

Розрахункове завдання за темою:

"Границі. Диференціальне числення функцій однієї змінної"

**Завдання 1.** Обчислити границю:

$$1. \lim_{x \rightarrow \infty} \frac{x(x+5)(x-9)}{(3x^2+2)^2 - x^4}$$

$$2. \lim_{x \rightarrow \infty} \frac{(x+3)^2 - (2x-3)^2}{(2x+4)(x-3)}$$

$$3. \lim_{x \rightarrow \infty} \frac{x^4 - 2(x^2+2)^2}{(x+3)^3 - 8x^3}$$

$$4. \lim_{x \rightarrow \infty} \frac{(3x+2)^2 - (x-1)^2}{(2x+3)(7x-1)}$$

$$5. \lim_{x \rightarrow \infty} \frac{(2x+3)^3(x+1) - 9x^4}{(3x+2)^2(x-4)}$$

$$6. \lim_{x \rightarrow \infty} \frac{x^2(x+1)^2 - 2x^4}{(2x+3)^3 + 3x^3}$$

$$7. \lim_{x \rightarrow \infty} \frac{(x+5)^3 - (2x+3)^3}{(2x-3)(4x+5) - 9x^2}$$

$$8. \lim_{x \rightarrow \infty} \frac{(2-x)^3 - x^3 - 6x^2}{(2x+5)(3x-1)}$$

$$9. \lim_{x \rightarrow \infty} \frac{(2x+5)^3 - x^3}{9x^2 + 5x - 4}$$

$$10. \lim_{x \rightarrow \infty} \frac{(3x+1)^3 - (2x+1)^3}{x + 100x^2}$$

$$11. \lim_{x \rightarrow \infty} \frac{(x-2)^2 - (x+3)^2}{x^2 + 7x - 4}$$

$$12. \lim_{x \rightarrow \infty} \frac{(2x+1)^3 - (x+1)^3}{5 - x^3}$$

$$13. \lim_{x \rightarrow \infty} \frac{(2x+1)^2 - (x-1)^2}{(9x+1)(x+3)}$$

$$14. \lim_{x \rightarrow \infty} \frac{x^2 + 3x - 4}{(2x+1)^2 - 4(x+2)^2}$$

$$15. \lim_{x \rightarrow \infty} \frac{(x+2)^2 - (3x+4)^2}{(4x+5)(x-2)}$$

$$16. \lim_{x \rightarrow \infty} \frac{(5x-4)(2x+3)}{5x^2 - x + 7}$$

$$17. \lim_{x \rightarrow \infty} \frac{x^2 + 3x + 4}{(2x+1)^2 - 4(x+3)^2}$$

$$18. \lim_{x \rightarrow \infty} \frac{(x+1)^2 + (2x-3)^2}{9x^2 + 4x + 1}$$

$$19. \lim_{x \rightarrow \infty} \frac{(x+2)^3 - (2x-1)^3}{3 + 2x^3}$$

$$20. \lim_{x \rightarrow \infty} \frac{(x+2)^3 - x^3}{7x^2 - 5x + 4}$$

$$21. \lim_{x \rightarrow \infty} \frac{(x+5)^3 - (2x+3)^3}{x(x+5)^2}$$

$$22. \lim_{x \rightarrow \infty} \frac{(2+x)^2 + (x+3)^2}{(x^2+1)^2 - x^4}$$

$$23. \lim_{x \rightarrow \infty} \frac{(x^2 + 3)^2 - x^4}{(2x + 3)(x + 7)}$$

$$24. \lim_{x \rightarrow \infty} \frac{(4x + 5)(x^3 - 4)}{(x^2 + 1)^2 + 7}$$

$$25. \lim_{x \rightarrow \infty} \frac{(3x + 1)^2 + (x + 1)^2}{x^5 + 3}$$

$$26. \lim_{x \rightarrow \infty} \frac{(x + 1)^2 + (4x + 3)^2}{(2 - x)^3 + (x + 1)^3}$$

$$27. \lim_{x \rightarrow \infty} \frac{(3x + 1)^2 - (2x^2 + 1)^2}{x^4 + 5}$$

$$28. \lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 4}{(2x + 1)^2 - 4(x + 3)^2}$$

$$29. \lim_{x \rightarrow \infty} \frac{(2x + 3)^2 + (x - 2)^2}{(x + 3)(2x - 3)}$$

$$30. \lim_{x \rightarrow \infty} \frac{x^4 - (x^2 + 2)^2}{(2x + 3)^3 - 8x^3}$$

**Завдання 2.** Обчислити границю:

$$1. \lim_{x \rightarrow 2} \frac{2x^2 - 5x + 2}{x^3 - 8}$$

$$16. \lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2}$$

$$2. \lim_{x \rightarrow -1} \frac{x^3 + 1}{2x^2 + 3x + 1}$$

$$17. \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 12x + 20}$$

$$3. \lim_{x \rightarrow 3} \frac{x^4 - 81}{x^2 - 5x + 6}$$

$$18. \lim_{x \rightarrow 0} \frac{x^4 + x^3}{x^4 - 2x^3}$$

$$4. \lim_{x \rightarrow -2} \frac{x^3 + 8}{3x^2 - 2x - 16}$$

$$19. \lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^3 + 1}$$

$$5. \lim_{x \rightarrow -2} \frac{x^3 - 2x + 4}{x + 2}$$

$$20. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3}$$

$$6. \lim_{x \rightarrow \frac{1}{3}} \frac{6x^2 + x - 1}{3x^2 + 17x - 6}$$

$$21. \lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x^2 - 8x + 12}$$

$$7. \lim_{x \rightarrow 2} \frac{3x^2 - 5x - 2}{x^3 - x - 6}$$

$$22. \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - x}$$

$$8. \lim_{x \rightarrow -3} \frac{x^2 + 3x}{x^3 - 5x + 12}$$

$$23. \lim_{x \rightarrow -2} \frac{x^3 + 3x^2 + 2x}{x^2 - x - 6}$$

$$9. \lim_{x \rightarrow \frac{1}{2}} \frac{6x^2 - 5x + 1}{4x^2 - 1}$$

$$10. \lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^3 - 2x + 4}$$

$$11. \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{2x^2 - 3x - 5}$$

$$12. \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{3x^2 - 5x - 12}$$

$$13. \lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 4x - 12}$$

$$14. \lim_{x \rightarrow 2} \frac{3x^2 - 2x - 8}{x^2 + 5x - 14}$$

$$15. \lim_{x \rightarrow 1} \frac{3x^2 - x - 2}{x^3 - 1}$$

$$24. \lim_{x \rightarrow -1} \frac{x^4 + x}{2x^4 - x^2 - 1}$$

$$25. \lim_{x \rightarrow 1} \frac{x^3 - 1}{3x^2 - x - 2}$$

$$26. \lim_{x \rightarrow \frac{2}{3}} \frac{9x^2 - 3x - 2}{3x^2 - 2x}$$

$$27. \lim_{x \rightarrow 2} \frac{3x^2 - 2x - 8}{x^2 + 5x - 14}$$

$$28. \lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 4x - 12}$$

$$29. \lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 8x + 3}{2x - 1}$$

$$30. \lim_{x \rightarrow -1} \frac{x^3 + 3x^2 + 3x + 1}{3x^2 + x - 2}$$

**Завдання 3.** Обчислити границю:

$$1. \lim_{x \rightarrow 10} \frac{\sqrt{x-1} - 3}{x - 10}$$

$$2. \lim_{x \rightarrow -7} \frac{\sqrt{2-x} - 3}{x^2 - 49}$$

$$3. \lim_{x \rightarrow 0} \frac{\sqrt{9+x^2} - 3}{5x^2}$$

$$4. \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4} - 2}$$

$$5. \lim_{x \rightarrow 5} \frac{\sqrt{3x+1} - 4}{x^2 - 25}$$

$$6. \lim_{x \rightarrow 1} \frac{1 - x^2}{1 - \sqrt{x}}$$

$$7. \lim_{x \rightarrow 4} \frac{\sqrt{x-3} - 1}{16 - x^2}$$

$$8. \lim_{x \rightarrow 2} \frac{x^2 - 4}{2 - \sqrt{x+2}}$$

$$9. \lim_{x \rightarrow -1} \frac{x+1}{\sqrt{6x^2+3}+3x}$$

$$10. \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{7x+2} - 2x}$$

$$11. \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - \sqrt{9-x}}{x}$$

$$12. \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$$

$$13. \lim_{x \rightarrow 2} \frac{\sqrt{4x+1} - 3}{x^2 - 3x + 2}$$

$$14. \lim_{x \rightarrow -3} \frac{x^2 + 3x}{\sqrt{12+x} - 3}$$

$$15. \lim_{x \rightarrow 1} \frac{\sqrt{5x+11} - 4}{x^2 - 1}$$

$$16. \lim_{x \rightarrow 10} \frac{\sqrt{x-1} - 3}{x - 10}$$

$$17. \lim_{x \rightarrow -7} \frac{\sqrt{2-x} - 3}{x^2 - 49}$$

$$18. \lim_{x \rightarrow 0} \frac{\sqrt{9+x^2} - 3}{5x^2}$$

$$19. \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4} - 2}$$

$$20. \lim_{x \rightarrow 5} \frac{\sqrt{3x+1} - 4}{x^2 - 25}$$

$$21. \lim_{x \rightarrow 1} \frac{1-x^2}{1-\sqrt{x}}$$

$$22. \lim_{x \rightarrow 4} \frac{\sqrt{x-3} - 1}{16-x^2}$$

$$23. \lim_{x \rightarrow 2} \frac{x^2 - 4}{2 - \sqrt{x+2}}$$

$$24. \lim_{x \rightarrow -1} \frac{x+1}{\sqrt{6x^2+3} + 3x}$$

$$25. \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{7x+2} - 2x}$$

$$26. \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - \sqrt{9-x}}{x}$$

$$27. \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$$

$$28. \lim_{x \rightarrow 2} \frac{\sqrt{4x+1} - 3}{x^2 - 3x + 2}$$

$$29. \lim_{x \rightarrow -3} \frac{x^2 + 3x}{\sqrt{12+x} - 3}$$

$$30. \lim_{x \rightarrow 1} \frac{\sqrt{5x+11} - 4}{x^2 - 1}$$

**Завдання 4.** Обчислити границю:

$$1. \lim_{x \rightarrow 0} \frac{x \cdot \operatorname{tg} 3x}{\cos x - \cos 3x}$$

$$16. \lim_{x \rightarrow 0} \frac{x \cdot (\operatorname{tg} \sqrt{7x})^2}{1 - \cos 2x}$$

$$2. \lim_{x \rightarrow -1} \frac{\arcsin(x^3 + 1)}{x^2 + 3x + 2}$$

$$17. \lim_{x \rightarrow 2} \frac{x \cdot (\operatorname{tg} \sqrt{x-2})^2}{1 - \cos(x-2)}$$

$$3. \lim_{x \rightarrow 5} \frac{\sin(x-5)}{\operatorname{tg}(x^2-25)}$$

$$4. \lim_{x \rightarrow 1} \frac{\sin(x^2-1)}{\operatorname{tg}(x^2-3x+2)}$$

$$5. \lim_{x \rightarrow 0} \frac{\sin 2x + \sin 3x}{x \cdot \operatorname{tg} 2x}$$

$$6. \lim_{x \rightarrow 0} (x^2 + 7x) \cdot \frac{1}{\operatorname{tg} 4x}$$

$$7. \lim_{x \rightarrow 1} \frac{\sin(x^2-2x-3)}{1-\cos(x-1)}$$

$$8. \lim_{x \rightarrow 3} \frac{\sin(x^2-3x)}{\operatorname{tg}(x-3)}$$

$$9. \lim_{x \rightarrow 0} \frac{\sin 2x - \sin 4x}{\operatorname{arctg}(x^2+x)}$$

$$10. \lim_{x \rightarrow 0} \frac{\cos 5x - \cos 3x}{1-\cos 4x}$$

$$11. \lim_{x \rightarrow 0} \frac{1-\cos 4x}{1-\cos 7x}$$

$$12. \lim_{x \rightarrow 0} \frac{\cos 2x - \cos x}{1-\cos\left(\frac{x}{3}\right)}$$

$$13. \lim_{x \rightarrow 3} \frac{1-\cos(x-3)}{\operatorname{tg}(x^2-3x)}$$

$$14. \lim_{x \rightarrow -3} \frac{1-\cos(x+3)}{x^2+4x+3}$$

$$15. \lim_{x \rightarrow 0} \frac{\operatorname{arctg}(3x^2-6x)}{\sin(x^2-4x)}$$

$$18. \lim_{x \rightarrow 0} \frac{x \cdot \sin 3x}{1-(\cos 4x)^2}$$

$$19. \lim_{x \rightarrow 0} \frac{x \cdot (\operatorname{tg} \sqrt{5x})^2}{1-(\cos 3x)^2}$$

$$20. \lim_{x \rightarrow 0} \frac{x^2+x}{\operatorname{tg} 3x}$$

$$21. \lim_{x \rightarrow 0} \frac{1-\cos 2x}{\operatorname{tg} 4x}$$

$$22. \lim_{x \rightarrow 0} \frac{1-\cos\left(\frac{x}{2}\right)}{x^2}$$

$$23. \lim_{x \rightarrow 0} \frac{x \cdot \operatorname{tg} x}{x^2-x}$$

$$24. \lim_{x \rightarrow 2} \frac{4 \operatorname{arctg}(x-2)}{\sin(2x-4)}$$

$$25. \lim_{x \rightarrow 0} \frac{\sin 3x - \sin 8x}{\operatorname{arctg}(4x^2+x)}$$

$$26. \lim_{x \rightarrow 0} \frac{x^2+9x}{\operatorname{tg}(5x)}$$

$$27. \lim_{x \rightarrow -1} \frac{\operatorname{arc} \sin(2x^2-x-3)}{\operatorname{tg}(x^2+x)}$$

$$28. \lim_{x \rightarrow 0} \frac{1-\cos(4x)}{x \cdot \sin 5x}$$

$$29. \lim_{x \rightarrow 0} \frac{1-\cos(9x)}{(\operatorname{tg} \sqrt{7x})^2}$$

$$30. \lim_{x \rightarrow 3} \frac{5 \operatorname{arcsin}(x-3)}{\sin(3x-9)}$$

**Завдання 5.** Обчислити границю:

$$1. \lim_{x \rightarrow \infty} \left( \frac{x^2 + 3}{x^2 - 4} \right)^{x^2 + 1}$$

$$2. \lim_{x \rightarrow \infty} \left( \frac{3x - 2}{3x + 4} \right)^{5 + x}$$

$$3. \lim_{x \rightarrow \infty} \left( \frac{5x - 1}{5x + 3} \right)^{2x - 1}$$

$$4. \lim_{x \rightarrow \infty} \left( \frac{2 - 3x}{1 - 3x} \right)^{x^2 + 1}$$

$$5. \lim_{x \rightarrow \infty} \left( \frac{2x - 7}{2x - 2} \right)^{x - 5}$$

$$6. \lim_{x \rightarrow \infty} \left( \frac{x^2 + 1}{x^2 - 1} \right)^{x^2 + x}$$

$$7. \lim_{x \rightarrow \infty} \left( \frac{x - 2}{x + 5} \right)^{\frac{x}{3} + 1}$$

$$8. \lim_{x \rightarrow \infty} \left( \frac{3x - 1}{3x - 2} \right)^{x^2 - 1}$$

$$9. \lim_{x \rightarrow \infty} \left( \frac{x + 1}{x - 1} \right)^{x + 1}$$

$$10. \lim_{x \rightarrow \infty} \left( \frac{3x + 6}{1 + 3x} \right)^x$$

$$11. \lim_{x \rightarrow \infty} \left( \frac{2x - 5}{2x - 2} \right)^{x - 3}$$

$$12. \lim_{x \rightarrow \infty} \left( \frac{2x + 1}{2x + 2} \right)^{2x - 1}$$

$$16. \lim_{x \rightarrow \infty} \left( \frac{5x - 4}{5x} \right)^{x - 1}$$

$$17. \lim_{x \rightarrow \infty} \left( \frac{2x + 3}{2x + 2} \right)^{3 - x}$$

$$18. \lim_{x \rightarrow \infty} \left( \frac{2x}{2x - 3} \right)^{x - 3}$$

$$19. \lim_{x \rightarrow \infty} \left( \frac{x^2 + 4x - 3}{x^2 - 5x + 9} \right)^{2x}$$

$$20. \lim_{x \rightarrow 1} \left( \frac{3x + 1}{3x - 3} \right)^{5x}$$

$$21. \lim_{x \rightarrow \infty} \left( \frac{x^2 + 2}{x^2 - 6} \right)^{x^2}$$

$$22. \lim_{x \rightarrow \infty} \left( \frac{2x - 5}{2x + 3} \right)^{x - 9}$$

$$23. \lim_{x \rightarrow \infty} \left( \frac{5x - 2}{5x + 3} \right)^{x^2 - 5x + 6}$$

$$24. \lim_{x \rightarrow \infty} \left( \frac{3x - 1}{3x - 2} \right)^{x - 1}$$

$$25. \lim_{x \rightarrow \infty} \left( \frac{5x - 1}{5x + 9} \right)^{3x}$$

$$26. \lim_{x \rightarrow \infty} \left( \frac{x^2 + 3}{x^2 - 1} \right)^{7x - 4}$$

$$27. \lim_{x \rightarrow \infty} \left( \frac{2x + 3}{2x + 2} \right)^{3x - 2}$$

$$13. \lim_{x \rightarrow \infty} \left( \frac{x-5}{x+3} \right)^{5x-1}$$

$$28. \lim_{x \rightarrow \infty} \left( \frac{3-x}{4-x} \right)^{x+4}$$

$$14. \lim_{x \rightarrow \infty} \left( \frac{2x+1}{2x-2} \right)^{3x}$$

$$29. \lim_{x \rightarrow \infty} \left( \frac{3x-1}{3x+1} \right)^{3x+4}$$

$$15. \lim_{x \rightarrow \infty} \left( \frac{2x-1}{2x+11} \right)^{3x+2}$$

$$30. \lim_{x \rightarrow \infty} \left( \frac{3x^2+5}{3x^2-7} \right)^{x^2+x-1}$$

**Завдання 6.** Дослідити на неперервність функції та класифікувати точки розриву.  
Зобразити поведінку поблизу точок розриву

<b>1.</b>	a) $f(x) = \frac{x+3}{x+5}$	b) $f(x) = 2^{\frac{x}{1-x}}$	<b>16.</b>	a) $f(x) = \frac{3x}{x+1}$	b) $f(x) = 5^{\frac{x}{x+2}}$
<b>2.</b>	a) $f(x) = \frac{x}{x+2}$	b) $f(x) = 3^{\frac{1-x}{x}}$	<b>17.</b>	a) $f(x) = \frac{x-4}{x+1}$	b) $f(x) = e^{\frac{1}{x+2}}$
<b>3.</b>	a) $f(x) = \frac{2x-1}{x+1}$	b) $f(x) = 4^{\frac{x}{2+x}}$	<b>18.</b>	a) $f(x) = \frac{2}{x-3}$	b) $f(x) = 8^{\frac{x}{3x-6}}$
<b>4.</b>	a) $f(x) = \frac{x+3}{3-x}$	b) $f(x) = 5^{\frac{2+x}{2-x}}$	<b>19.</b>	a) $f(x) = \frac{x}{x-3}$	b) $f(x) = 7^{\frac{4}{2x-1}}$
<b>5.</b>	a) $f(x) = \frac{x-1}{x+2}$	b) $f(x) = e^{\frac{1}{2x-1}}$	<b>20.</b>	a) $f(x) = \frac{x-5}{x-2}$	b) $f(x) = e^{\frac{3x}{2-x}}$
<b>6.</b>	a) $f(x) = \frac{2}{3-x}$	b) $f(x) = 6^{\frac{x}{x+1}}$	<b>21.</b>	a) $f(x) = \frac{x-5}{x+1}$	b) $f(x) = 9^{\frac{1}{2-x}}$
<b>7.</b>	a) $f(x) = \frac{3}{x+2}$	b) $f(x) = 7^{\frac{x}{2x+4}}$	<b>22.</b>	a) $f(x) = \frac{6}{x+3}$	b) $f(x) = 12^{\frac{3}{x-1}}$
<b>8.</b>	a) $f(x) = \frac{2x}{x+4}$	b) $f(x) = 8^{\frac{x}{3x-3}}$	<b>23.</b>	a) $f(x) = \frac{3x}{x+1}$	b) $f(x) = 8^{\frac{x}{5-x}}$
<b>9.</b>	a) $f(x) = \frac{x-1}{x}$	b) $f(x) = 9^{\frac{x}{2x-2}}$	<b>24.</b>	a) $f(x) = \frac{2x}{x+1}$	b) $f(x) = 14^{\frac{1}{16-x}}$
<b>10.</b>	a) $f(x) = \frac{2x-1}{x^2}$	b) $f(x) = 10^{\frac{1}{2+x}}$	<b>25.</b>	a) $f(x) = \frac{12}{x+5}$	b) $f(x) = 4^{\frac{x}{3-x}}$
<b>11.</b>	a) $f(x) = \frac{3x}{x+2}$	b) $f(x) = 11^{\frac{3}{x-3}}$	<b>26.</b>	a) $f(x) = \frac{4}{2-x}$	b) $f(x) = 3^{\frac{2x}{4-x}}$
<b>12.</b>	a) $f(x) = \frac{2x}{x+3}$	b) $f(x) = 12^{\frac{2}{1+x}}$	<b>27.</b>	a) $f(x) = \frac{x+3}{x-4}$	b) $f(x) = 10^{\frac{2}{7-x}}$
<b>13.</b>	a) $f(x) = \frac{x+1}{x}$	b) $f(x) = 2^{\frac{2x}{1-x}}$	<b>28.</b>	a) $f(x) = \frac{3x}{x+4}$	b) $f(x) = 11^{\frac{2x}{1+x}}$
<b>14.</b>	a) $f(x) = \frac{x^2-1}{x+2}$	b) $f(x) = 3^{\frac{1-x}{2x}}$	<b>29.</b>	a) $f(x) = \frac{2x}{3-x}$	b) $f(x) = e^{\frac{x}{2x+2}}$
<b>15.</b>	a) $f(x) = \frac{x^2}{x-3}$	b) $f(x) = 4^{\frac{x}{x-2}}$	<b>30.</b>	a) $f(x) = \frac{2x}{x+1}$	b) $f(x) = 3^{\frac{2x}{4-x}}$



**Завдання 8.** Знайти похідну функції.

- |   |  |  |
|---|--|--|
| 1. $\begin{cases} a) y = \frac{\arcsin 2x}{\ln(4-3x)}; \\ б) y = (3\sin x + 5^{\cos x})^3; \\ в) y = \ln \sin^3(e^{3x+1}). \end{cases}$   | 2. $\begin{cases} a) y = e^{2x} \cdot \operatorname{arctg} x; \\ б) y = (2^{x^2} - \sin 4x)^2; \\ в) y = \arcsin^2(\ln(x^3 + 4)). \end{cases}$                   | 3. $\begin{cases} a) y = \frac{\ln(5x-1)}{\cos(2x+3)}; \\ б) y = (e^{\sin 3x} + 3x)^7; \\ в) y = \sin^4(\operatorname{arctg}(3x^2)); \end{cases}$  |
| 4. $\begin{cases} a) y = \sqrt{x^2 + 3} \cdot \sin 3x; \\ б) y = (2^{\operatorname{tg} x} + \arcsin 3x)^2; \\ в) y = \ln^5(\operatorname{arctg} \sqrt{x^2 - 1}). \end{cases}$                     | 5. $\begin{cases} a) y = \frac{\arcsin 7x}{\sqrt{x^3 - 1}}; \\ б) y = (5^{\operatorname{ctg} x} - e^x)^3; \\ в) y = \sqrt{\sin(\arccos(x^2))}. \end{cases}$      | 6. $\begin{cases} a) y = \cos 3x \cdot \ln\left(\frac{x}{3} - 1\right); \\ б) y = \left(\arccos \frac{1}{x} + \sqrt[3]{x^2}\right)^4; \\ в) y = \operatorname{tg}^2(\operatorname{ctg} \sqrt{x}). \end{cases}$ |
| 7. $\begin{cases} a) y = \frac{\cos(1-5x)}{\sqrt{x-1}}; \\ б) y = (7^{\sin x} + \operatorname{tg} 7x)^3; \\ в) y = \ln^3(\ln \sin x). \end{cases}$  | 8. $\begin{cases} a) y = 2^x \cdot \sin 2x; \\ б) y = (\cos 3x + 9^{\cos x})^5; \\ в) y = \sqrt{\operatorname{tg}^2 x \left( e^{x+x^{21}} \right)}. \end{cases}$ | 9. $\begin{cases} a) y = \sqrt{x} \cdot \operatorname{arctg} 3x; \\ б) y = (2^{x^2} - \cos 6x)^4; \\ в) y = \frac{1}{\arccos^2(\ln x)}. \end{cases}$   |
| 10. $\begin{cases} a) y = \frac{\cos 5x}{\ln(2x-1)}; \\ б) y = (2^{\sin 2x} + \operatorname{arctg} x)^2; \\ в) y = \arcsin \sqrt{x^2 + 3x}. \end{cases}$  | 11. $\begin{cases} a) y = \sqrt{x^3} \cdot \ln(2x-1); \\ б) y = (2^{\sin x} + \cos 4x)^3; \\ в) y = \cos^2 \sqrt{x + \sqrt{x}}. \end{cases}$                     | 12. $\begin{cases} a) y = \frac{\sqrt{1+x^2}}{1-x}; \\ б) y = (3^{\cos x} + 2\sin^2 x)^5; \\ в) y = \ln^3(\operatorname{arctg}(x + \ln 3)). \end{cases}$   |
| 13. $\begin{cases} a) y = e^{-x} \cdot (x^4 + 2x^2 + 2); \\ б) y = (4^{\operatorname{tg} x} + 3\cos x)^3; \\ в) y = \ln \left( \cos \left( \arcsin \frac{e^{2x}}{3} \right) \right). \end{cases}$ | 14. $\begin{cases} a) y = \sqrt[4]{x} \cdot \cos 2x; \\ б) y = 5^{\sin \sqrt{3x}}; \\ в) y = \frac{1}{2} \operatorname{arctg}^3(e^{-2x}). \end{cases}$           | 15. $\begin{cases} a) y = 3x \cdot \sqrt{\cos x}; \\ б) y = (5^{\operatorname{ctg} x} + \cos^3 x)^5; \\ в) y = \ln \left( \sqrt{\arcsin^3(2x)} \right). \end{cases}$   |

$$16. \left\{ \begin{array}{l} a) y = 5x^2 + \sqrt[3]{\sin(5x+1)}; \\ \bar{b}) y = \left( 3^{\sin x} + \operatorname{arctg} 4x \right)^4; \\ \bar{e}) y = \cos^2(\ln \sin 3x). \end{array} \right. \quad \left\{ \begin{array}{l} a) y = x^2 \cdot \operatorname{arctg}(2x+1); \\ \bar{b}) y = \left( \arccos 2x - 2x^2 \right)^5; \\ \bar{e}) y = \ln \left( \sin \sqrt{1-3x^2} \right). \end{array} \right. \quad \left\{ \begin{array}{l} a) y = \frac{1 + \sin^2 x}{2 - \sqrt{x}}; \\ \bar{b}) y = \left( 5\sqrt{x} + \operatorname{tg}(2x+4) \right)^3; \\ \bar{e}) y = \operatorname{arctg}^5 \left( \sin \frac{\pi}{x} \right). \end{array} \right.$$

$$19. \left\{ \begin{array}{l} a) y = \sqrt{x} \cdot \arccos(x+2); \\ \bar{b}) y = \left( 2^{\cos x} - \sqrt[5]{x^2} \right)^4; \\ \bar{e}) y = \arcsin \sqrt{\sin^3 \left( \frac{x}{3} \right)}. \end{array} \right. \quad 20. \left\{ \begin{array}{l} a) y = \frac{1}{\sin 2} \left( \frac{3x}{x^2 + 4x + 4} \right); \\ \bar{b}) y = \left( \sqrt[7]{x^6} + e^{\sin 3x} \right)^3; \\ \bar{e}) y = \ln \sqrt{x + e^{\operatorname{tg}(\ln x)}} \end{array} \right. \quad 21. \left\{ \begin{array}{l} a) y = x \cdot \ln \left( 2x + \frac{1}{2} \right); \\ \bar{b}) y = \left( \arcsin \frac{1}{x} + \sqrt[4]{x^3} \right)^9; \\ \bar{e}) y = \cos^3 \sqrt{4 + 5^{2x}}. \end{array} \right.$$

$$22. \left\{ \begin{array}{l} a) y = \frac{\arccos x}{x^2 + 4}; \\ \bar{b}) y = \left( 9x^2 - \operatorname{tg}(e^{3x}) \right)^2; \\ \bar{e}) y = \ln^4 \left( \arcsin \left( \ln \frac{7}{x} \right) \right). \end{array} \right. \quad 23. \left\{ \begin{array}{l} a) y = e^{2x+1} \cdot \arccos 3x; \\ \bar{b}) y = \left( 7^{\cos x} + \operatorname{tg} 4x \right)^3; \\ \bar{e}) y = \operatorname{arctg} \left( \cos^4 \sqrt{x} \right). \end{array} \right. \quad 24. \left\{ \begin{array}{l} a) y = x^2 \cdot e^{\frac{1}{x}}; \\ \bar{b}) y = \left( \arcsin \sqrt{x} + 2x^2 \right)^7; \\ \bar{e}) y = \frac{7 \operatorname{arctg} \sqrt{x^2-1}}{9 + \ln^2 3}. \end{array} \right.$$

$$25. \left\{ \begin{array}{l} a) y = \frac{\operatorname{arctg} x}{2x + x^2}; \\ \bar{b}) y = \left( 2\sqrt{x} + \sin 4x \right)^5; \\ \bar{e}) y = \ln \sqrt{\operatorname{arctg} \sqrt{x}}. \end{array} \right. \quad 26. \left\{ \begin{array}{l} a) y = \cos^2 \frac{x}{2} + e^{2x-3}; \\ \bar{b}) y = \left( 5^{\operatorname{ctg} x} + \sqrt[7]{x^3} \right)^4; \\ \bar{e}) y = \arcsin \sqrt{e^x + 5^x}. \end{array} \right. \quad 27. \left\{ \begin{array}{l} a) y = \frac{\sqrt{1-x^2}}{e^{3x}}; \\ \bar{b}) y = \left( 5^{\operatorname{tg} x} + \operatorname{arctg} 4x \right)^3; \\ \bar{e}) y = \sqrt{\ln(\arccos \sqrt{2x})}. \end{array} \right.$$

$$28. \left\{ \begin{array}{l} a) y = x^3 \cdot \arcsin \sqrt{x}; \\ \bar{b}) y = \left( \sqrt[3]{x^2} + \sin^3 2x \right)^6; \\ \bar{e}) y = \arccos^3(5^{2x-1}). \end{array} \right. \quad 29. \left\{ \begin{array}{l} a) y = \sqrt{x} \cdot \operatorname{arctg} 2x; \\ \bar{b}) y = \left( 7^{\sin x} + \operatorname{tg} 3x \right)^4; \\ \bar{e}) y = \ln^2(\sin x) + \operatorname{tg}^2(\operatorname{ctg} \sqrt{x}). \end{array} \right. \quad 30. \left\{ \begin{array}{l} a) y = \frac{2}{\arccos 3x}; \\ \bar{b}) y = \left( 2^{3x} + \cos \frac{1}{x} \right)^3; \\ \bar{e}) y = \sqrt[3]{\sin(\cos x^2)}. \end{array} \right.$$

**Завдання 9.** Дослідити функцію та побудувати її графік.

$$1. y = x^2 + \frac{2}{x}$$

$$2. y = \frac{x}{x-1}$$

$$3. y = \frac{x^2 - 2x + 2}{x-1}$$

$$4. y = \frac{3x^4 - 1}{x^3}$$

$$5. y = \frac{x^4 - 3}{x}$$

$$6. y = \frac{8}{x^2 - 4}$$

$$7. y = \frac{x}{5 + x^2}$$

$$8. y = \frac{x^3}{9 - x^2}$$

$$9. y = \frac{1}{1 - x^2}$$

$$10. y = \frac{x+2}{x^3}$$

$$11. y = \frac{2+x}{(x+1)^2}$$

$$12. y = \frac{(x-1)^2}{x};$$

$$13. y = \frac{4x^3 + 5}{2x};$$

$$14. y = \frac{(x-1)^2}{x^2};$$

$$15. y = \frac{x}{2 - x^2};$$

$$16. y = \frac{2x^3 + 1}{x^2};$$

$$17. y = \frac{x^4 - 81}{3x^2};$$

$$18. y = \frac{16}{x^2(x-4)};$$

$$19. y = \frac{x^2 - 4}{x^2 - 9};$$

$$20. y = \frac{x}{\sqrt[3]{x^2 - 1}};$$

$$21. y = \frac{x^2 - 1}{x^2 + 1};$$

$$22. y = \frac{2}{x^2 + 2x + 3};$$

$$23. y = \frac{x^2 + 1}{x-1};$$

$$24. y = \left( \frac{x+1}{x-1} \right)^2;$$

$$25. y = \frac{3}{x^2 + 4x + 4};$$

$$26. y = \frac{27 + 8x^3}{8x};$$

$$27. y = \frac{(x-1)^2}{x^2 + 1};$$

$$28. y = \frac{x^2 - 1}{x^2 + 2};$$

$$29. y = \sqrt[3]{1 - x^3};$$

$$30. y = \frac{x^3}{1 - x^2};$$