1. CALCULATION OF DIRECT CURRENT ELECTRIC CIRCUITS PRACTICE TOPIC № 1

1.1. Formulation of tasks

To calculate the practice topic № 1, it is given a variant number for all tasks.

Task 1. Calculation of an electric circuit with one source of electromotive force (EMF) with a series-parallel connected resistors.

For a given variant from in table 1.1:

a) solve a direct task: draw a given electrical circuit scheme and evaluate the currents in all branches of this circuit. Use the reducing and transformations and verify the calculation by checking power balance of the sources and loads;

b) solve an inverse task: evaluate EMF of the source in the circuit if you know the current *I*, and all of the resistances.

Task 2. Calculation of an electric circuit with one source of EMF by the method of "Wye" – "Delta" or ("star-delta" or "Y-Δ") transformation.

In the given variant in table 1.2, draw a given electrical circuit with the EMF in the branch, which is specified in the table (last column). The polarity of the EMF can be random selected. Then you need to evaluate the currents in all the branches by the method of equivalent transformations ("Y-Δ" transformation).

Task 3. Calculation of an electric circuit by the mesh-current method.

In the given variant in table 1.3, draw a given multi-loop circuit and evaluate the currents in all branches by the mesh-current method. Then you need to verify the calculation by checking power balance of the sources and load.

Task 4. Calculation of an electric circuit by the method of Thevenin's theorem.

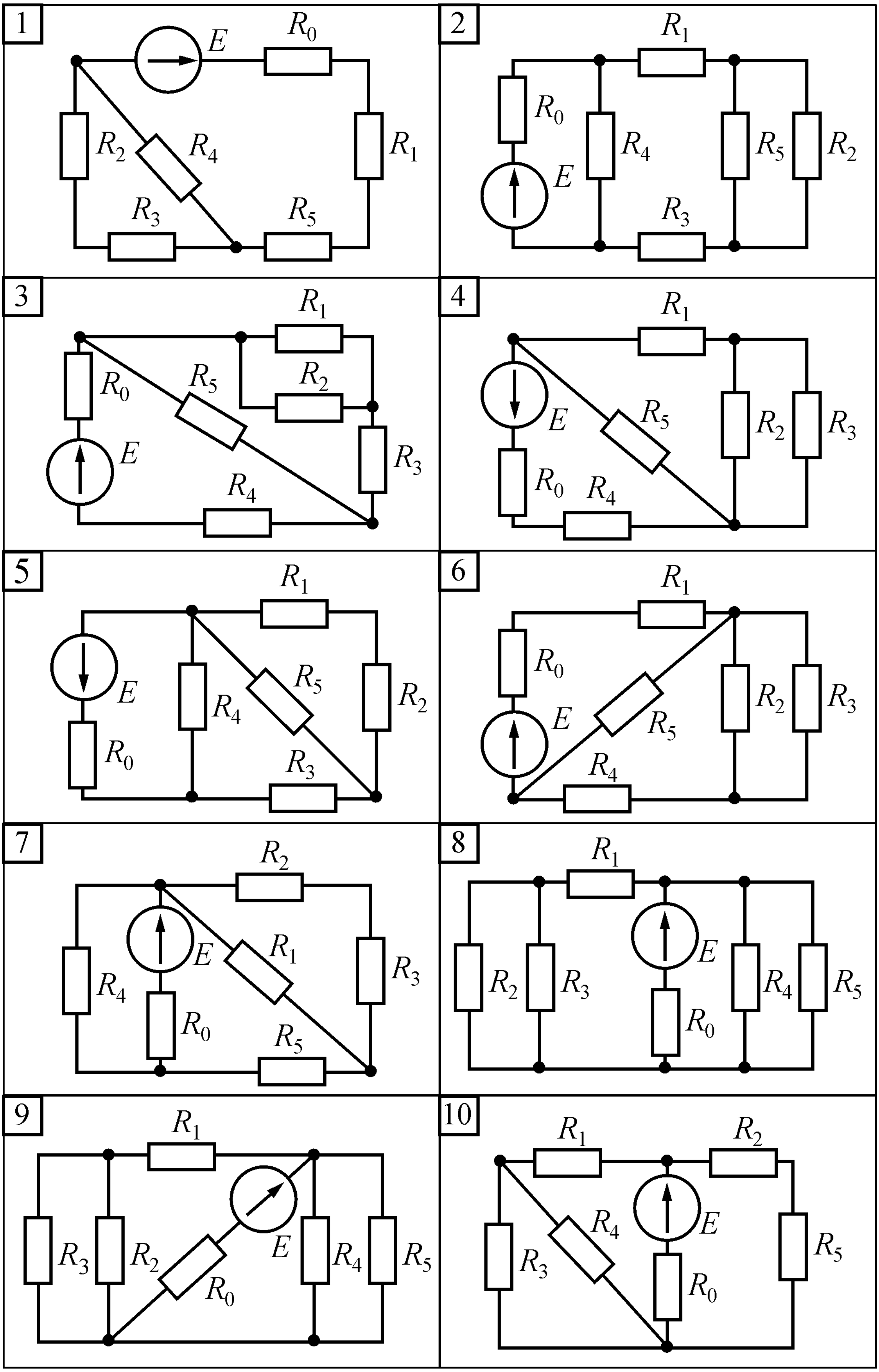
According to the given variant, which is given in table 1.3, draw a electrical circuit scheme and determine the current in the branch with resistance by the Thevenin's theorem method.

Table 1.1 – Input data to Task 1

Variant	Scheme number from Fig.1.1	Resistance, Ohm						Direct task	The reverse task
		R_0	R_1	R_2	R_3	R_4	R_5	E, V	I_1, A
1	1	1	5	10	20	15	4	120	12
2	2	2	40	20	10	40	20	130	10
3	3	1	10	15	14	7	30	200	1,8
4	4	0,5	15	30	15	17	25	180	1,5
5	5	0,5	20	20	10	36	10	90	1,6
6	6	1	4	30	70	9	30	100	10
7	7	2	20	10	10	45	20	240	1,8
8	8	1	6	45	30	24	12	140	2,5
9	9	1	4	60	15	60	40	130	10
10	10	1	6	20	10	15	16	120	6
11	11	2,5	10	30	12	60	5	270	4
12	12	2,5	4	50	30	20	15	150	3
13	13	2,5	15	8	13	10	9	90	2
14	14	1	25	15	8	5	8	135	6
15	15	2	7	18	5	10	15	90	12
16	16	1,5	7	4	3	6	5	220	3
17	17	2,5	20	35	25	5	60	140	9
18	18	1	10	6	4	12	8	100	6
19	19	2,5	24	15	10	30	6	210	3
20	20	1,2	15	9	4	21	6	135	8
21	21	1	20	10	20	8	12	110	10
22	22	2,5	20	20	24	10	25	240	1,5
23	23	1	20	15	60	20	7	100	6
24	24	2	12	30	20	30	120	200	2,5
25	25	2	15	10	17	15	7	180	5
26	26	1	15	10	40	10	50	80	9
27	27	2,5	20	20	20	5	5	150	4
28	28	2	11	5	9	8	24	150	2
29	29	1	40	15	20	10	60	220	1
30	30	1	22	10	45	40	18	200	3
31	1	0,9	6	3	4	3	7	160	5
32	2	2	6	10	5	20	90	240	2,5
33	3	1	5	20	14	3	12	140	8
34	4	2,5	10	30	60	5	50	165	8
35	5	2	20	30	25	200	50	210	1
36	6	0,5	4	10	30	5	30	240	6
37	7	2	40	20	20	100	5	110	9

Variant	Scheme number from Fig.1.1	Resistance, Ohm						Direct task	The reverse task
		R_0	R_1	R_2	R_3	R_4	R_5	E, V	I_1, A
38	8	2	9	24	24	70	30	100	8
39	9	0,5	7	30	60	30	70	190	6
40	10	2	16	12	20	5	18	70	9
41	11	1	5	6	30	6	7	180	3
42	12	1	8	10	14	12	48	130	15
43	13	2	60	10	7	15	7	150	4
44	14	2,5	50	90	21	60	21	125	1,8
45	15	2	12	60	13	80	80	220	4
46	16	0,5	30	10	20	30	5	250	4,8
47	17	1	10	5	10	6	4	80	6
48	18	1	3	4	5	10	10	130	10
49	19	1	5	19	70	30	5	90	4
50	20	0,5	20	40	10	10	20	100	6
51	21	1	12	3	4	5	7	90	4
52	22	2,5	30	40	30	19	40	45	1,2
53	23	1	30	6	30	120	4	200	2,5
54	24	1	6	15	10	15	60	100	10
55	25	2,5	20	40	6	10	6	75	6
56	26	2	40	40	30	60	30	240	1,5
57	27	1	40	30	70	14	5	210	2,5
58	28	2	30	40	30	16	40	220	1
59	29	2,5	5	12	40	8	15	200	4
60	30	1	8	20	100	30	25	110	10
61	1	1	4	5	7	12	4	90	3
62	2	0,5	7	10	5	6	15	100	10
63	3	1,5	48	16	3	6	15	120	2
64	4	1	10	100	25	7	20	150	4
65	5	1	40	80	52	25	80	105	0,8
66	6	1	4	3	6	8	10	120	6
67	7	2,5	12	16	20	15	6	200	9
68	8	1	10	15	10	20	80	180	5
69	9	1	10	10	15	80	20	90	10
70	10	2,5	5	10	30	30	10	150	8
71	11	2	20	45	30	90	10	85	2,4
72	12	4	50	45	15	60	20	120	3
73	13	3	10	10	7	40	7	60	1,6
74	14	2,5	50	90	21	60	21	125	1,8
75	15	1	14	25	16	40	40	230	3

Variant	Scheme number from Fig.1.1	Resistance, Ohm						Direct task	The reverse task
		R_0	R_1	R_2	R_3	R_4	R_5	E, V	I_1, A
76	16	0,5	20	60	30	20	5	130	4,8
77	17	1	15	20	10	4	14	160	2
78	18	2,5	4	10	6	20	12	220	4
79	19	2	10	38	140	60	10	180	4
80	20	1	5	10	10	15	10	110	16
81	21	2	24	6	8	10	14	180	2
82	22	1	30	10	10	10	18	100	2,5
83	23	2	60	14	20	80	18	300	2
84	24	2,5	20	30	30	15	15	100	1,5
85	25	2,5	20	40	6	10	6	75	6
86	26	1	20	20	15	30	15	150	3
87	27	2,5	35	30	70	5	9	160	2
88	28	1	10	30	10	8	20	110	6
89	29	2,5	20	10	20	50	10	75	4
90	30	2	16	40	200	60	50	220	2,5
91	1	1	3	40	50	10	6	190	5
92	2	2	5	12	3	36	6	220	7,5
93	3	2	15	35	4,5	4	60	225	14
94	4	2	8	30	20	7	80	150	6
95	5	2,5	10	8	4	30	9	120	2,5
96	6	1,5	6	20	5	16	60	180	4
97	7	1	4	5	7	30	17	260	4,5
98	8	1	2	20	80	90	10	120	10
99	9	2	6	45	30	10	15	170	7,5
100	10	2	11,5	15	10	40	24	270	6


1
E
R0
R2
R4
R1
R3
R5
2
R1
R0
R4
R5
R2
E
R3
3
R1
R0
R5
R2
E
R3
R4
4
R1
E
R5
R2
R3
R0
R4
5
R1
R5
R2
E
R4
R0
R3
6
R1
R0
R2
R3
E
R5
R4
7
R2
R1
R3
E
R4
R0
R5
8
R1
E
R2
R3
R4
R5
R0
9
R1
E
R3
R2
R4
R5
R0
10
R1
R2
R4
E
R5
R3
R0
Figure 1.1

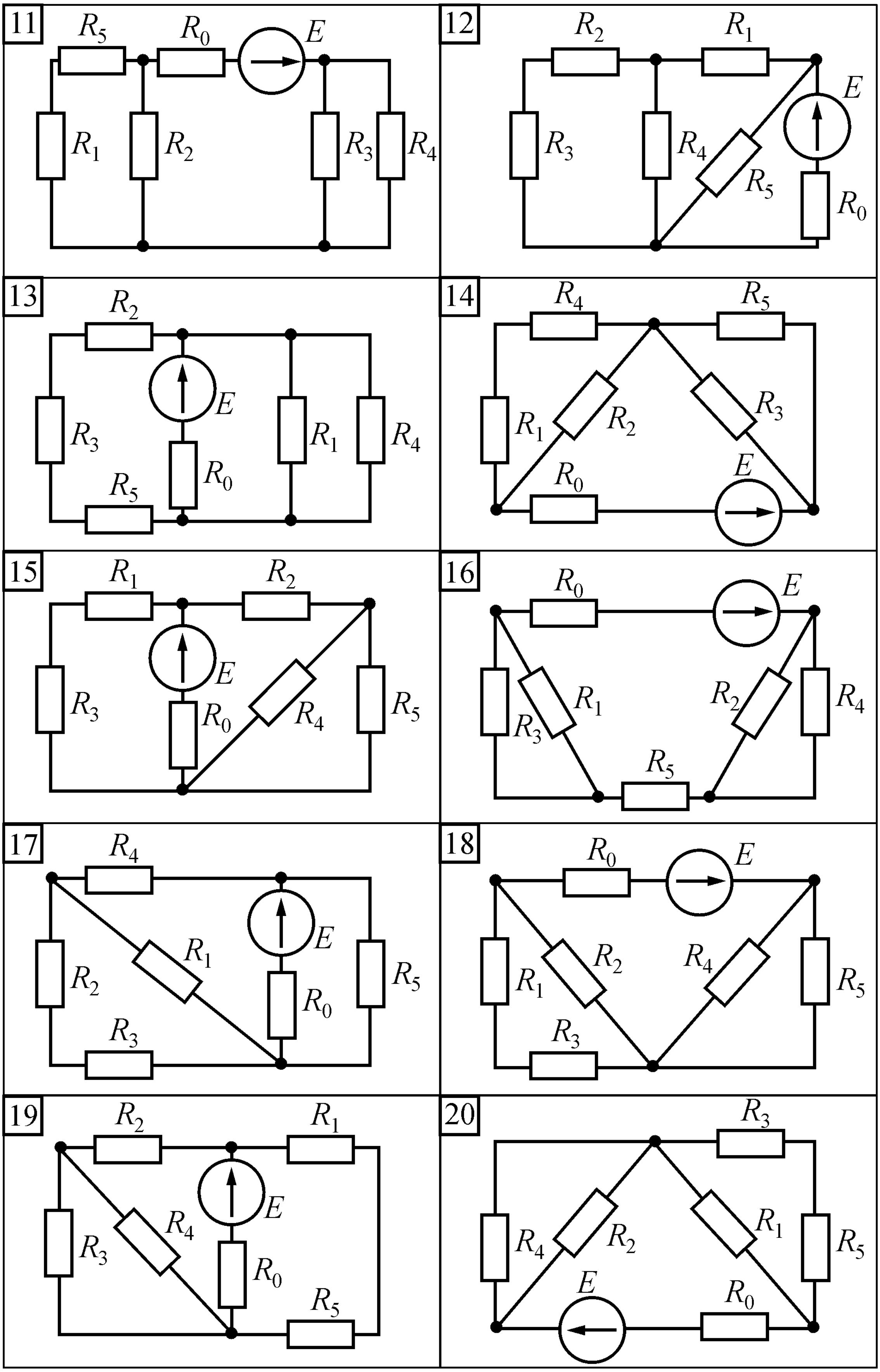


Figure 1.1 (continued)

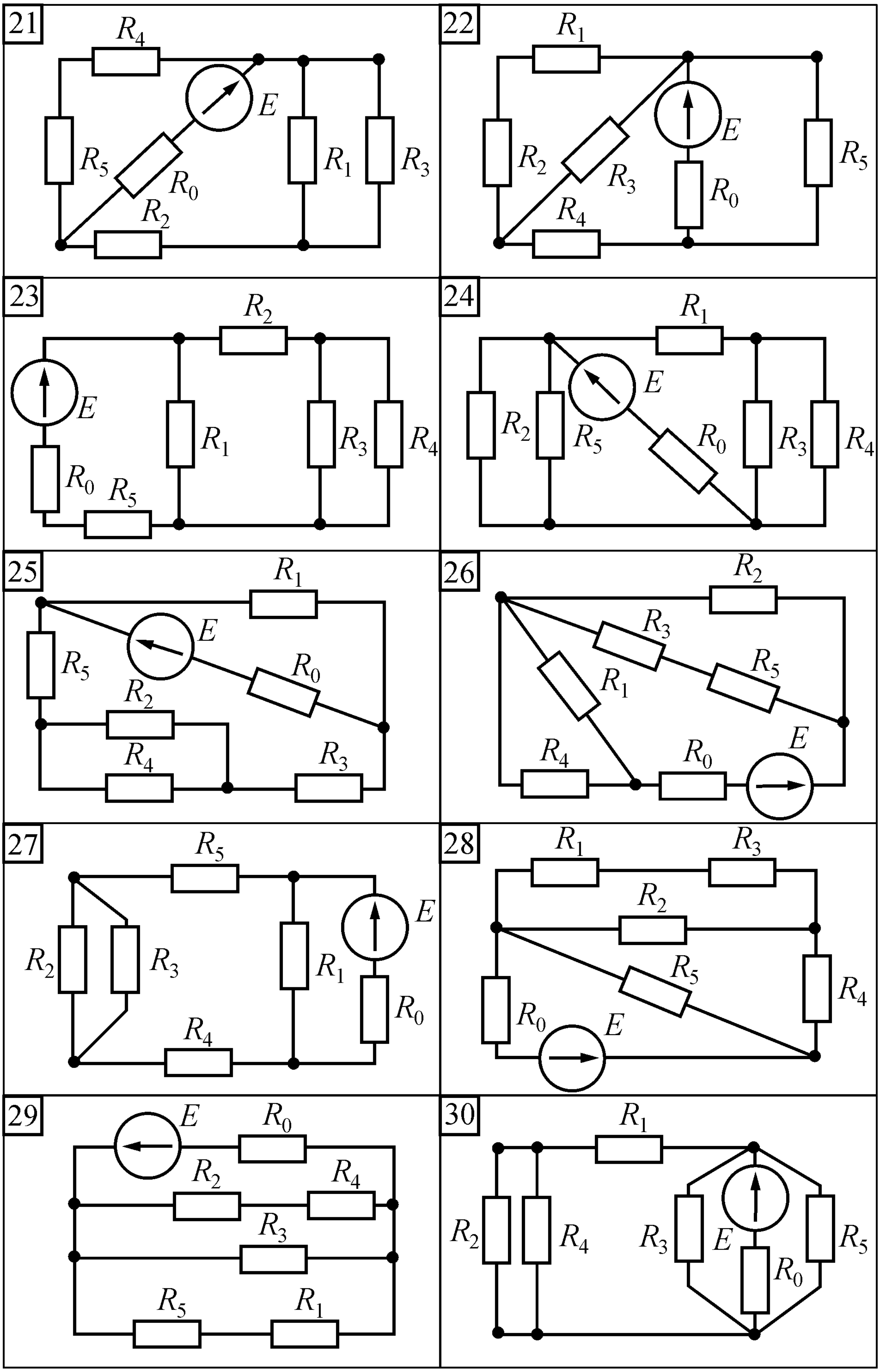


Figure 1.1 (continued)

Table 1.2 – Input data to Task 2

Variant	Scheme number from Fig.1.2	Resistance, Ohm						EMF	
		R_1	R_2	R_3	R_4	R_5	R_6	E, V	Branch
1	1	10	40	20	8	20	20	200	*bc*
2	2	30	14	18	30	7	15	250	*ac*
3	3	30	30	14	30	70	10	160	*bc*
4	4	12	24	36	7	74	16	210	*cd*
5	5	8	10	30	30	30	20	180	*bd*
6	6	12	5	40	40	20	14	250	*ad*
7	7	20	5	30	30	30	5	150	*cb*
8	8	14	40	7	80	54	80	180	*ac*
9	9	10	20	30	65	8	50	150	*bd*
10	10	4	7	18	12	6	8	90	*ac*
11	11	20	10	10	6	85	5	175	*cd*
12	12	3	9	33	33	33	69	165	*ad*
13	13	8	36	24	11	12	24	270	*вd*
14	14	18	4	48	48	48	64	200	*ab*
15	15	25	5	13	20	5	15	300	*вd*
16	16	5	10	8	2	10	10	150	*вc*
17	17	8	20	10	20	12	5	170	*вd*
18	18	14	20	30	50	25	30	200	*cd*
19	19	3	9	6	17	4	4	285	*ac*
20	20	8	2	6	6	6	13	135	*cd*
21	21	10	4	6	5	3	10	90	*ad*
22	22	12	24	12	7	17	24	150	*cd*
23	23	19	9	33	33	33	4	135	*ab*
24	24	10	6	50	40	25	10	250	*вd*
25	25	4	28	60	14	60	60	270	*вc*
26	26	9	6	3	2,5	37	9	180	*ad*
27	27	3	5	3	2	2	3	90	*ab*
28	28	6	2	7	20	10	20	160	*ad*
29	29	2	24	24	22	24	7	180	*ac*
30	30	11	30	3	8	30	40	300	*вc*
31	1	4	5	20	30	50	9	150	*ab*
32	2	8	7	6	12	18	4	180	*вc*
33	3	4	40	30	60	90	15	210	*ac*
34	4	18	10	12	12	4,6	3	180	*вd*
35	5	12	12	24	4,5	9	32	300	*ac*

Variant	Scheme number from Fig.1.2	Resistance, Ohm						EMF	
		R_1	R_2	R_3	R_4	R_5	R_6	E, V	Branch
36	6	75	5	75	4	45	75	200	*bd*
37	7	18	18	18	4	9	6	180	*ad*
38	8	14	5	28	3	8	14	145	*bd*
39	9	10	10	15	5	12	25	120	*cd*
40	10	14	6	12	36	24	8	240	*ad*
41	11	9	9	9	37	7	4	90	*bd*
42	12	8	18	36	36	36	12	240	*cd*
43	13	12	28	8	62	13	11	250	*bc*
44	14	10	20	30	30	30	3	300	*ad*
45	15	50	40	26	10	5	10	250	*bc*
46	16	51	51	13	51	43	13	225	*ab*
47	17	6	7	4	18	12	8	180	*bc*
48	18	14	20	8	50	15	30	210	*ac*
49	19	12	3	13	7	6	18	270	*ab*
50	20	1	9	6	6	24	6	120	*ac*
51	21	32	2	24	24	24	4	200	*bc*
52	22	12	3	12	4	7	24	140	*ac*
53	23	68	8	36	36	36	7	175	*bc*
54	24	5	15	10	9	15	15	300	*ad*
55	25	30	10	70	`30	4	30	240	*ab*
56	26	11	27	5,5	17	18	9	240	*bd*
57	27	6	3	6	4	18	6	100	*bc*
58	28	10	20	8	10	2	5	210	*bd*
59	29	20	7	4	12	40	40	150	*ab*
60	30	9	13	27	9	9	5	300	*ac*
61	1	20	6	32	20	8	10	300	*bd*
62	2	25	15	10	5	15	15	90	*cd*
63	3	10	10	17,5	60	7,5	5	240	*bd*
64	4	2	5	10	20	10	35	200	*ab*
65	5	20	5	30	30	30	5	150	*cb*
66	6	12	5	40	40	20	14	250	*ad*
67	7	45	5	10	20	30	15	300	*ac*
68	8	15	30	75	6	4,5	60	240	*ad*
69	9	6	42	24	24	24	5	220	*ac*
70	10	3	6	4	18	6	6	150	*вd*
71	11	18	5	12	3	12	42	215	*вс*

Variant	Scheme number from Fig.1.2	Resistance, Ohm						EMF	
		R_1	R_2	R_3	R_4	R_5	R_6	E, V	Branch
72	12	5	9	45	45	45	65	200	*вd*
73	13	9	18	10	8	6	12	190	*ad*
74	14	6	24	3	4	36	12	240	*вc*
75	15	10	2	6	8	10	55	180	*ab*
76	16	2	13	10	2	8	2	140	*cd*
77	17	51	11	57	10	57	57	150	*ac*
78	18	35	5	15	30	15	7	200	*вd*
79	19	12	4	8	2	6	18	120	*вc*
80	20	3	3	3	3	2	5	270	*вd*
81	21	20	5	3	5	13	25	260	*ab*
82	22	4	60	9	30	22	10	200	*вd*
83	23	5	20	25	7	2	10	155	*cd*
84	24	34	12	4	8	18	6	180	*ac*
85	25	7	12	6	12	12	86	300	*cd*
86	26	12	12	12	20	3	2	295	*ac*
87	27	20	40	15	10	20	5	200	*cd*
88	28	4	6	36	20	20	10	300	*ab*
89	29	7	24	24	7	24	22	225	*bc*
90	30	5	18	84	4	18	18	300	*bd*
91	1	1	20	2	6	20	10	225	*ac*
92	2	15	2,5	15	30	5	45	240	*ab*
93	3	65	5	45	45	45	5	170	*ad*
94	4	12	9	3	6	24	12	180	*bc*
95	5	10	7	8	4	4	11	180	*ab*
96	6	2	16	5	10	10	3	120	*ac*
97	7	15	20	15	9	2	4	125	*bd*
98	8	5	25	15	15	15	2,5	90	*bc*
99	9	26	1	10	30	10	2	235	*ab*
100	10	20	30	60	9	30	30	120	*bc*

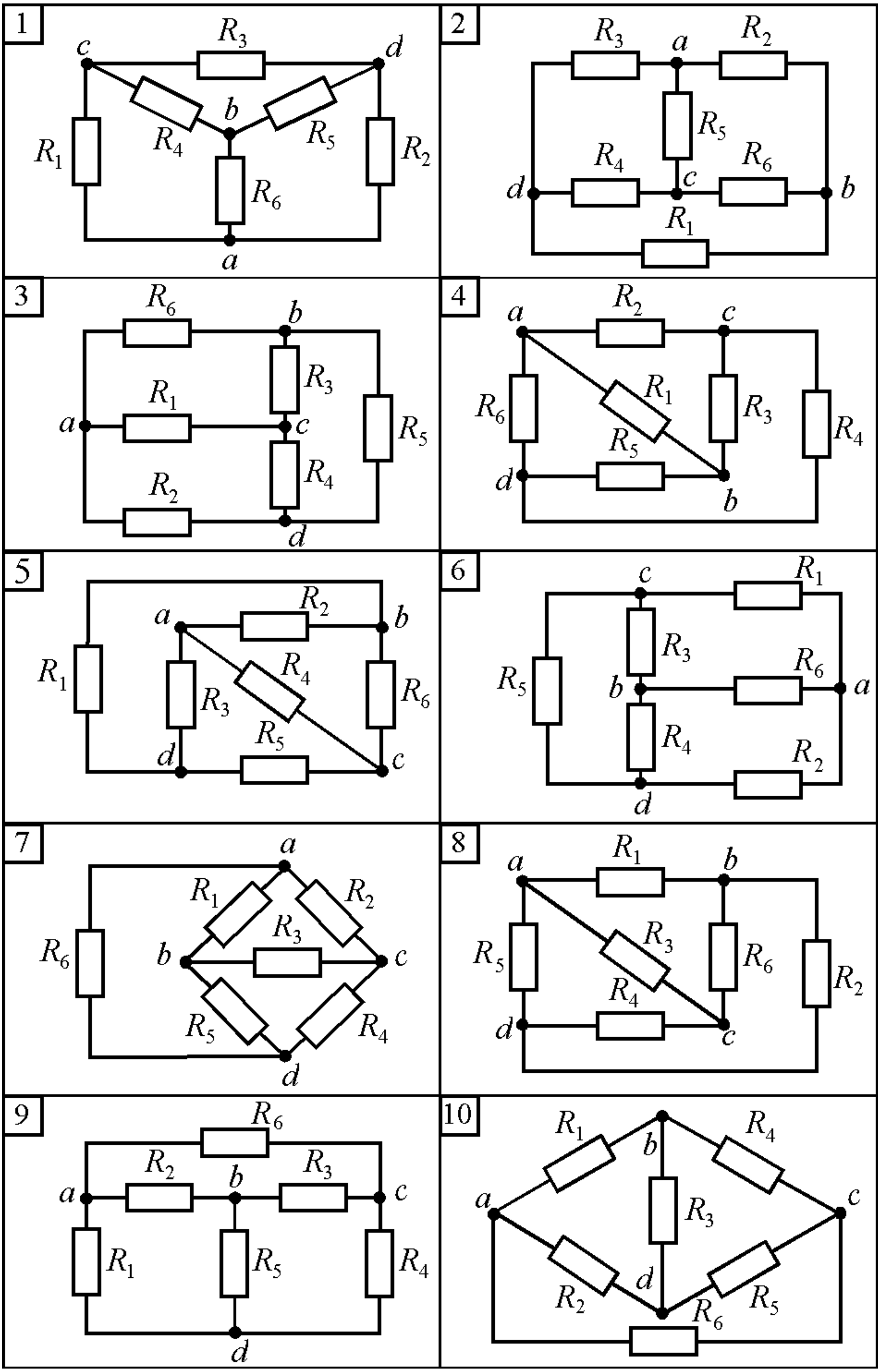

1
c
R3
d
b
R4
R5
R1
R6
R2
a
2
R3
a
R2
R5
R4
R6
d
c
b
R1
3
R6
b
R3
R1
a
c
R5
R4
R2
d
4
a
R2
c
R1
R6
R3
R4
R5
d
b
5
R2
a
b
R4
R1
R3
R6
R5
d
c
6
c
R1
R3
R6
R5
b
a
R4
R2
d
7
a
R1
R2
R3
R6
b
c
R5
R4
d
8
a
R1
b
R3
R5
R6
R2
R4
d
c
9
R6
R2
R3
a
b
c
R1
R5
R4
d
10
R1
R4
b
R3
a
c
d
R2
R6
R5

Figure 1.2

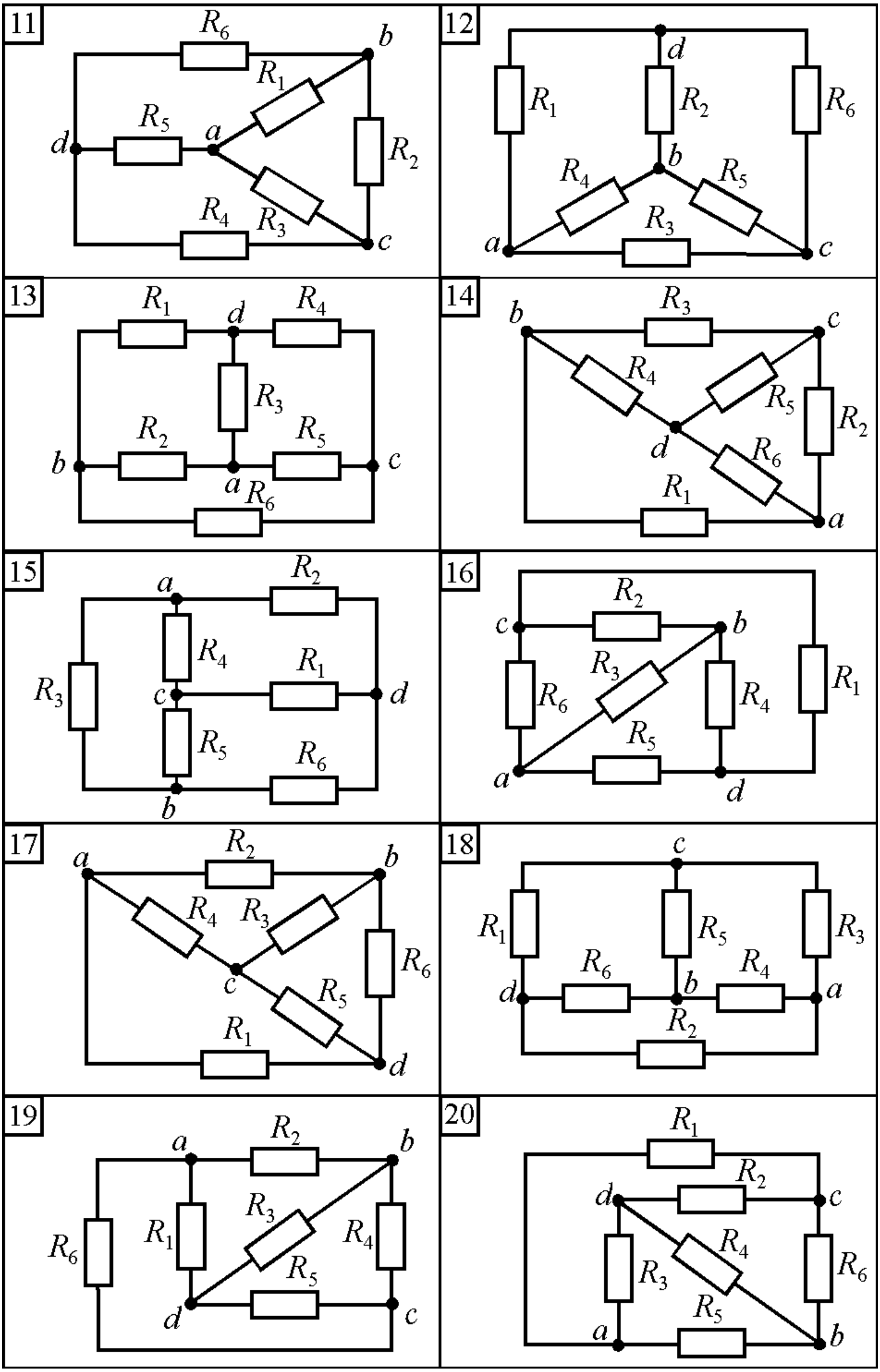


Figure 1.2 (continued)

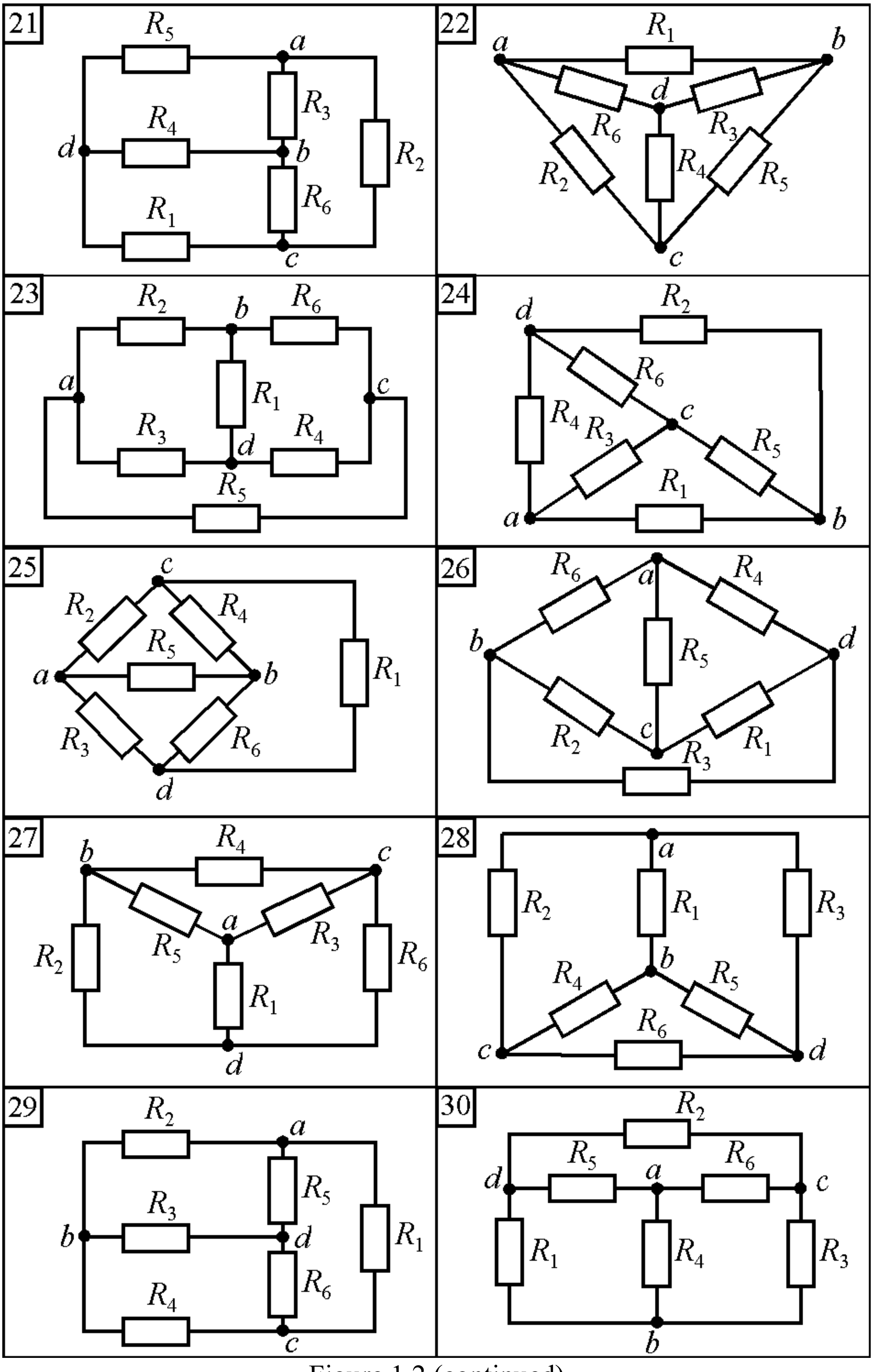


Figure 1.2 (continued)

Table 1.3 – Input data to Task 3

Variant	Scheme number for Fig.1.3	EMF, V		Resistance, Ohm					
		E_1, V	E_2, V	R_1	R_2	R_3	R_4	R_5	R_6
1	1	90	70	6	15	6	8	5	12
2	2	100	30	6	1	8	6	7	20
3	3	170	95	4	5	18	6	10	6
4	4	130	220	10	17	14	8	7	20
5	5	150	85	13	6	9	2	10	7
6	6	90	100	6	3	7	12	4	8
7	7	125	155	4	2	21	3	4	11
8	8	150	50	5	4	6	14	9	16
9	9	120	40	8	10	12	13	6	30
10	10	90	80	2	4	4	7	10	5
11	11	130	160	4	2	10	4	20	18
12	12	60	30	33	4	12	5	1	5
13	13	145	160	10	5	15	10	25	10
14	14	100	60	5	21	10	6	9	2
15	15	110	75	12	7	15	4	3	10
16	16	270	120	9	9	7	9	2	17
17	17	240	100	1,5	5	14	8	10	12
18	18	110	55	10	10	15	20	15	15
19	19	195	480	5	35	15	20	15	15
20	20	100	100	7,5	15	15	4	45	15
21	21	90	75	12	6	3	15	4	3
22	22	35	40	7	10	4	2	3,5	8
23	23	70	55	60	10	10	5	4	15
24	24	100	80	7	5	8	8	12	7
25	25	70	110	20	14	8	2	25	6
26	26	150	55	4	5	4,5	7	2	16
27	27	190	135	26	6	10	24	5	15
28	28	75	70	5	3	10	15	10	2
29	29	135	90	2	11	10	0,5	5	0,5
30	30	120	50	11	4	2	4	7	6
31	1	270	210	12	30	12	16	10	24
32	2	150	135	10	12	10	30	11	56
33	3	80	25	1	3	5,5	34	3	1,5
34	4	195	330	20	34	28	16	14	40

Variant	Scheme number for Fig.1.3	EMF, V		Resistance, Ohm					
		E_1, V	E_2, V	R_1	R_2	R_3	R_4	R_5	R_6
35	5	130	110	9	4	20	35	4	2
36	6	140	105	12	4	5	4	15	17
37	7	55	150	4	3	10	5	15	4
38	8	110	70	5	6	40	2	1	6
39	9	140	160	10	3	6	8	16	8
40	10	40	35	1	6	5	2,5	1	12
41	11	120	40	1	6	4	11	2	1,5
42	12	40	110	14	4	15	6	3	2
43	13	125	35	12	2	25	4	3	7
44	14	185	210	6	11	5	10	9	30
45	15	90	90	6	6	6	1	4	16
46	16	125	145	6	9	5	15	2	3
47	17	110	30	5	24	6	6	6	4
48	18	150	75	1	14	5	5	1	9
49	19	130	20	8	10	6	5	15	3
50	20	70	130	10	15	16	10	17	6
51	21	120	50	10	2	4	4	6	3
52	22	105	195	15	5	5	10	20	28
53	23	90	70	60	10	10	5	9	20
54	24	100	120	3	9	8	8	12	12
55	25	80	220	15	14	8	2	25	6
56	26	300	110	8	10	9	14	4	32
57	27	200	135	20	7	10	30	5	15
58	28	150	140	10	6	20	30	20	4
59	29	270	180	4	22	20	1	10	1
60	30	240	100	22	8	4	8	14	12
61	1	180	140	3	7,5	3	4	2,5	6
62	2	50	45	5	6	5	15	5,5	28
63	3	240	75	2	6	11	68	6	3
64	4	130	220	5	8,5	7	4	3,5	10
65	5	80	80	3	21	2	6	5	11
66	6	120	150	4	4	8	10	3	5
67	7	360	300	16	8	26	14	30	27
68	8	110	70	10	4	5	7	3	7
69	9	90	95	15	5	10	5	20	30

Variant	Scheme number for Fig.1.3	EMF, V		Resistance, Ohm					
		E_1, V	E_2, V	R_1	R_2	R_3	R_4	R_5	R_6
70	10	160	140	2	12	10	5	2	24
71	11	240	80	2	12	8	22	4	3
72	12	40	40	3	8	18	4	5	14
73	13	60	120	4	2	10	2	5	2
74	14	120	150	7	5	2	9	3	15
75	15	210	170	6	4	3	5	4	10
76	16	55	65	2	3	10	30	4	16
77	17	100	200	4	5	6	11	12	8
78	18	120	60	1	12	6	10	20	10
79	19	240	90	1	6	12	16	12	12
80	20	100	50	20	24	2	8	21	10
81	21	50	20	10	2	5	6	2,5	20
82	22	140	160	14	20	8	4	7	16
83	23	140	110	30	5	5	2,5	2	7,5
84	24	230	160	11	6	8	8	15	7
85	25	40	155	15	14	8	7	25	6
86	26	130	85	3	5	5	6	2	15
87	27	150	80	9	10	5	14	8	8
88	28	90	75	6	9	10	9	3	5
89	29	155	170	15	5	10	27	20	18
90	30	65	25	5,5	2	1	2	3,5	3
91	1	110	230	30	4	20	12	6	16
92	2	40	90	6	2	4	3	6	4
93	3	130	220	9	10	3	9	8	3
94	4	145	70	2	10	10	2	3	6
95	5	105	15	11	10	16	1	5	12
96	6	160	100	10	4	8	7	2	30
97	7	120	100	8	4	13	7	15	13,5
98	8	95	130	5	9	8	2	10	18
99	9	180	210	15	5	20	5	20	40
100	10	50	95	2	4	5	5	25	2

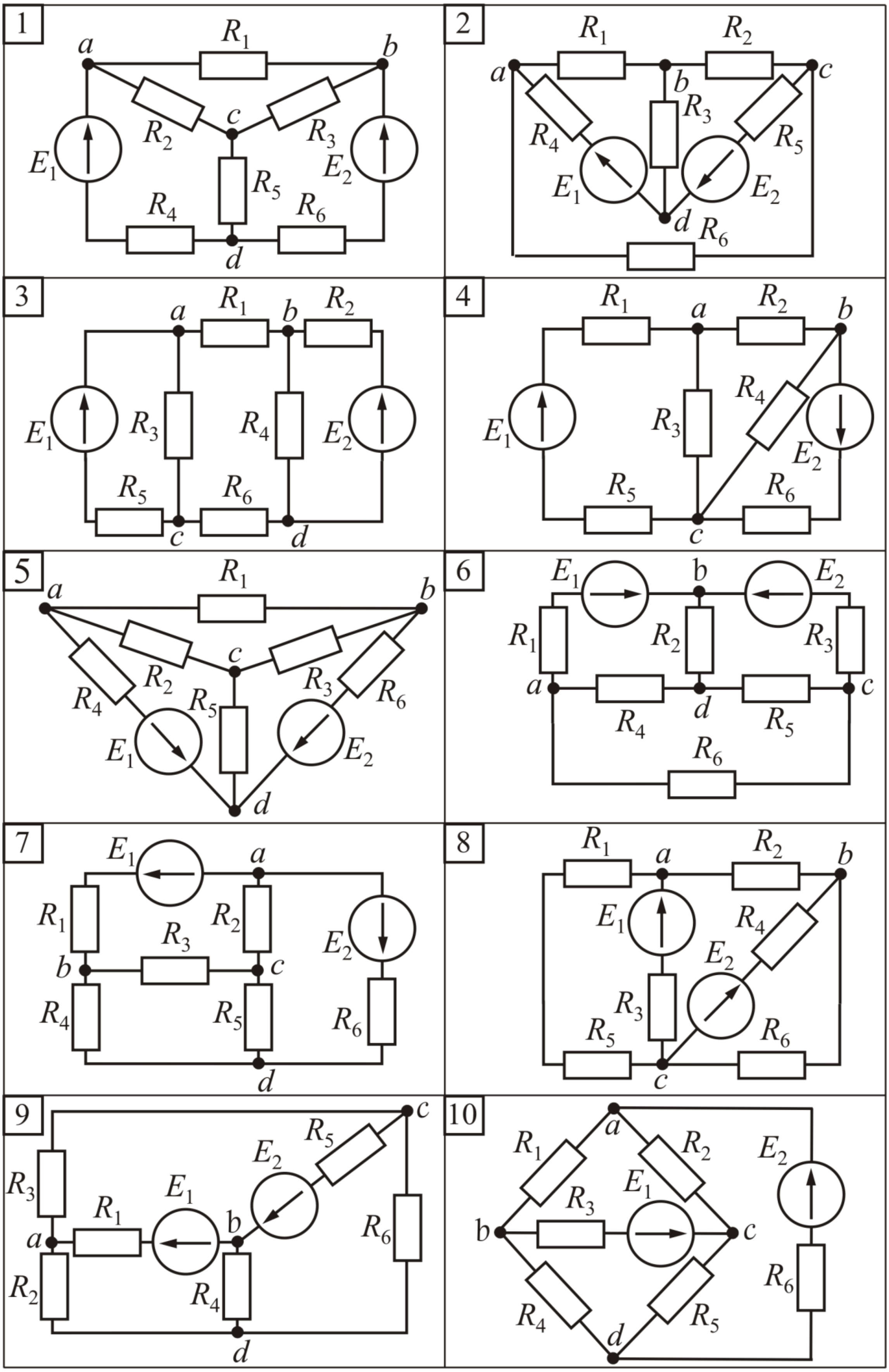

1
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
2
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
3
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
4
a b c
R1 R2 R3 R4 R5 R6
E1 E2
5
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
6
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
7
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
8
a b c
R1 R2 R3 R4 R5 R6
E1 E2
9
a b c d
R1 R2 R3 R4 R5 R6
E1 E2
10
a b c d
R1 R2 R3 R4 R5 R6
E1 E2

Figure 1.3

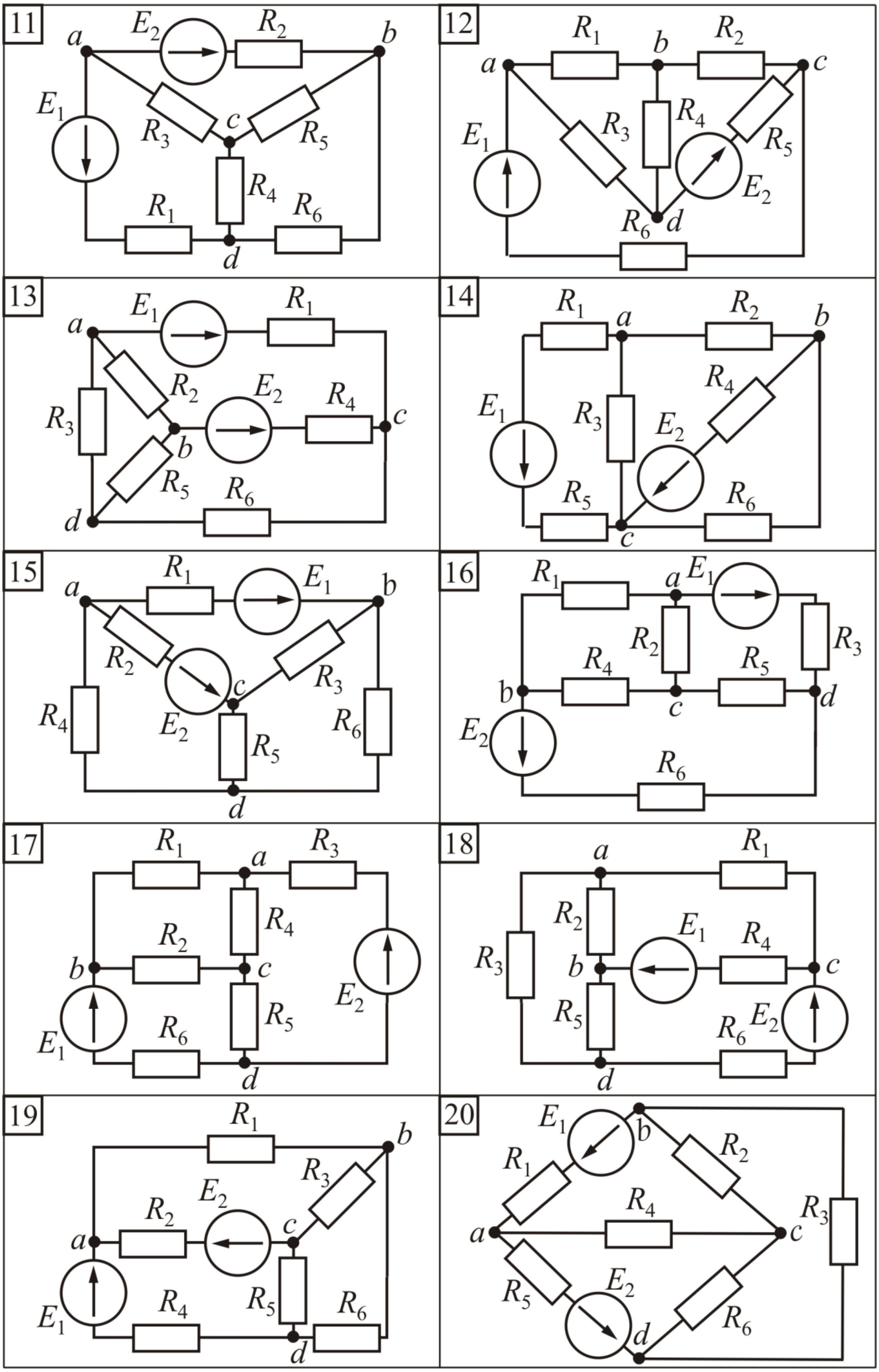


Figure 1.3 (continued)

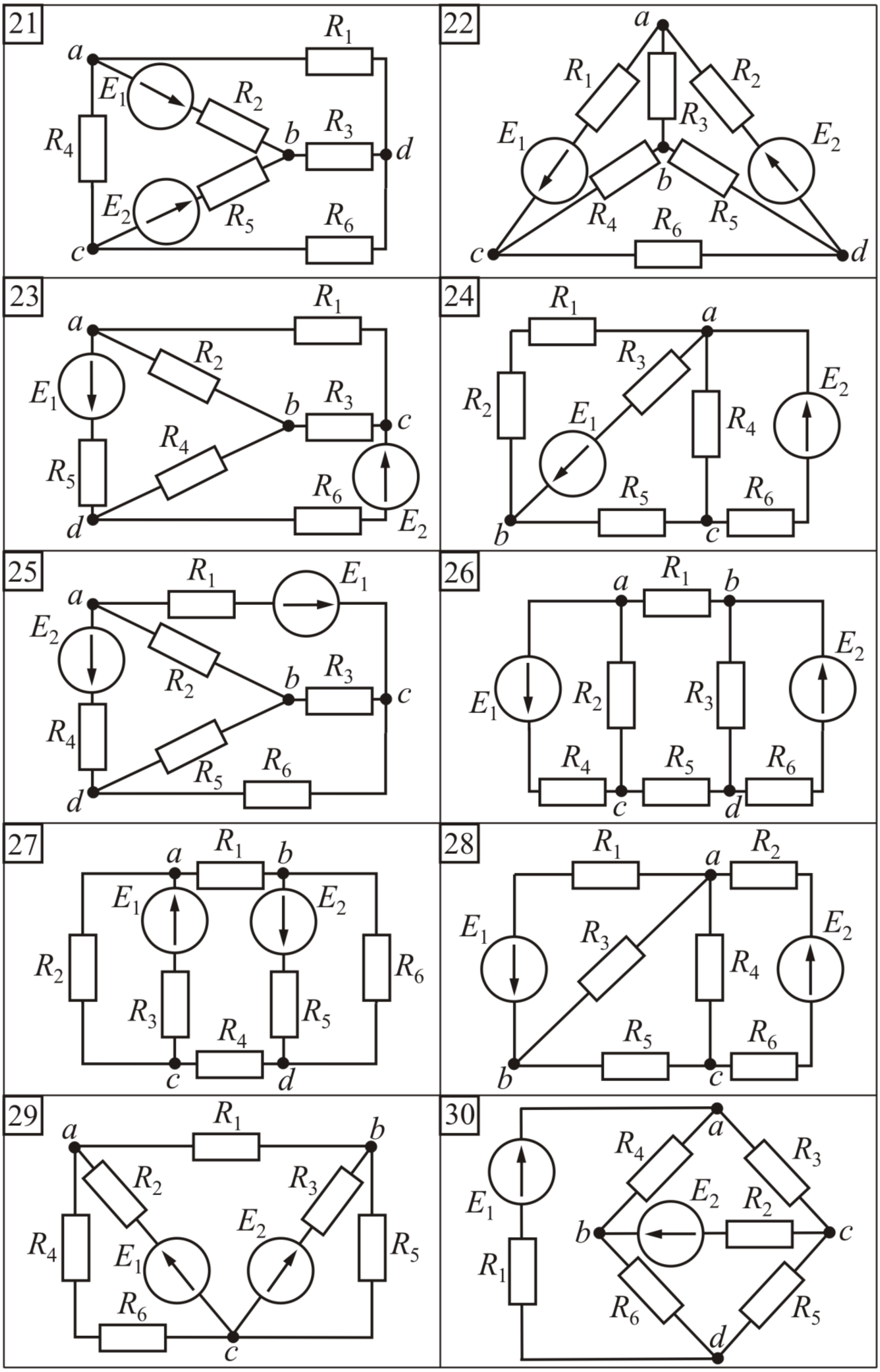


Figure 1.3 (continued)

1.2. Examples of tasks solving

Example 1. Calculation of an electric circuit with one source of EMF at the series-parallel connected resistances.

a) determine the currents in the branches of an electric circuit, the scheme of which is shown in Fig. 1.4, and it is given EMF $E = 120$ V; resistances $R_0 = 2$ Ohm; $R_1 = 3$ Ohm; $R_2 = 25$ Ohm; $R_3 = 5$ Ohm; $R_4 = 20$ Ohm; $R_5 = 30$ Ohm. Write the equation of the power balance of the circuit and calculate it.

b) determine the EMF of energy sources E and currents in the branches of an electric circuit, the scheme of which is shown in Fig. 1.4, if the values of resistance of resistors and the current $I_1 = 4$ A.

Solution.

a) direct task

We choose the positive directions of the currents in the all branches of the electric circuit, according to the direction of the EMF, shown on the scheme. We use the method of consistent reducing of the initial scheme into non-branched by the method of equivalent replacement of some parts of the scheme by another, as shown in Fig. 1.4, *b*, *c*, *d*.

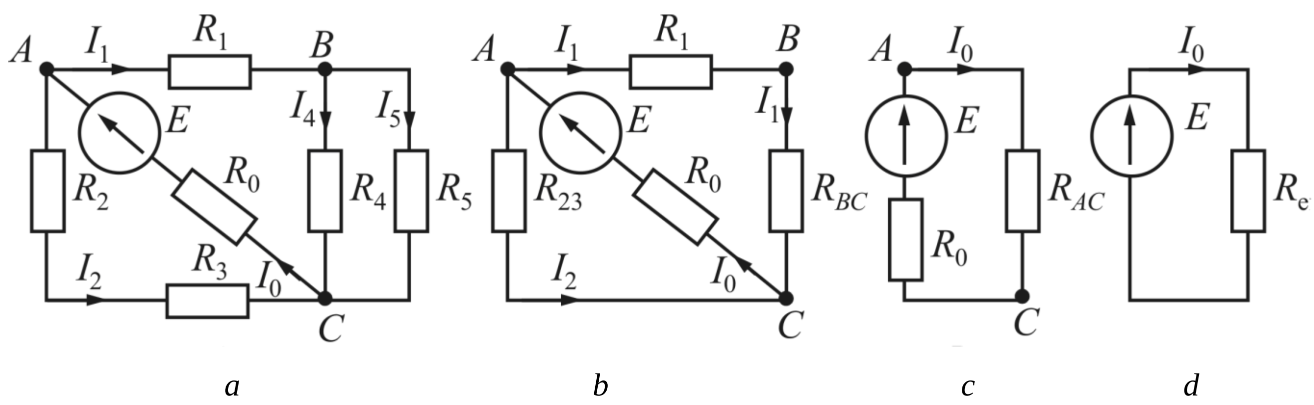


Figure 1.4

We determine the equivalent resistances of the parts and of the whole circuit after reducing:

$$R_{23} = R_2 + R_3 = 25 + 5 = 30 \text{ Ohm}; \quad R_{BC} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{20 \cdot 30}{20 + 30} = 12 \text{ Ohm};$$

$$R_{AC} = \frac{(R_1 + R_{BC}) \cdot R_{23}}{R_1 + R_{BC} + R_{23}} = \frac{(3+12) \cdot 30}{3 + 12 + 30} = 10 \text{ Ohm}; \quad R_\text{e} = R_0 + R_{AC} = 2 + 10 = 12 \text{ Ohm}.$$

Having received R_e, we can determine the current I_0 through the EMF source (Fig. 1.4, *d*) $I_0 = \dfrac{E}{R_\text{екв}} = \dfrac{120}{12} = 10$ A, and then the voltage in the part *AC* (Fig. 1.4, *c*) $U_{AC} = R_{AC} \cdot I_0 = 10 \cdot 10 = 100$ V and currents in parallel branches in the same part (Fig. 1.4, *b*) are equal to:

$$I_1 = \frac{U_{AC}}{R_1 + R_{BC}} = \frac{100}{3+12} = 6{,}67 \text{ A}; \quad I_2 = \frac{U_{AC}}{R_{23}} = \frac{100}{30} = 3{,}33 \text{ A}.$$

Using current I_1, we can determine the voltage in the part *BC* (Fig. 1.4, *b*) $U_{BC} = R_{BC} \cdot I_1 = 12 \cdot 6{,}67 = 80$ V and currents in its parallel branches (Fig. 1.4, *a*):

$$I_4 = \frac{U_{BC}}{R_4} = \frac{80}{20} = 4 \text{ A}; \quad I_5 = \frac{U_{BC}}{R_5} = \frac{80}{30} = 2{,}67 \text{ A}.$$

The equation of the power balance of the circuit in Fig. 1.4, *a* is:

$$P_\text{source} = \sum_{k=1}^{6} P_{l,k},$$

where the power of the EMF source $P_\text{source} = E \cdot I_0 = 120 \cdot 10 = 1200$ W.

Arithmetic sum of loads power:

$$\sum_{k=1}^{6} P_{\text{load},k} = R_0 I_0^2 + R_1 I_1^2 + R_2 I_2^2 + R_3 I_2^2 + R_4 I_4^2 + R_2 I_5^2 =$$
$$= 2 \cdot 10^2 + 3 \cdot 6{,}67^2 + 25 \cdot 3{,}33^2 + 5 \cdot 3{,}33^2 + 20 \cdot 4^2 + 30 \cdot 2{,}67^2 =$$
$$= 200 + 133{,}47 + 277{,}22 + 55{,}44 + 320 + 213{,}87 = 1200 \text{ W}.$$

If the powers values are not equal, then we calculate the relative error, which should be less than 5 percent

$$\delta = \left| \frac{P_{\text{source}} - \sum_{k=1}^{n} P_{load,k}}{P_{\text{source}}} \right| \cdot 100\% < 5\%$$

If this condition is not met, the task is not solved correctly.

b) reverse task

Voltage in the part BC (Fig. 1.4, *b*) is equal to:

$$U_{BC} = R_{BC} \cdot I_1 = 12 \cdot 4 = 48 \text{ V.}$$

Currents in parallel branches in this part (Fig. 1.4, *b*) are equal to:

$$I_4 = \frac{U_{BC}}{R_4} = \frac{48}{20} = 2{,}4 \text{ A}; \quad I_5 = \frac{U_{BC}}{R_5} = \frac{48}{30} = 1{,}6 \text{ A.}$$

Voltage in the part AC (Fig. 1.4, *b*) is:

$$U_{AC} = I_1 \cdot R_1 + U_{BC} = 4 \cdot 3 + 48 = 60 \text{ V.}$$

The current in the branch with resistance R_{23} (Fig. 1.4, *b*) is:

$$I_2 = \frac{U_{AC}}{R_{23}} = \frac{60}{30} = 2 \text{ A.}$$

According to Kirchhoff's current law (Kirchhoff's first law), we can determine the source current for node *A* (Fig. 1.4, *b*)

$$I_0 = I_1 + I_2 = 4 + 2 = 6 \text{ A.}$$

We can determine the voltage of the source EMF (Fig. 1.4, *d*)

$$E = R_{\text{e}} \cdot I_0 = 12 \cdot 6 = 72 \text{ V.}$$

Example 2. Calculation of an electric circuit with one EMF source by the method of transformation "delta" - "wye".

EMF source *E* is connected to the branch *D-A* in the electric circuit shown in Fig. 1.5. The direction of the EMF is pointed from node *D* to node *A*. Parameters of the elements of the circuit: $E = 60$ V; $R_1 = 8$ Ohm; $R_2 = 10$ Ohm; $R_3 = 20$ Ohm; $R_4 = 15$ Ohm; $R_5 = 7$ Ohm; $R_6 = 9$ Ohm.

Determine the currents in the branches of an electric circuit using the method of converting the resistances connected by a “Δ” into an equivalent “Y” and vice versa. Write the equation of the power balance equation of the circuit and calculate it.

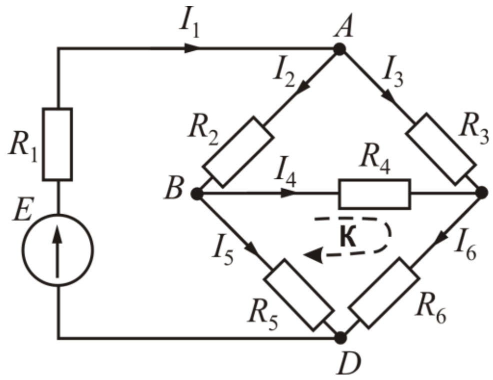


Figure 1.5

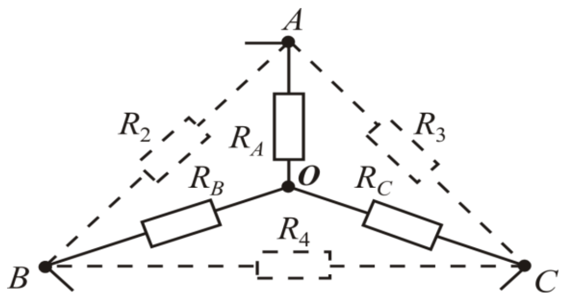


Figure 1.6

Solution. Previously we must choose the positive directions of currents in the branches of the circuit, as it is shown in Fig. 1.5. If the current direction will be different from the one we have chosen, then in the calculation of the task there will be a sign “–”.

To solve the task we use transformations, for example, of three resistances R_2, R_3 and R_4, which are delta connected between nodes *A*, *B* and *C* (Fig. 1.5), in an equivalent “Y” with resistances R_A, R_B and R_C and the central node *O* (Fig. 1.6).

Determine the equivalent resistances of the “Y” connection:

$$R_A = \frac{R_2 \cdot R_3}{R_2 + R_3 + R_4} = \frac{10 \cdot 20}{10 + 20 + 15} = 4{,}44 \text{ Ohm};$$

$$R_B = \frac{R_2 \cdot R_4}{R_2 + R_3 + R_4} = \frac{10 \cdot 15}{10 + 20 + 15} = 3{,}33 \text{ Ohm};$$

$$R_C = \frac{R_3 \cdot R_4}{R_2 + R_3 + R_4} = \frac{20 \cdot 15}{10 + 20 + 15} = 6{,}67 \text{ Ohm}.$$

Equivalent electrical circuit scheme of the replacement is given on the Fig. 1.7, in which it is shown the directions of the currents through the resistances that are not taken to the transformation.

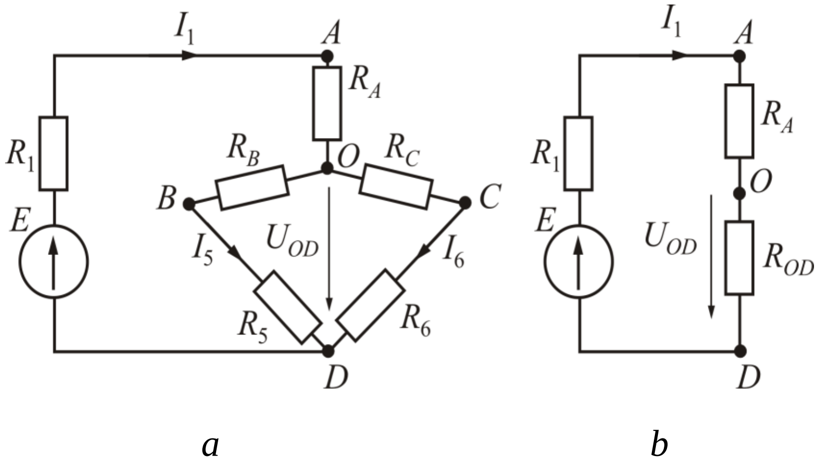


Figure 1.7

Determine the equivalent resistance between nodes *O* and *D* and transform the electric circuit in Fig. 1.7, *a* to the equivalent electric circuit shown in Fig. 1.7, *b*.

$$R_{OD} = \frac{(R_B + R_5)\cdot(R_C + R_6)}{R_B + R_5 + R_C + R_6} = \frac{(3{,}33 + 7)\cdot(6{,}67 + 9)}{3{,}33 + 7 + 6{,}67 + 9} = 6{,}23 \text{ Ohm.}$$

Equivalent resistance of the whole circuit (Fig. 1.7, *b*):

$$R_e = R_1 + R_A + R_{OD} = 8 + 4{,}44 + 6{,}23 = 18{,}67 \text{ Ohm.}$$

Current through the EMF source

$$I_1 = \frac{E}{R_e} = \frac{60}{18{,}67} = 3{,}21 \text{ A.}$$

Voltage between nodes *OD* (Fig. 1.8, *b*):

$$U_{OD} = R_{OD}\cdot I_1 = 6{,}23\cdot 3{,}21\cdot = 20 \text{ V.}$$

We can determine the currents I_5 and I_6 in the parallel branches (Fig. 1.7, *a*):

$$I_5 = \frac{U_{OD}}{R_B + R_5} = \frac{20}{3{,}33 + 7} = 1{,}94 \text{ A;}\quad I_6 = \frac{U_{OD}}{R_C + R_6} = \frac{20}{6{,}67 + 9} = 1{,}28 \text{ A.}$$

According to Kirchhoff's voltage law, for the loop K (Fig. 1.5), when bypassing it clockwise, we receive the equation:

$$R_4\cdot I_4 + R_6\cdot I_6 - R_5\cdot I_5 = 0;$$

from this equation we can determine the current

$$I_4 = \frac{R_5 \cdot I_5 - R_6 \cdot I_6}{R_4} = \frac{7 \cdot 1{,}94 - 9 \cdot 1{,}28}{15} = 0{,}14 \text{ A.}$$

According to Kirchhoff's current law for currents in branches which are connected in node *B* (Fig. 1.5), we have the following equation $I_2 - I_4 - I_5 = 0$; from this equation we determine the current $I_2 = I_4 + I_5 = 0{,}14 + 1{,}94 = 2{,}08$ A.

According to Kirchhoff's current law for currents of branches which are connected in node *C*, we have the equation $I_3 + I_4 - I_6 = 0$; from this equation we determine the current $I_3 = I_6 - I_4 = 1{,}28 - 0{,}14 = 1{,}14$ A.

The equation of the power balance for the circuit in Fig. 1.5:

$$P_{\text{source}} = \sum_{k=1}^{6} P_{l,k} ,$$

where the power of the EMF source $P_{\text{source}} = E \cdot I_1 = 60 \cdot 3{,}21 = 192{,}6$ W; sum of load powers:

$$\sum_{k=1}^{6} P_{l,k} = R_1 \cdot I_1^2 + R_2 \cdot I_2^2 + R_3 \cdot I_3^2 + R_4 \cdot I_4^2 + R_5 \cdot I_5^2 + R_6 \cdot I_6^2 =$$
$$= 8 \cdot 3{,}21^2 + 10 \cdot 2{,}08^2 + 20 \cdot 1{,}14^2 + 15 \cdot 0{,}14^2 + 7 \cdot 1{,}94^2 + 9 \cdot 1{,}28^2 =$$
$$= 82{,}43 + 43{,}26 + 25{,}99 + 0{,}29 + 26{,}34 + 14{,}75 = 193{,}06 \text{ W.}$$

Calculating percentage error:

$$\delta = \left| \frac{P_{\text{source}} - \sum_{k=1}^{6} P_{l,k}}{P_{\text{source}}} \cdot 100\% \right| = \left| \frac{192{,}6 - 193{,}06}{192{,}6} \cdot 100\% \right| = 0{,}24\%$$

would be considered as acceptable.

<u>Example 3.</u> Calculation of an electrical circuit by the mesh-current method.

The scheme for an electric circuit is shown in Fig.1.8, determine the currents in all branches, using the mesh current method of calculating loop currents, if $E_1 = 100$ V; $E_2 = 50$ V; $R_1 = R_5 = 8$ Ohm; $R_2 = 10$ Ohm; $R_3 = 7$ Ohm; $R_4 = 9$ Ohm; $R_6 = 13$ Ohm.

Solution. Selection of the independent loops I, II, III, designation of the circuit currents direction I_{11}, I_{22}, I_{33} and the direction of bypassing the loops are shown in Fig. 1.8.

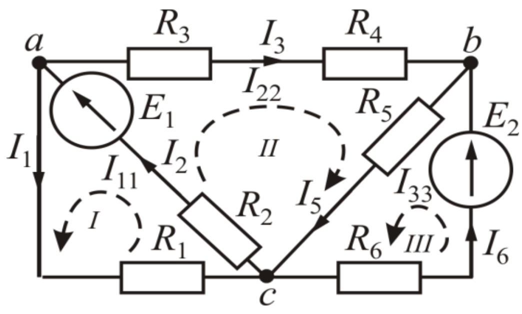


Figure 1.8

The system of equations for loops I, II and III is as follows:

$$\begin{cases} R_{11} \cdot I_{11} + R_{12} \cdot I_{22} + R_{13} \cdot I_{33} = E_{11}; \\ R_{21} \cdot I_{11} + R_{22} \cdot I_{22} + R_{23} \cdot I_{33} = E_{22}; \\ R_{31} \cdot I_{11} + R_{32} \cdot I_{22} + R_{33} \cdot I_{33} = E_{33}, \end{cases} \qquad (1)$$

where the loop EMF: $E_{11} = E_1 = 100$ V; $E_{22} = E_1 = 100$ V; $E_{33} = E_2 = 50$ V,

total loop resistances of the loops:

$R_{11} = R_1 + R_2 = 8 + 10 = 18$ Ohm; $R_{22} = R_2 + R_3 + R_4 + R_5 = 10 + 7 + 9 + 8 = 34$ Ohm;

$R_{33} = R_5 + R_6 = 8 + 13 = 21$ Ohm,

mutual resistance of loops: $R_{12} = R_{21} = R_2 = 10$ Ohm; $R_{23} = R_{32} = R_5 = 8$ Ohm;

$R_{13} = R_{31} = 0$ (loops I and III are not connected). Resistances $R_{12} = R_{21}$, $R_{23} = R_{32}$ written with a "+" sign because the loop currents in them coincide.

After substituting the numerical values, the system of equations (1) has the form:

$$\begin{cases} 18 \cdot I_{11} + 10 \cdot I_{22} + 0 \cdot I_{33} = 100; \\ 10 \cdot I_{11} + 34 \cdot I_{22} + 8 \cdot I_{33} = 100; \\ 0 \cdot I_{11} + 8 \cdot I_{22} + 21 \cdot I_{33} = 50, \end{cases} \qquad (2)$$

the common determinant of which $\Delta = \begin{vmatrix} 18 & 10 & 0 \\ 10 & 34 & 8 \\ 0 & 8 & 21 \end{vmatrix}$.

Solving it by the elements of the first line, we find

$$\Delta = \begin{vmatrix} 18 & 10 & 0 \\ 10 & 34 & 8 \\ 0 & 8 & 21 \end{vmatrix} = (-1)^{1+1} \cdot 18 \cdot \begin{vmatrix} 34 & 8 \\ 8 & 21 \end{vmatrix} + (-1)^{1+2} \cdot 10 \cdot \begin{vmatrix} 10 & 8 \\ 0 & 21 \end{vmatrix} + (-1)^{1+3} \cdot 0 \cdot \begin{vmatrix} 10 & 34 \\ 0 & 8 \end{vmatrix} =$$

$$= 18 \cdot (34 \cdot 21 - 8 \cdot 8) - 10 \cdot (10 \cdot 21 - 0 \cdot 8) + 0 = 11700 - 2100 = 9600.$$

Auxiliary determinants of the system are obtained from the general determinant by replacing columns 1, 2 and 3 with the EMF column of the system of equations (2), namely:

$$\Delta_{11}=\begin{vmatrix}100 & 10 & 0\\ 100 & 34 & 8\\ 50 & 8 & 21\end{vmatrix}=(-1)^{1+1}\cdot 100\cdot\begin{vmatrix}34 & 8\\ 8 & 21\end{vmatrix}+(-1)^{1+2}\cdot 10\cdot\begin{vmatrix}100 & 8\\ 50 & 21\end{vmatrix}=$$

$$=100\cdot(34\cdot 21-8\cdot 8)-10\cdot(100\cdot 21-50\cdot 8)=65000-17000=48000.$$

$$\Delta_{22}=\begin{vmatrix}18 & 100 & 0\\ 10 & 100 & 8\\ 0 & 50 & 21\end{vmatrix}=(-1)^{1+1}\cdot 18\cdot\begin{vmatrix}100 & 8\\ 50 & 21\end{vmatrix}+(-1)^{1+2}\cdot 100\cdot\begin{vmatrix}10 & 8\\ 0 & 21\end{vmatrix}=$$

$$=18\cdot(100\cdot 21-50\cdot 8)-100\cdot(10\cdot 21-0\cdot 8)=30600-21000=9600.$$

$$\Delta_{33}=\begin{vmatrix}18 & 10 & 100\\ 10 & 34 & 100\\ 0 & 8 & 50\end{vmatrix}=(-1)^{1+1}\cdot 18\cdot\begin{vmatrix}34 & 100\\ 8 & 50\end{vmatrix}+(-1)^{1+2}\cdot 10\cdot\begin{vmatrix}10 & 100\\ 0 & 50\end{vmatrix}+$$

$$+(-1)^{1+3}\cdot 100\cdot\begin{vmatrix}10 & 34\\ 0 & 8\end{vmatrix}=18\cdot(34\cdot 50-100\cdot 8)-10\cdot(10\cdot 50-0\cdot 100)+$$

$$+100\cdot(10\cdot 8-0\cdot 34)=16200-5000+8000=19200.$$

Loop currents are equal to:

$$I_{11}=\frac{\Delta_{11}}{\Delta}=\frac{48000}{9600}=5\ \text{A};\ I_{22}=\frac{\Delta_{22}}{\Delta}=\frac{9600}{9600}=1\ \text{A};\ I_{33}=\frac{\Delta_{33}}{\Delta}=\frac{19200}{9600}=2\ \text{A}.$$

They determine the currents in the branches as follows:

$$I_1=I_{11}=5\,\text{A};\qquad I_2=I_{11}+I_{22}=5+1=6\,\text{A};$$

$$I_3=I_{22}=1\,\text{A};\qquad I_5=I_{22}+I_{33}=1+2=3\,\text{A};\qquad I_6=I_{33}=2\,\text{A}.$$

Equation of the power balance

$$E_1\cdot I_2+E_2\cdot I_6=R_1\cdot I_1^2+R_2\cdot I_2^2+R_3\cdot I_3^2+R_4\cdot I_3^2+R_5\cdot I_5^2+R_6\cdot I_6^2;$$

$$100\cdot 6+50\cdot 2=8\cdot 5^2+10\cdot 6^2+7\cdot 1^2+9\cdot 1^2+8\cdot 3^2+13\cdot 2^2\ ;$$

700 W = 700 W.

Example 4. Calculation of an electrical circuit by the Thevenin's theorem method.

Determine the current I_1 in a branch with resistance R_1 in the electric circuit, which is shown in Fig. 1.9, if $E_1 = 120$ V; $E_2 = 60$ V; $R_1 = 8$ Ohm; $R_2 = 5$ Ohm; $R_3 = 4$ Ohm; $R_4 = 6$ Ohm; $R_5 = 10$ Ohm; $R_6 = 15$ Ohm.

Solution. We replace the electric circuit without a section of the circuit with resistance R_1 with Thevenin's generator (in Fig. 1.9 it is circled by a dashed line). Thevenin's generator (Fig. 1.10) is characterized by the value of EMF (EMF of Thevenin's generator E_{eg}) and internal (equivalent resistance) R_{eg}.

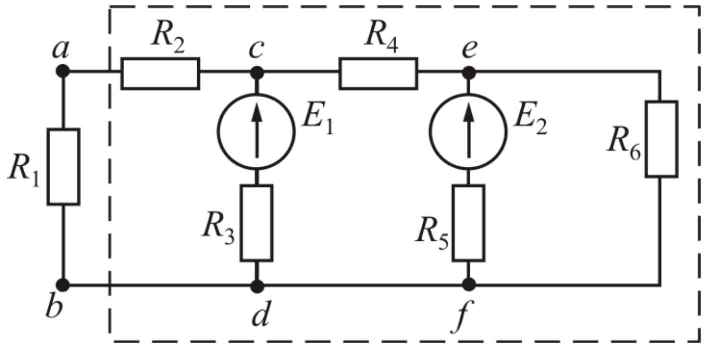


Figure 1.9

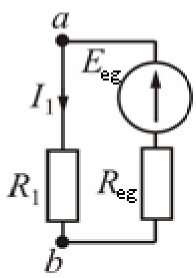


Figure 1.10

The EMF is equal to the no-load voltage $U_{ab\,nl}$ across the open terminals *a-b* of the circuit (Fig. 1.11), $E_{eg}=U_{ab\,nl}$. The total equivalent resistance R_{eg} is equal to the input resistance of the circuit according to Fig. 1.9 between the terminals *a-b,* when the circuit becomes passive (Fig. 1.12), i.e. all EMF are equal zero, and in the branches instead of real EMF their internal resistances are stored.

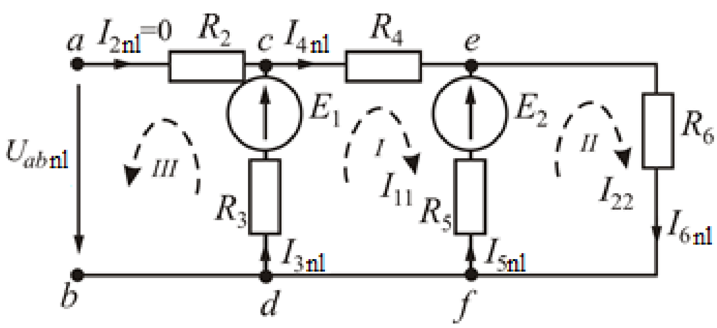


Figure 1.11

Current I_1 in the given circuit (Fig. 1.16) is determined by the formula

$$I_1 = \frac{E_{eg}}{R_1 + R_{eg}} = \frac{U_{ab\,nl}}{R_1 + R_{eg}}. \qquad (1)$$

For the circuit shown in Fig. 1.11, we determine the voltage $U_{ab\,nl}$ according to Kirchhoff's voltage law for the loop *a-c-d-b-a* (circuit III) when bypassing it counterclockwise

$E_1 = I_{3nl} \cdot R_3 + U_{ab\,nl}$, where

$$U_{ab\,nl} = E_1 - I_{3nl} \cdot R_3. \qquad (2)$$

To determine the current, I_{3nl} we use the mesh current method.

For two independent loops I and II, by bypassing them clockwise, we can write the equation according to Kirchhoff's voltage law in general:

$$\begin{cases} R_{11} \cdot I_{11} + R_{12} \cdot I_{22} = E_{11}; \\ R_{21} \cdot I_{11} + R_{22} \cdot I_{22} = E_{22}, \end{cases} \qquad (3)$$

where the loop EMF:

$E_{11} = E_1 - E_2 = 120 - 60 = 60$ V;

$E_{22} = E_2 = 60$ V,

total (own) resistances of loops:

$R_{11} = R_3 + R_4 + R_5 = 4 + 6 + 10 = 20$ Ohm;

$R_{22} = R_5 + R_6 = 10 + 15 = 25$ Ohm,

mutual resistance of loops: $R_{12} = R_{21} = -R_5 = -10$ Ohm.

After substituting the numerical values, the system of equations (3) has the form:

$$\begin{cases} 20 \cdot I_{11} - 10 \cdot I_{22} = 60; \\ -10 \cdot I_{11} + 25 \cdot I_{22} = 60. \end{cases}$$

The general determinant of the system

$$\Delta = \begin{vmatrix} 20 & -10 \\ -10 & 25 \end{vmatrix} = 20 \cdot 25 - (-10) \cdot (-10) = 400.$$

The determinant of the system needed to find the current $I_{11} = I_{3nl}$:

$$\Delta_1 = \begin{vmatrix} 60 & -10 \\ 60 & 25 \end{vmatrix} = 60 \cdot 25 - 60 \cdot (-10) = 2100.$$

Then $I_{11} = I_{3nl} = \frac{\Delta_1}{\Delta} = \frac{2100}{400} = 5{,}25$ A.

After substituting the numerical values in equation (2):

$$U_{ab\,nl} = E_1 - I_{3nl} \cdot R_3 = 120 - 5{,}25 \cdot 4 = 99 \text{ V}.$$

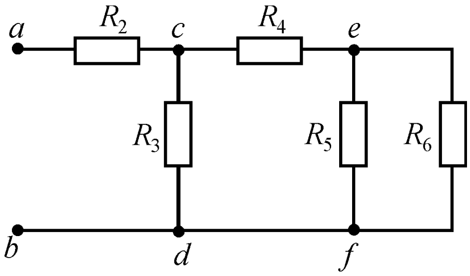


Figure 1.12

We determine the equivalent resistance R_{eg} for Fig. 1.12:

$$R_{56} = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{10 \cdot 15}{10 + 15} = 6 \text{ Ohm};$$

$$R_{456} = R_4 + R_{56} = 6 + 6 = 12 \text{ Ohm};$$

$$R_{eg} = R_2 + \frac{R_3 \cdot R_{456}}{R_3 + R_{456}} = 5 + \frac{4 \cdot 12}{4 + 12} = 8 \text{ Ohm}.$$

We determine the current I_1

$$I_1 = \frac{U_{ab\,nl}}{R_1 + R_{eg}} = \frac{99}{10 + 8} = 5{,}5 \text{ A}.$$