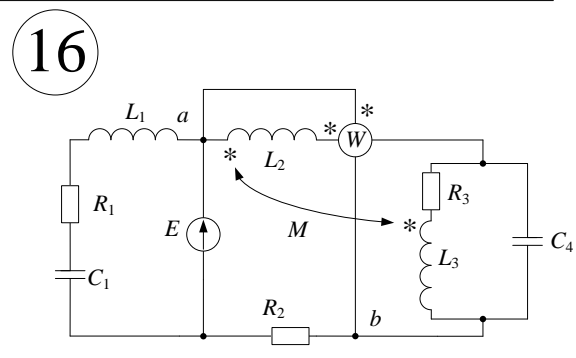
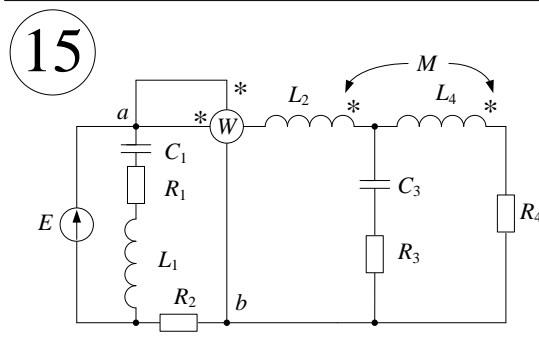
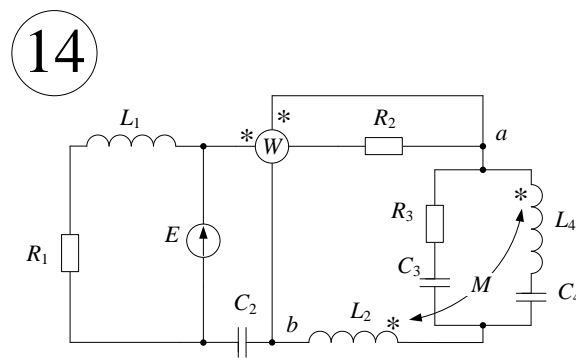
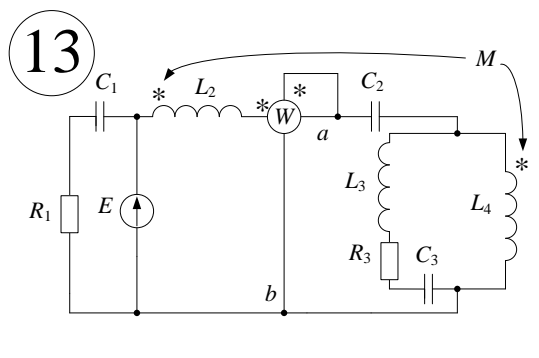
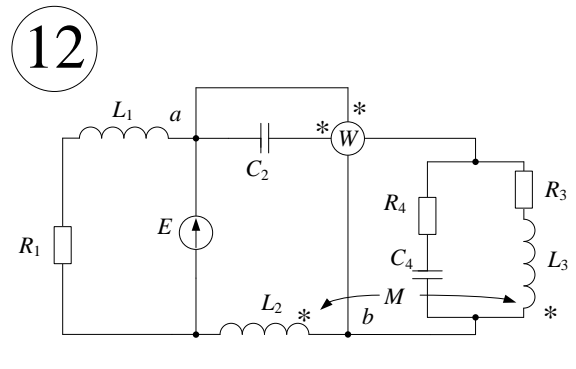
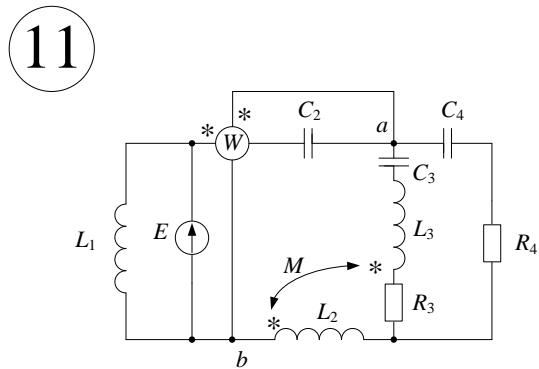
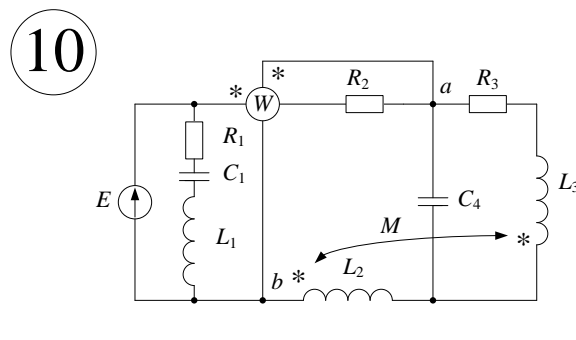
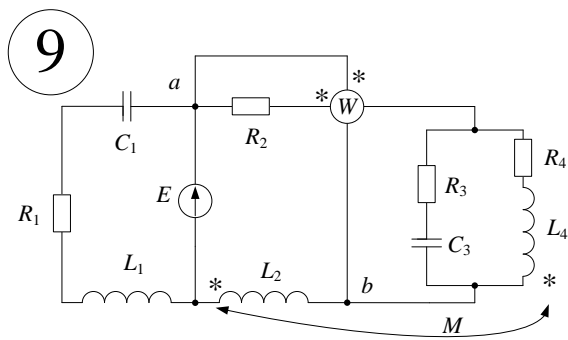
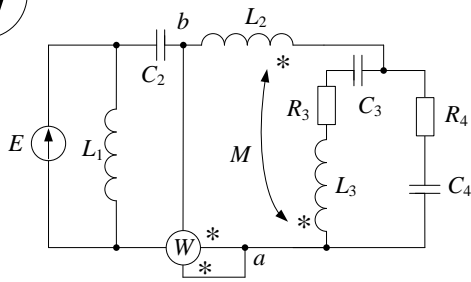
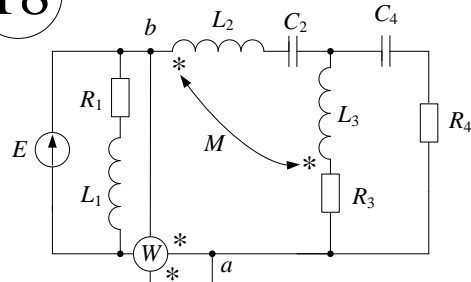


Figure A.1 (sheet 2)

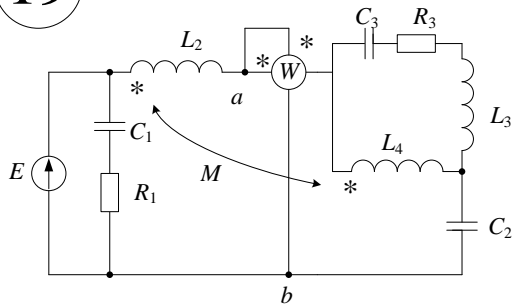
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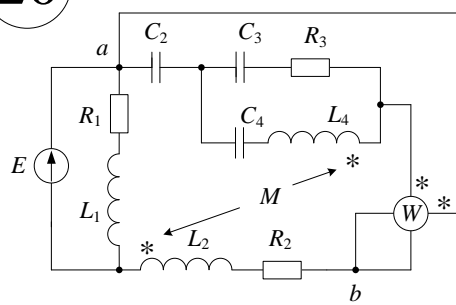
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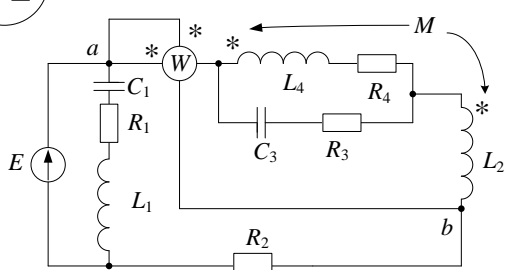
19



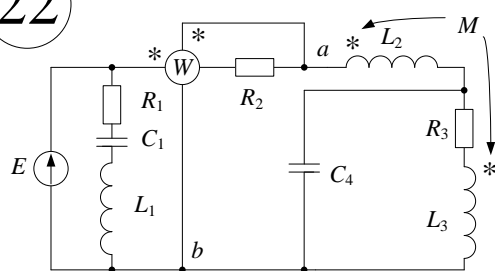
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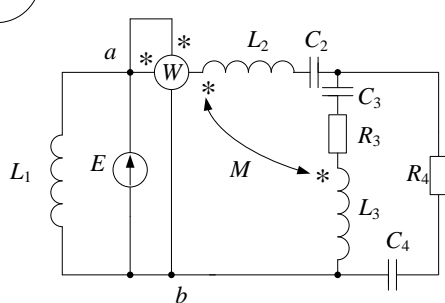
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22



23



24

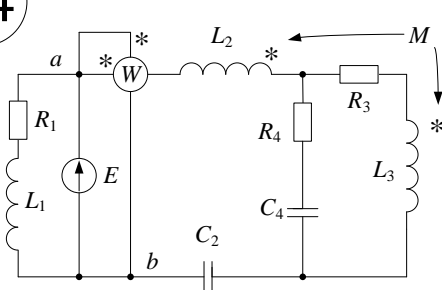
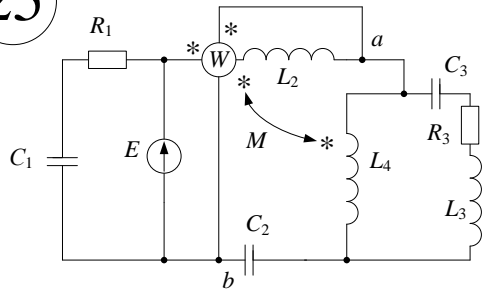
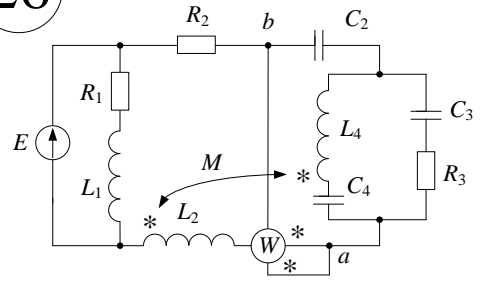


Figure A.1 (sheet 3)

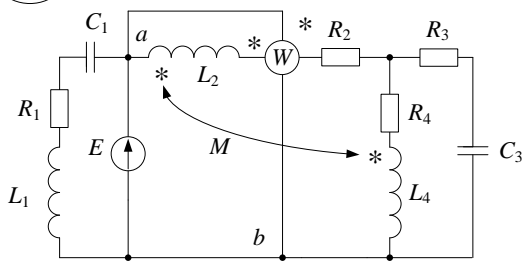
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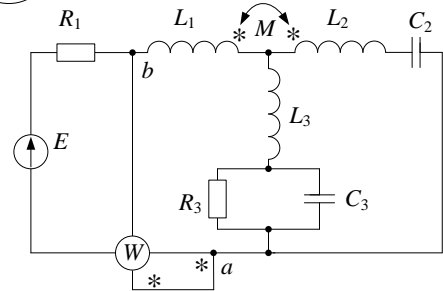
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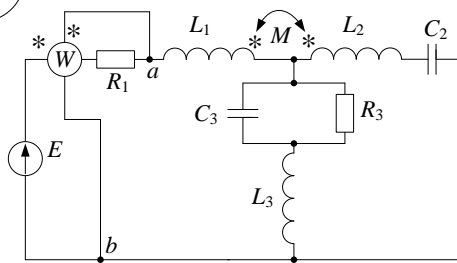
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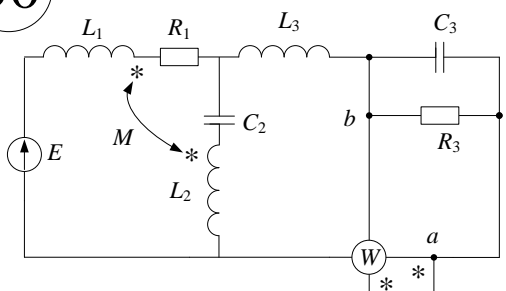
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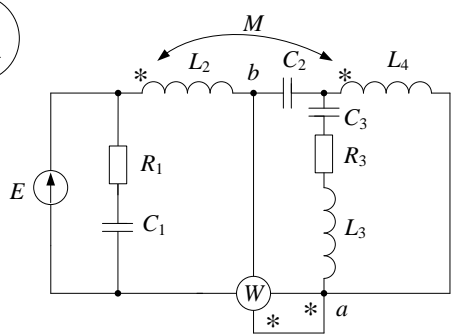
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30



31



32

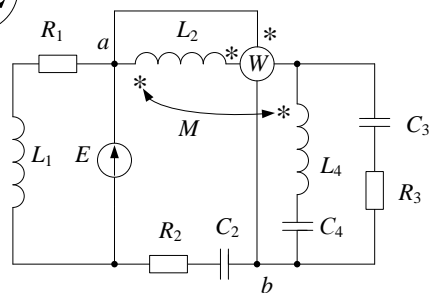


Figure A.1 (sheet 4)

Variants of the numerical data of the AC circuits parameters.

Table A.2 – Circuit elements parameters

Data number	E V	ψ_e rad	f Hz	R ₁ Ohm	L ₁ mH	C ₁ μ F	R ₂ Ohm	L ₂ mH	C ₂ μ F	R ₃ Ohm	L ₃ mH	C ₃ μ F	R ₄ Ohm	L ₄ mH	C ₄ μ F	M mH
1	120	$\pi/2$	500	4	–	20	–	1	30	5	10,13	10	–	2	–	0,5
2	60	$\pi/6$	100	4	40	–	2	25,5	50	10	–	125	–	50	102	10
3	140	$\pi/3$	200	15	25,32	25	5	10	–	6	–	100	3	5	–	2
4	220	$\pi/4$	250	35	32	40	20	4	–	40	40	–	–	–	20	10
5	400	$\pi/6$	100	–	50	–	–	50	200	30	63,7	39,8	10	–	75	25
6	50	$-\pi/4$	400	6	10	–	–	12,75	6,25	40	12,5	–	60	–	5	5
7	240	$\pi/2$	50	4	–	200	–	10	300	5	101,3	100	–	20	–	5
8	300	$\pi/6$	500	4	8	–	2	5,1	10	10	–	25	–	10	20,4	2
9	280	$\pi/3$	250	15	20,26	20	5	8	–	6	–	80	3	4	–	1,6
10	110	$\pi/4$	200	35	40	50	20	5	–	40	50	–	–	–	25	12,5
11	40	$\pi/6$	200	–	25	–	–	25	100	30	31,85	19,9	10	–	37,5	12,5
12	100	$-\pi/4$	250	6	16	–	–	20,4	10	40	20	–	60	–	8	8
13	360	$\pi/2$	250	4	–	40	–	2	60	5	20,26	20	–	4	–	1
14	360	$\pi/6$	50	4	80	–	2	51	100	10	–	250	–	100	204	20
15	70	$\pi/3$	500	15	10,13	10	5	4	–	6	–	40	3	2	–	0,8
16	220	$\pi/4$	50	35	160	200	20	20	–	40	200	–	–	–	100	50
17	200	$\pi/6$	250	–	20	–	–	20	80	30	25,48	15,92	10	–	30	10
18	500	$-\pi/4$	100	6	40	–	–	51	25	40	50	–	60	–	20	20
19	240	$\pi/2$	100	4	–	100	–	5	150	5	50,65	50	–	10	–	2,5
20	120	$\pi/6$	200	4	20	–	2	12,75	25	10	–	62,5	–	25	51	5
21	140	$\pi/3$	50	15	101,3	100	5	40	–	6	–	400	3	20	–	8
22	110	$\pi/4$	100	35	80	100	20	10	–	40	100	–	–	–	50	25
23	400	$\pi/6$	50	–	100	–	–	100	400	30	127,4	79,6	10	–	150	50

Continuation of Table A.2.

Data number	E V	ψ_e rad	f Hz	R_1 Ohm	L_1 mH	C_1 μ F	R_2 Ohm	L_2 mH	C_2 μ F	R_3 Ohm	L_3 mH	C_3 μ F	R_4 Ohm	L_4 mH	C_4 μ F	M mH
26	60	$\pi/6$	250	4	16	–	2	10,2	20	10	–	50	–	20	40,8	4
24	100	$-\pi/4$	200	6	20	–	–	25,5	12,5	40	25	–	60	–	10	10
25	120	$\pi/2$	200	4	–	50	–	2,5	75	5	25,32	25	–	5	–	1,25
27	70	$\pi/3$	100	15	50,65	50	5	20	–	6	–	200	3	10	–	4
28	1000	$\pi/2$	50	20	200	–	–	30	160	5	2	160	–	–	–	50
29	1200	$\pi/2$	400	20	25	–	–	3,75	20	5	2,5	20	–	–	–	6,25
30	600	$\pi/2$	200	20	50	–	–	7,5	40	5	5	40	–	–	–	12,5
31	240	$\pi/2$	50	4	–	200	–	10	300	5	101,3	100	–	20	–	5
32	300	$\pi/6$	500	4	8	–	2	5,1	10	10	–	25	–	10	20,4	2
33	140	$\pi/3$	200	15	25,32	25	5	10	–	6	–	100	3	5	–	2
34	110	$\pi/4$	200	35	40	50	20	5	–	40	50	–	–	–	25	12,5