2. CALCULATION OF ELECTRIC CIRCUITS OF SINUSOID CURRENT PRACTICE TOPIC № 2

2.1. Formulation of tasks

2.1. The formulation of the tasks of practice topic 2 consists of the next tasks.

The content of the assignment, i.e. the total number of tasks and their specific numbers, may be varied by the teacher depending on the scope of the training course.

Each student is given a variant of input for three loads (Z_α, Z_β, Z_γ), used in all tasks (Table 2.1).

Task 1. Calculation of an electric circuit of a single-phase sinusoidal current with one source of electrical energy at the serial connection of loads.

To calculate the electrical circuit in Fig. 2.1 with a series connection of loads Z_α, Z_β, Z_γ, parameters which must be taken from the Table 2.1. The energy source voltage is U = 220 V at a frequency f = 50 Hz.

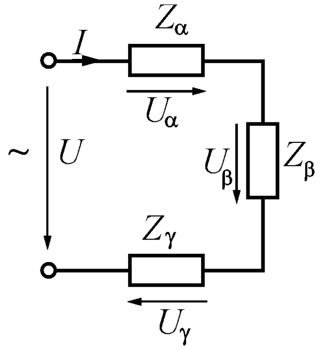

Figure 2.1

Calculation volume: a) draw the electrical scheme in Fig. 2.1 with the ideal elements of substitution of each load; b) determine the current in the circuit, the voltages across each load, active and reactive power of the source and loads; c) build a phasor diagram of the current and voltages of the source and all loads; d) check the solution of the task with the help of the phasor diagram and balance of active and reactive power; e) build the sinusoidal time functions of the current and voltage of the energy source and plot these functions.

Task 2. Calculation of an electric circuit of a single-phase sinusoidal current with one source of electrical energy at the parallel connection of loads.

Calculate the electrical circuit in Fig. 2.2 with parallel connection of loads Z_α, Z_β, Z_γ, the parameters of which are defined in Task 1. Source voltage U = 127 V at a frequency f = 50 Hz.

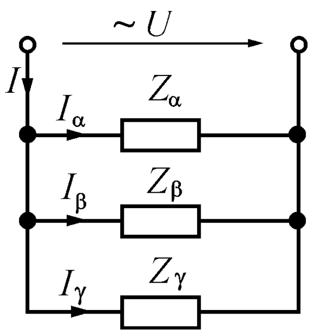

Figure 2.2

Calculation volume: a) draw the electrical circuit in Fig. 2.2 with the ideal elements of replacement for each load; b) determine all currents in the circuit, active and reactive power of the source and loads; c) build a phasor diagram of voltage and currents of all loads; d) check the solution of the task by means of the phasor diagram and balance of active and reactive power.

Task 3. Calculation of an electric circuit of single-phase sinusoidal current with one energy source using the symbolic method.

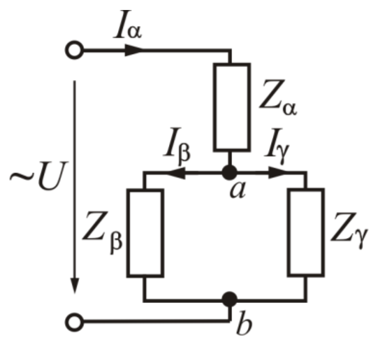

Figure 2.3

Carry out the calculation of the electrical circuit in Fig. 2.3 with a mixed connection of loads Z_α, Z_β, Z_γ, which parameters are defined in Task 1. The voltage of the energy source is U = 220 V at frequency f = 50 Hz.

Calculation volume: a) draw the equivalent electrical scheme in Fig. 2.3 with ideal elements instead of each load; b) determine the currents in the circuit, active and reactive power of the energy source and loads; c) build a phasor diagram of voltages and currents; d) check the solution of the task using a phasor diagram and the balance of active and reactive powers.

Task 4. Calculation of a three-phase electric circuit with a symmetrical wye connected load.

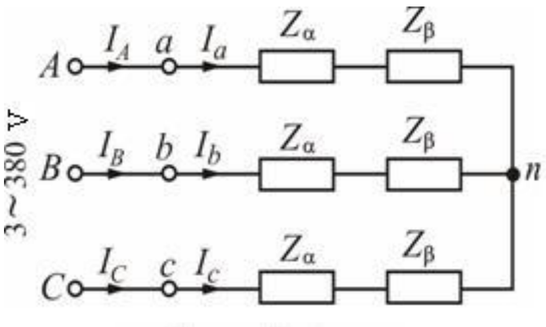

Figure 2.4

In a three-phase circuit with linear voltage U_L = 380 V a symmetrical load is switched at series connection of the loads Z_α , Z_β in each phase (Fig. 2.4). The loads Z_α , Z_β are the same as in from Task 1.

Calculation volume: a) draw the electrical scheme of replacement of three-phase circuit with ideal elements instead of loads; b) determine phase and linear currents, as well as active and reactive power; c) construct a phasor diagram.

Task 5. Calculation of a three-phase electric circuit at a symmetrical delta connected load.

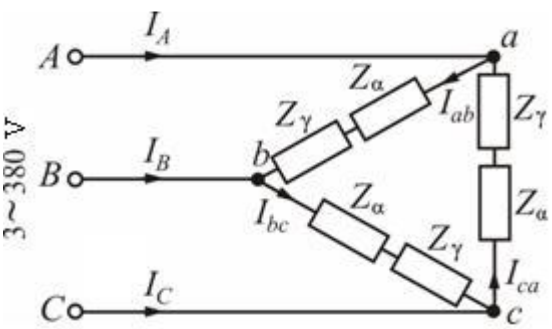

Figure 2.5

In a three-phase circuit with linear voltage U_L = 380 V a symmetrical load is switched on at serial connection of the loads Z_α, Z_γ in each phase (Fig. 2.5). The loads Z_α , Z_γ take from Task 1.

Calculation volume: a) draw the electric equivalent scheme of three-phase electric circuit with ideal elements instead of the loads; b) determine phase and linear currents, as well as active and reactive power; c) build a phasor diagram.

Task 6. Calculation of a three-phase electric circuit at asymmetric load and wye connection of loads with a neutral wire.

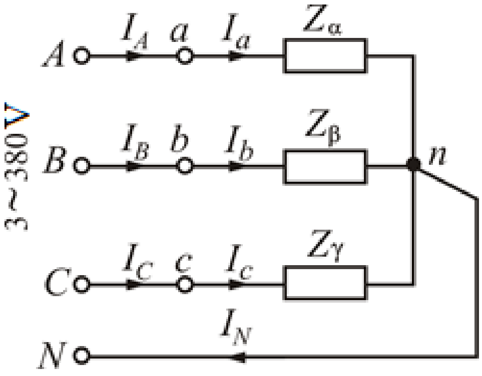

Figure 2.6

In a three-phase network with linear voltage $U_L = 380$ V at frequency $f = 50$ Hz loads the Z_α, Z_β, Z_γ are switched on in phases (Fig. 2.6), which due to their inequality leads to an asymmetric load. The loads parameters Z_α, Z_β, Z_γ are taken from Task 1.

Calculation volume: a) draw the electrical circuit of a three-phase circuit with ideal elements instead of loads; b) determine the phase and line currents, as well as the current in the neutral wire; c) determine the active and reactive power; d) build a phasor diagram.

Table 2.1 – Input data of AC load parameters

Variant	Z_α			Z_β			Z_γ		
	R_α, Ohm	L_α, mH	C_α, uF	R_β, Ohm	L_β, mH	C_β, uF	R_γ, Ohm	L_γ, mH	C_γ, uF
1	12	50,96	-	12	-	199,04	-	76,43	-
2	7	76,43	-	5	-	265,39	20	-	-
3	20	-	212,31	10	95,54	-	-	-	127,39
4	25	-	-	12	50,96	-	10	-	90,99
5	8	19,11	-	-	-	176,93	10	63,69	-
6	-	79,62	-	20	47,77	-	-	-	127,4
7	12	50,96	-	12	-	79,62	-	152,87	-
8	5	-	265,39	15	63,69	-	25	-	-
9	16	38,22	-	7	76,43	-	-	-	63,69
10	12	50,96	-	12	-	199,04	-	-	133
11	-	-	144,76	24	101,91	-	12	15,92	-
12	7	76,43	-	12	-	636,94	-	-	83,81
13	-	63,69	-	24	22,29	-	16	-	265,39
14	10	-	-	12	-	199,04	7	76,43	-
15	12	50,96	-	8	-	530,79	-	-	106,16
16	24	-	176,93	6	25,48	-	-	101,91	-
17	20	-	-	15	63,69	-	5	-	265,39
18	-	63,69	-	10	-	318,47	10	31,85	-
19	18	76,43	-	-	-	79,62	12	-	199,04
20	17	25,48	-	5	38,22	-	-	-	199,52
21	-	-	96,51	30	41,4	-	15	63,69	-
22	15	25,48	-	5	38,22	-	-	-	159,24

Variant	$Z_α$			$Z_β$			$Z_γ$		
	$R_α$, Ohm	$L_α$, mH	$C_α$, uF	$R_β$, Ohm	$L_β$, mH	$C_β$, uF	$R_γ$, Ohm	$L_γ$, mH	$C_γ$, uF
23	27	114,65	-	-	-	63,69	13	44,59	-
24	9	41,4	-	13	28,66	-	-	-	106,16
25	7	-	132,7	7	76,43	-	25	-	-
26	-	105,1	-	20	-	212,31	24	-	454,96
27	12	-	199,04	12	50,96	-	20	-	-
28	-	-	90,99	20	47,77	-	20	-	212,31
29	16	38,22	-	12	50,96	-	-	-	199,04
30	-	-	79,62	20	63,69	-	20	63,69	-
31	20	-	-	7	-	132,7	5	38,22	-
32	12		199,04	12	50,96	-	22	-	-
33	15	63,69	-	20	47,77	-	-	-	144,76
34	30	-	79,62	30	127,39	-	-	159,24	-
35	-	79,62	-	10	-	212,31	15	-	318,47
36	6	25,48	-	16	38,22	-	-	-	72,38
37	-	-	95,51	15	41,4	-	15	63,69	-
38	24	57,32	-	15	31,85	-	-	-	113,74
39	18	-	132,7	10	-	212,31	-	124,2	-
40	11	-	167,17	11	60,67	-	-	121,34	-
41	24	57,32	-	24	-	176,93	-	-	114,65
42	18	-	132,7	18	76,43	-	14	-	-
43	-	127,39	-	30	-	79,62	15	-	159,24
44	5	-	265,39	15	63,69	-	10	-	-
45	12	50,96	-	8	-	530,78	-	-	106,16
46	6	25,48	-	16	38,22	-	-	-	159,24
47	7	76,43	-	-	-	66,35	12	-	636,94
48	-	-	106,16	20	47,77	-	10	143,31	-
49	8	19,11	-	12	50,96	-	-	-	144,76
50	14	-	66,35	14	152,87	-	50	-	-
51	20	95,54	-	10	31,85	-	-	-	159,24
52	-	-	127,39	20	-	212,31	20	127,39	-
53	24	-	99,52	24	101,92	-	40	-	-
54	-	105,1	-	30	-	244,98	20	-	159,24
55	30	-	212,31	10	47,77	-	-	79,62	-
56	-	63,69	-	20	63,69	-	20	-	212,31
57	16	-	132,7	-	-	132,7	24	76,43	-
58	18	82,8	-	26	57,32	-	-	-	53,08
59	20	-	-	18	-	132,7	18	76,43	-
60	-	-	117,96	10	117,83	-	12	-	318.47
61	40	95,54	-	7	76,43	-	-	-	318,47

Variant	Z_α			Z_β			Z_γ		
	R_α, Ohm	L_α, mH	C_α, uF	R_β, Ohm	L_β, mH	C_β, uF	R_γ, Ohm	L_γ, mH	C_γ, uF
62	20	-	-	12	-	199,04	12	50,96	-
63	25	111,46	-	20	-	212,31	22	-	-
64	22	-	-	5	38,22	-	5	-	265,39
65	-	159,42	-	40	95,54	-	14	-	66,35
66	18	76,43	-	18	-	-	9	-	79,62
67	-	-	159,24	30	111,46	-	20	-	212,31
68	18	-	132,7	18	-	-	9	127,39	-
69	20	-	212,31	30	111,46	-	-	-	159,24
70	40	-	106,16	7	-	132,7	-	31,85	-
71	20	-	-	12	50,96	-	12	-	199,04
72	-	127,39	-	20	-	159,24	20	63,69	-
73	15	-	66,35	15	146,5	-	50	-	-
74	-	114,65	-	20	-	212,31	20	-	88,46
75	20	63,69	-	25	-	159,24	-	114,65	-
76	8	19,11	-	8	-	530,79	20	-	-
77	-	-	127,39	25	15,92	-	25	-	636,94
78	40	-	79,62	-	127,39	-	20	63,69	-
79	17	31,85	-	12	15,92	-	-	-	127,39
80	-	-	117,95	27	104,14	-	15	63,69	-
81	-	159,24	-	24	-	132,7	20	-	79,62
82	-	-	95,64	20	58,28	-	20	-	174,03
83	10	-	183,87	10	55,16	-	30	-	-
84	12	38,6	-	18	-	-	30	-	79,62
85	-	-	83,81	19	60,51	-	11	63,69	-
86	12	-	199,04	25	-	-	12	50,96	-
87	25	-	-	12	50,96	-	9	-	79,62
88	22	-	-	22	-	83,59	22	121,34	-
89	-	89,17	-	28	-	113,74	12	50,96	-
90	35	-	90,99	30	-	-	10	31,85	-
91	12	50,96	-	20	-	212,31	-	-	144,76
92	18	-	132,7	-	63,69	-	20	47,77	-
93	-	70,06	-	24	-	454,96	25	-	127,39
94	-	-	144,76	27	88,46	-	15	-	398,09
95	16	28,66	-	-	-	144,76	12	-	636,94
96	-	-	127,39	20	47,77	-	15	-	159,24
97	-	79,62	-	15	-	159,24	7	76,43	-
98	-	-	144,76	18	76,43	-	12	50,96	-
99	20	-	212,31	-	70,06	-	18	-	132,76
100	15	63,69	-	22	-	83,59	25	-	-

2.2. Examples of tasks solution

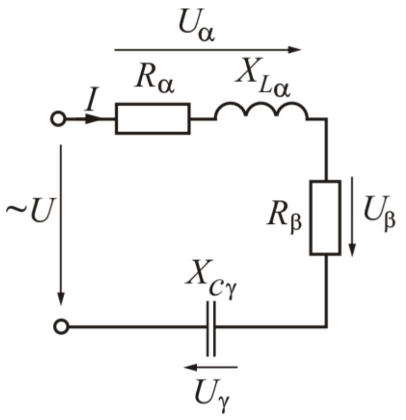

Figure 2.7

Example 1. Calculate an electrical circuit with a series connection of loads Z_α, Z_β and Z_γ, the scheme of which is shown in Fig. 2.1. The current voltage value $U = 200$ V, its frequency $f = 50$ Hz. Parameters of ideal elements of the loads: $R_\alpha = 20$ Ohm; $L_\alpha = 47{,}77$ mH; $R_\beta = 23{,}3$ Ohm; $C_\gamma = 79{,}62$ uF.

Calculation volume: a) draw the electrical scheme in Fig. 2.1 with the ideal elements of substitution of each load; b) determine current in the circuit, voltages across each load, active and reactive power of the source and loads; c) build a phasor diagram of the current and voltages of the source and all loads; d) check the solution of the task with the help of phasor diagram and balance of active and reactive power; e) construct the sinusoidal time functions of the current and voltage of the energy source and draw these functions.

Angular frequency of current and voltage:

$$\omega = 2\pi \cdot f = 2 \cdot 3{,}14 \cdot 50 = 314 \text{ c}^{-1}.$$

Inductive reactance of the load Z_α:

$$X_{L\alpha} = \omega \cdot L_\alpha = 314 \cdot 47{,}77 \cdot 10^{-3} = 15 \text{ Ohm}.$$

Capacitive reactance of the load Z_γ:

$$X_{C\gamma} = \frac{1}{\omega \cdot C_\gamma} = \frac{1}{314 \cdot 79{,}62 \cdot 10^{-6}} = 40 \text{ Ohm}.$$

Resistance, reactance and impedance:

$$R = R_\alpha + R_\beta = 20 + 23{,}3 = 43{,}3 \text{ Ohm}; \quad X = X_{L\alpha} - X_{C\gamma} = 15 - 40 = -25 \text{ Ohm};$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{43{,}3^2 + (-25)^2} = 50 \text{ Ohm}.$$

Current in the electrical circuit:

$$I = \frac{U}{Z} = \frac{200}{50} = 4 \text{ A}.$$

The phase shift angle between the current phasors and the source voltage:

$$\varphi = \text{arctg}\frac{X}{R} = \text{arctg}\frac{-25}{43{,}3} = -30^\circ.$$

Impedances of loads:

$$Z_\alpha = \sqrt{R_\alpha^2 + X_{L\alpha}^2} = \sqrt{20^2 + 15^2} = 25 \text{ Ohm};$$

$$Z_\beta = R_\beta = 23{,}3 \text{ Ohm}; \quad Z_\gamma = X_{C\gamma} = 40 \text{ Ohm}.$$

Voltages across the loads:

$U_\alpha = I\,Z_\alpha = 4 \cdot 25 = 100$ V; $U_\beta = I\,Z_\beta = 4 \cdot 23{,}3 = 93{,}2$ V;

$U_\gamma = I\,Z_\gamma = 4 \cdot 40 = 160$ V.

Angles of phase shift between current phasor $\underline{I}$ and voltages phasors $\underline{U}_\alpha, \underline{U}_\beta, \underline{U}_\gamma$ across the loads:

$$\varphi_\alpha = \text{arctg}\frac{X_{L\alpha}}{R_\alpha} = \text{arctg}\frac{15}{20} = 36{,}87^\circ;$$

$$\varphi_\beta = \text{arctg}\frac{0}{R_\beta} = \text{arctg}\frac{0}{23{,}3} = 0^\circ;$$

$$\varphi_\gamma = \text{arctg}\frac{-X_{C\gamma}}{0} = \text{arctg}\frac{-40}{0} = -90^\circ.$$

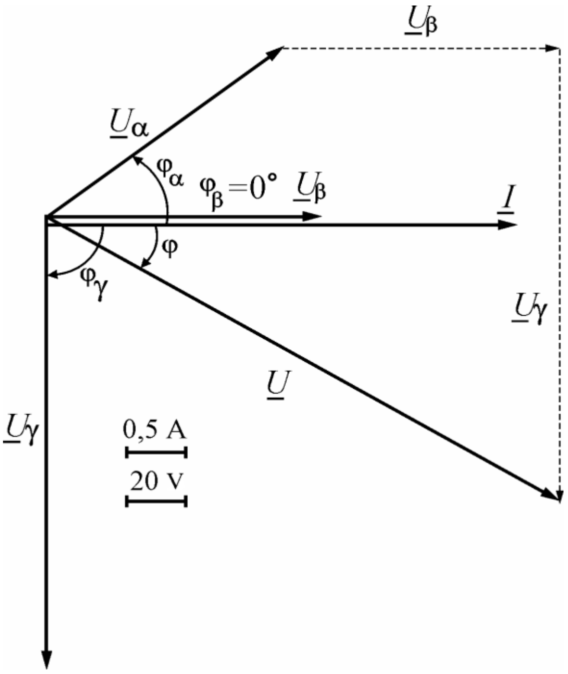

Figure 2.8

Figure 2.8 shows a phasor diagram. Let's explain the order of solving.

For the circuit in Fig. 2.7 common is the current for which we take the initial phase $\psi_i = 0^\circ$. Choose the current scale and draw a phasor $\underline{I}$ along the horizontal axis. Next, choose the voltage scale and draw the voltage phasor of the source at an angle φ clockwise relative to the current phasor $\underline{I}$, as well as voltage phasors across the loads $\underline{U}_\alpha, \underline{U}_\beta, \underline{U}_\gamma$ at the appropriate phase shift angles φ_α, φ_β, φ_γ relative to the phasor $\underline{I}$.

Using the phasor diagram, we check the solution based on Kirchhoff's voltage law for a loop in Fig. 2.7. The phasor sum of load's voltages gives the voltage phasor of the energy source, i.e. $\underline{U} = \underline{U}_\alpha + \underline{U}_\beta + \underline{U}_\gamma$ (Fig. 2.8), which on the accepted scale must be equal to 200 V.

Total, active and reactive power of the energy source:

$$S = U \cdot I = 200 \cdot 4 = 800 \text{ VA};$$

$$P = U \cdot I \cdot \cos\varphi = 200 \cdot 4 \cdot \cos(-30^\circ) = 692{,}8 \text{ W};$$

$$Q = U \cdot I \cdot \sin\varphi = 200 \cdot 4 \cdot \sin(-30^\circ) = -400 \text{ VAr}.$$

The ratio of these powers is shown in Fig. 2.9 using their triangle.

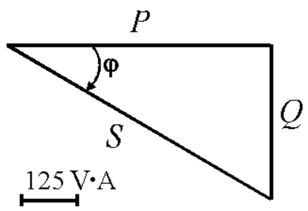

Figure 2.9

According to the balance of active powers

$$P = R_\alpha \cdot I^2 + R_\beta \cdot I^2 = 20 \cdot 4^2 + 23{,}3 \cdot 4^2 = 692{,}8 \text{ W},$$

which coincides with the active power of the source.

According to the balance of active powers

$$P = R_\alpha \cdot I^2 + R_\beta \cdot I^2 = 20 \cdot 4^2 + 23{,}3 \cdot 4^2 = 692{,}8 \text{ W},$$

which coincides with the active power of the source.

According to the balance of reactive powers

$$Q = X_{L\alpha} \cdot I^2 - X_{C\gamma} \cdot I^2 = 15 \cdot 4^2 - 40 \cdot 4^2 = -400 \text{ VAr},$$

which coincides with the reactive power of the source.

Amplitude values of current and voltage of the energy source

$$I_m = \sqrt{2} \cdot I = \sqrt{2} \cdot 4 = 5{,}66 \text{ A}; \quad U_m = \sqrt{2} \cdot U = \sqrt{2} \cdot 200 = 282 \text{ V}.$$

Sinusoidal time functions of current and voltage of the energy source:

$$i = I_m \cdot \sin(\omega t + \psi_i) = 5{,}66 \cdot \sin 314\, t \text{ A};$$

$$u = U_m \sin(\omega t + \psi_u) = 282{,}8 \cdot \sin(314t - 30^\circ) \text{ V},$$

where the initial phase of the current is already accepted ($\psi_i = 0^\circ$), and the initial phase ψ_u find, remembering that the angle $\varphi = \psi_u - \psi_i$: $\psi_u = \varphi + \psi_i = -30^\circ + 0^\circ = -30^\circ$.

Graphs of current and voltage sine curves are drawn when using no time scale *t*, and a multiple of its scale ω*t*, which is more convenient when constructing these graphs. In the Table 2.2 shows the calculations of current and voltage for different values ω*t* and in Fig. 2.10 shows the graphs of functions $i(t)$ and $u(t)$.

Table 2.2 – The value of current and voltage of the energy source depending on time

ωt, rad	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
ωt, grad	0	30	60	90	120	150	180	210	240	270	300	330	360
i,A	0	2,83	4,9	5,66	4,9	2,83	0	-2,83	-4,9	-5,66	-4,9	-2,83	0
u,V	-141	0	141	245	283	245	141	0	-141	-245	-283	-245	-141

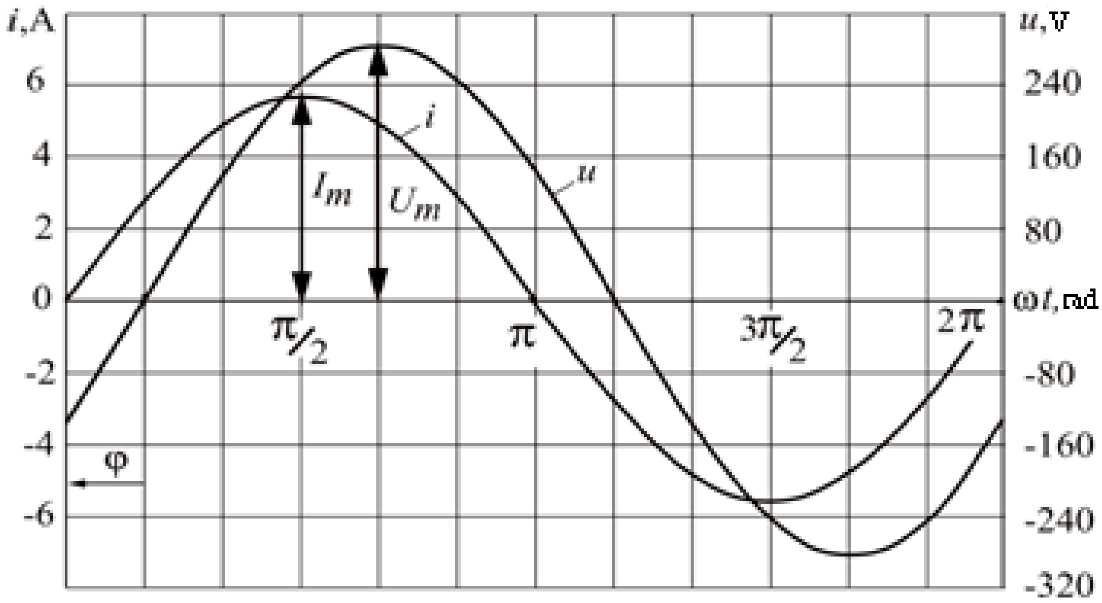

Figure 2.10

Example 2. Calculate the electrical circuit with a parallel connection of the loads Z_α, Z_β and Z_γ, the electrical scheme of which is shown in Fig. 2.2. The voltage value $U = 220$ V. Parameters of ideal elements of the loads: $R_\alpha = 24$ Ohm; $X_{C\alpha} = 32$ Ohm; $R_\beta = 20$ Ohm; $X_{L\beta} = 20$ Ohm; $X_{L\gamma} = 80$ Ohm.

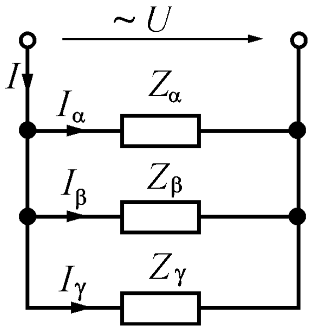

Figure 2.2

Calculate the electrical circuit in Fig. 2.2 with parallel connection of loads Z_α, Z_β, Z_γ, the parameters of which are defined in Task 1. The source voltage U = 220 V at a frequency of f = 50 Hz.

Calculation volume: a) draw the electrical scheme in Fig. 2.2 with the ideal elements of replacement for each load; b) determine currents in the circuit, active and reactive power of the source and loads; c) draw a phasor diagram of voltage and currents of all loads; d) check the solution of the task by means of a phasor diagram and balance of active and reactive power.

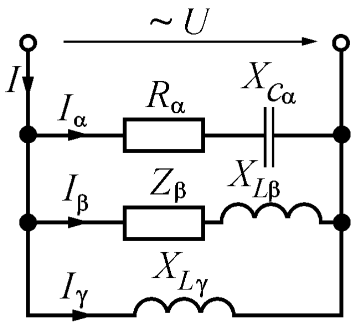

Figure 2.11

Solution. In the Fig. 2.11 is shown the electrical scheme of the substitution of the electricalal circuit in Fig. 2.2 with ideal elements instead Z_α, Z_β, Z_γ.

The impedances value of the parallel branches are equal to:

$$Z_\alpha = \sqrt{R_\alpha^2 + (-X_{C\alpha})^2} = \sqrt{24^2 + (-32)^2} = 40 \text{ Ohm};$$

$$Z_\beta = \sqrt{R_\beta^2 + X_{L\beta}^2} = \sqrt{20^2 + 20^2} = 28{,}28 \text{ Ohm};$$

$$Z_\gamma = X_{L\gamma} = 80 \text{ Ohm}.$$

The currents in the parallel branches:

$$I_\alpha = \frac{U}{Z_\alpha} = \frac{220}{40} = 5{,}5 \text{ A}; \quad I_\beta = \frac{U}{Z_\beta} = \frac{220}{28{,}28} = 7{,}78 \text{ A};$$

$$I_\gamma = \frac{U}{Z_\gamma} = \frac{220}{80} = 2{,}75 \text{ A}.$$

Angles of phase shift between the current phasors $\underline{I}_\alpha, \underline{I}_\beta, \underline{I}_\gamma$ and voltage phasor $\underline{U}$:

$$\varphi_\alpha = \text{arctg}\frac{-X_{C\alpha}}{R_\alpha} = \text{arctg}\frac{-32}{24} = -53{,}13^\circ;$$

$$\varphi_\beta = \text{arctg}\frac{X_{L\beta}}{R_\beta} = \text{arctg}\frac{20}{20} = 45^\circ;$$

$$\varphi_\gamma = 90^\circ \text{ (inductive element)}.$$

To determine the source current, we use a phasor diagram (Fig. 2.12), where the phasors are built at these scales. We are the first to build a voltage phasor $\underline{U}$. Its initial phase is accepted arbitrarily ($\psi_u = 0^\circ$). Current phasors $\underline{I}_\alpha, \underline{I}_\beta$ and $\underline{I}_\gamma$ we construct using the initial phases. Because $\psi_u = 0^\circ$, the initial phases are determined by formulas:

$$\psi_{i_\alpha} = -\varphi_\alpha = 53{,}13^\circ; \qquad \psi_{i_\beta} = -\varphi_\beta = -45^\circ; \qquad \psi_{i_\gamma} = -\varphi_\gamma = -90^\circ.$$

According to Kirchhoff's current law (Fig. 2.11), the phasor of the source current is the phasor sum of the currents of the parallel branches, i.e. $\underline{I} = \underline{I}_{\alpha} + \underline{I}_{\beta} + \underline{I}_{\gamma}$ (Fig. 2.12). The geometric addition of these phasors gives the value of the source current on the basis of measurements $I \approx 9{,}7\,\text{A}$ and the phase shift angle $\varphi \approx 24^{\circ}$.

More accurate results can be obtained directly by calculations, using the decomposition of current phasors into active and reactive components: the first – parallel to the phasor $\underline{U}$, and the second is perpendicular to it. The diagram of phasors on components is given on the phasor diagram (Fig. 2.12).

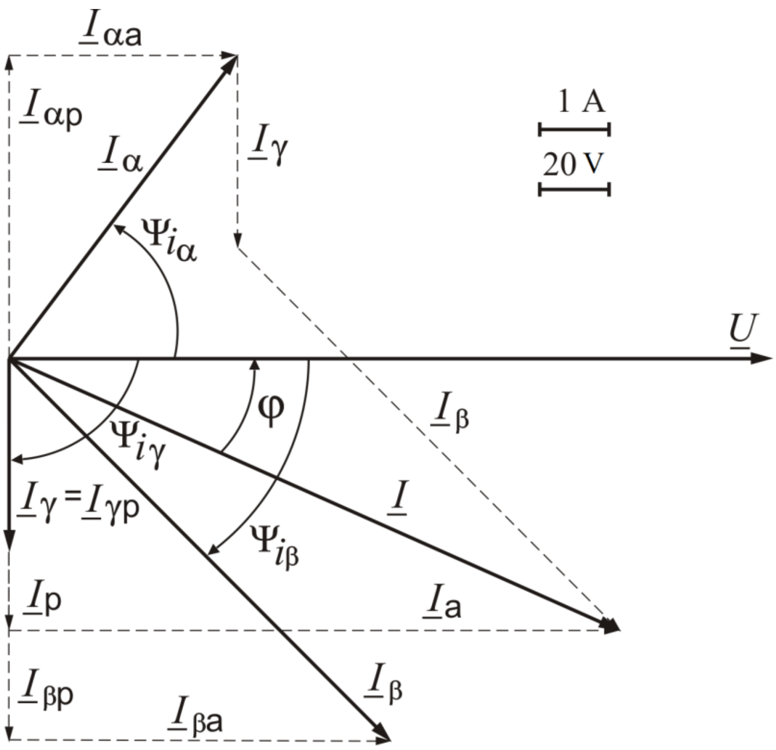

Figure 2.12

Active and reactive components of the load currents:

$$I_{\alpha a} = I_{\alpha} \cos \psi_{i_{\alpha}} = 5{,}5 \cos 53{,}13^{\circ} = 3{,}3 \ \text{A};$$

$$I_{\beta a} = I_{\beta} \cos \psi_{i_{\beta}} = 7{,}78 \cos(-45^{\circ}) = 5{,}5 \ \text{A};$$

$$I_{\gamma a} = I_{\gamma} \cos \psi_{i_{\gamma}} = 2{,}75 \cos(-90^{\circ}) = 0 \ \text{A};$$

$$I_{\alpha p} = I_{\alpha} \sin \psi_{i_{\alpha}} = 5{,}5 \sin 53{,}13^{\circ} = 4{,}4 \ \text{A};$$

$$I_{\beta p} = I_{\beta} \sin \psi_{i_{\beta}} = 7{,}78 \sin(-45^{\circ}) = -5{,}5 \ \text{A};$$

$$I_{\gamma p} = I_{\gamma} \sin \psi_{i_{\gamma}} = 2{,}75 \sin(-90^{\circ}) = -2{,}75 \ \text{A}.$$

Active and reactive components of the source current and its rms(root-mean-square) value:

$$I_{a} = I_{\alpha a} + I_{\beta a} + I_{\gamma a} = 3{,}3 + 5{,}5 + 0 = 8{,}8\,\text{A};$$

$$I_p = I_{\alpha p} + I_{\beta p} + I_{\gamma p} = 4{,}4 - 5{,}5 - 2{,}75 = -3{,}85\,\text{A};$$

$$I = \sqrt{I_a^2 + I_p^2} = \sqrt{8{,}8^2 + (-3{,}85)^2} = 9{,}61\,\text{A}.$$

The phase shift angle of the current phasor $\underline{I}$ relative to the voltage phasor $\underline{U}$:

$$\varphi = -\psi_i = -\text{arctg}\frac{I_p}{I_a} = -\text{arctg}\frac{-3{,}85}{8{,}8} = 23{,}63^{\text{o}}.$$

Total, active and reactive source powers:

$$S = U \cdot I = 220 \cdot 9{,}61 = 2114{,}2\ \text{VA};\ P = S\cos\varphi = 2114{,}2 \cdot \cos 23{,}63^{\text{o}} = 1936{,}9\ \text{W};$$

$$Q = S\sin\varphi = 2114{,}2 \cdot \sin 23{,}63^{\circ} = 847{,}4\,\text{VAr}.$$

According to the balance of active and reactive powers:

$$P = R_\alpha \cdot I_\alpha^2 + R_\beta \cdot I_\beta^2 = 24 \cdot 5{,}5^2 + 20 \cdot 7{,}78^2 = 1936{,}6\ \text{W},$$

$$Q = -X_{C\alpha} \cdot I_\alpha^2 + X_{L\beta} \cdot I_\beta^2 + X_{L\gamma} \cdot I_\gamma^2 = -32 \cdot 5{,}5^2 + 20 \cdot 7{,}78^2 + 80 \cdot 2{,}75^2 = 847{,}6\ \text{VAr}.$$

Example 3. Calculate the electrical circuit in Fig. 2.3 at the mixed connection of the loads Z_α, Z_β and Z_γ by a symbolic method, i.e. using complex numbers. The energy source voltage $U = 127$ V, its frequency $f = 50$ Hz. Parameters of the ideal elements of loads: $R_\alpha = 8$ Ohm; $X_{L\alpha} = 15$ Ohm; $R_\beta = 16$ Ohm; $X_{C\beta} = 12$ Ohm; $X_{L\gamma} = 42$ Ohm.

In a three-phase circuit with linear voltage U_L = 380 V symmetrical load is activated when connected in series with the loads Z_α, Z_β in each phase (Fig. 2.3). The loads Z_α, Z_β will be taken from Task 1.

Calculation volume: a) draw the electrical scheme of replacement of three-phase circuit with ideal elements instead of the loads $Z_\alpha, Z_\beta, Z_\gamma$; b) determine phase and linear currents, as well as active and reactive powers; c) draw a phasor diagram.

Solution. The electrical scheme of substitution of the electrical circuit in Fig. 2.3 with ideal elements is shown in Fig. 2.13. Currents, voltages and also loads will be defined by a symbolic method therefore ideal elements which are connected in series, unite in total complex impedances. From the initial electrical scheme according to Fig. 2.13 we come to the electrical scheme with complex impedances of branches (Fig. 2.14, *a*) where currents and voltages are also represented in a complex form.

Values of complex impedances of branches in algebraic and demonstrative forms:

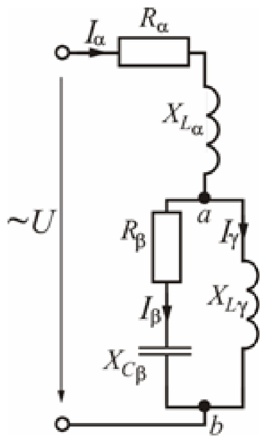

Figure 2.13

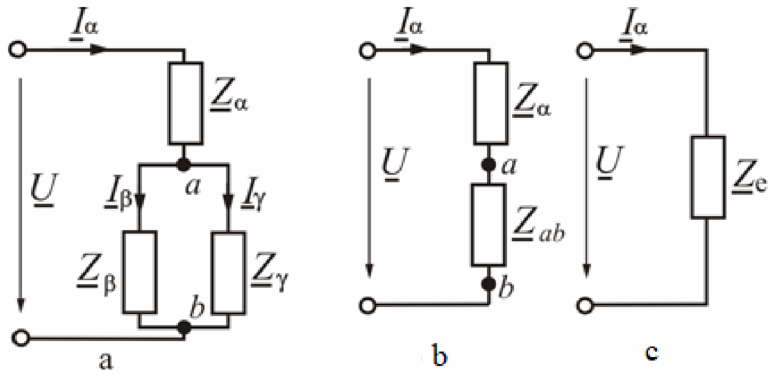

Figure 2.14

$$\underline{Z}_\alpha = R_\alpha + jX_{L\alpha} = 8 + j15 = \sqrt{8^2 + 15^2}\cdot e^{j\,\mathrm{arctg}\frac{15}{8}} = 17\cdot e^{j61{,}93^\circ};$$

$$\underline{Z}_\beta = R_\beta - jX_{C\beta} = 16 - j12 = \sqrt{16^2 + (-12)^2}\cdot e^{j\,\mathrm{arctg}\frac{-12}{16}} = 20\cdot e^{-j36{,}87^\circ}\ \text{Ohm};$$

$$\underline{Z}_\gamma = jX_{L\gamma} = j42 = 42\cdot e^{j\,\mathrm{arctg}\frac{42}{0}} = 42\cdot e^{j90^\circ}\ \text{Ohm}.$$

A part of electrical circuit with a parallel connection $\underline{Z}_\beta$ and $\underline{Z}_\gamma$ replace with equivalent complex impedance

$$\underline{Z}_{ab} = \frac{\underline{Z}_\beta\cdot\underline{Z}_\gamma}{\underline{Z}_\beta + \underline{Z}_\gamma} = \frac{20\cdot e^{-j36{,}87^\circ}\cdot 42\cdot e^{j90^\circ}}{16 - j12 + j42} = \frac{840\cdot e^{j53{,}13^\circ}}{16 + j30} = \frac{840\cdot e^{j53{,}13^\circ}}{\sqrt{16^2 + 30^2}\cdot e^{j\mathrm{arctg}\frac{30}{16}}} =$$

$$= \frac{840\cdot e^{j53{,}13^\circ}}{34\cdot e^{j61{,}93^\circ}} = 24{,}71\cdot e^{-j8{,}8^\circ} = 24{,}71\left[\cos(-8{,}8^\circ) + j\sin(-8{,}8^\circ)\right] = 24{,}42 - j3{,}78\ \text{Ohm}$$

and we obtain a simpler scheme for Fig. 2.14, *b*.

Two loads connected in series $\underline{Z}_\alpha$ and $\underline{Z}_{ab}$ for Fig. 2.14, *b* is replaced by one impedance, which is equivalent to the impedance of the whole circuit (Fig. 2.14, *c*)

$$\underline{Z}_e = \underline{Z}_\alpha + \underline{Z}_{ab} = 8 + j15 + 24{,}42 - j3{,}78 = 32{,}42 + j11{,}22 =$$

$$= \sqrt{32{,}42^2 + 11{,}22^2}\cdot e^{j\mathrm{arctg}\frac{11{,}22}{32{,}42}} = 34{,}3\cdot e^{j19{,}09^\circ}\ \text{Ohm}.$$

The complex value of source voltage $\underline{U} = U\cdot e^{j\psi_u} = 127\cdot e^{j0^\circ} = 127$ V, if the initial phase of this voltage is accepted $\psi_u = 0^\circ$.

Current source according to Ohm's law (Fig. 2.14, *c*)

$$\underline{I}_\alpha = \frac{\underline{U}}{\underline{Z}_e} = \frac{127\cdot e^{j0^\circ}}{34{,}3\cdot e^{j19{,}09^\circ}} = 3{,}7\cdot e^{-j19{,}09^\circ} = 3{,}5 - j1{,}21\,\text{A}.$$

The voltage across the load Z_α (Fig. 2.14, *b*):

$$\underline{U}_\alpha = \underline{Z}_\alpha\cdot\underline{I}_\alpha = 17\cdot e^{j61{,}93^\circ}\cdot 3{,}7\cdot e^{-j19{,}09^\circ} = 62{,}9\cdot e^{j42{,}84^\circ} = 46{,}1 + j42{,}8\ \text{V}.$$

The voltage value on the terminals *a-b* (Fig. 2.14, *b*) and, accordingly, on the loads $\underline{Z}_\beta$ and $\underline{Z}_\gamma$ (Fig. 2.14, a):

$$\underline{U}_{ab} = \underline{Z}_{ab} \cdot \underline{I}_\alpha = 24{,}71 \cdot e^{-j8{,}8^\circ} \cdot 3{,}7 \cdot e^{-j19{,}09^\circ} = 91{,}4 \cdot e^{-j27{,}89^\circ} = 80{,}8 - j42{,}8 \text{ V.}$$

The currents in the parallel branches (Fig. 2.14, *a*):

$$\underline{I}_\beta = \frac{\underline{U}_{ab}}{\underline{Z}_\beta} = \frac{91{,}4 \cdot e^{-j27{,}89^\circ}}{20 \cdot e^{-j36{,}87^\circ}} = 4{,}57 \cdot e^{j8{,}98^\circ} = 4{,}51 + j0{,}71 \text{ A;}$$

$$\underline{I}_\gamma = \frac{\underline{U}_{ab}}{\underline{Z}_\gamma} = \frac{91{,}4 \cdot e^{-j27{,}89^\circ}}{42 \cdot e^{j90^\circ}} = 2{,}18 \cdot e^{-j117{,}89^\circ} = -1{,}02 - j1{,}93 \text{ A.}$$

Checking currents according to Current law (Fig. 2.14, a): $\underline{I}_\beta + \underline{I}_\gamma = \underline{I}_\alpha$; $4{,}51 + j0{,}71 - 1{,}02 - j1{,}93 = 3{,}49 - j1{,}22$ A (found earlier $I_\alpha = 3{,}5 - j1{,}21$ A).

Checking voltages according to Kirchhoff's voltage law (Fig. 2.14, *a*):

$$\underline{U}_\alpha + \underline{U}_{ab} = \underline{U};$$

$$46{,}1 + j42{,}8 + 80{,}8 - j42{,}8 = 126{,}9 \text{ V (accepted } U = 127 \text{ V).}$$

A small difference in values is due to truncating in the calculations.

Voltage ratio $\underline{U} = \underline{U}_\alpha + \underline{U}_{ab}$ and currents $\underline{I}_\alpha = \underline{I}_\beta + \underline{I}_\gamma$ shown in Fig. 2.15 using a phasor diagram at the specified scale. The structure of the diagram is performed according to the rms values of currents and voltages and their initial phases. The rms value of voltage and current is defined as a module, and the initial phase is an argument in the form of each complex number that determines the current or voltage.

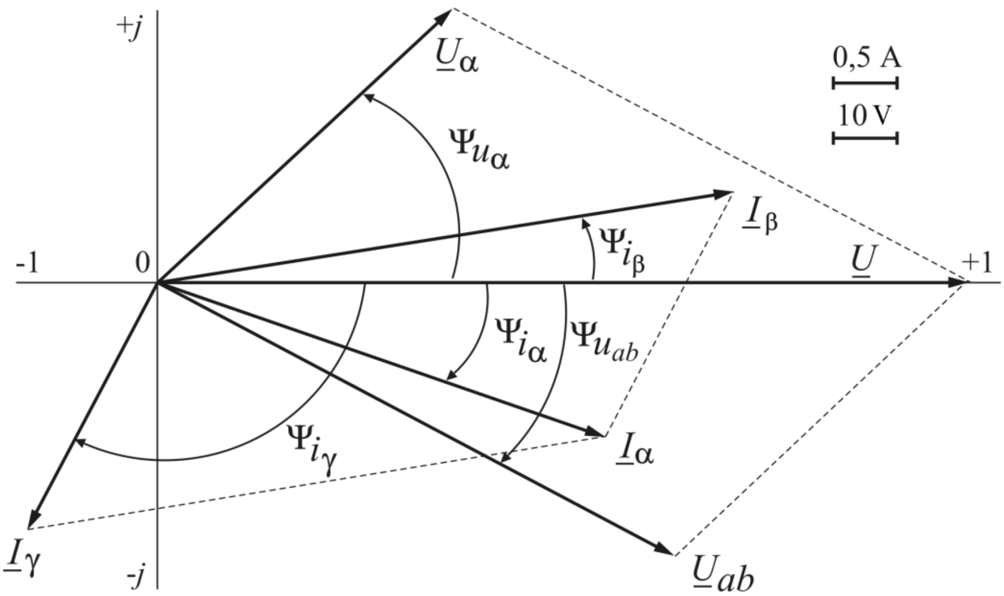

Figure 2.15

Rms values of currents and voltages and their initial phases: $I_\alpha = 3{,}7$ A; $\psi_{i_\alpha} = -19{,}09^\circ$; $I_\beta = 4{,}57$ A; $\psi_{i_\beta} = 8{,}98^\circ$; $I_\gamma = 2{,}18$ A; $\psi_{i_\gamma} = -117{,}89^\circ$; $U_\alpha = 62{,}9$ V; $\psi_{u_\alpha} = 42{,}84^\circ$; $U_{ab} = 91{,}4$ V; $\psi_{U_{ab}} = -27{,}89^\circ$; $U = 127$ V; $\psi_u = 0^\circ$.

The phasor diagram can also be construct from real and imaginary parts of complex current values of currents and voltages:

$$\underline{I}_\alpha = 3{,}5 - j1{,}21\ \text{A};\ \underline{I}_\beta = 4{,}51 + j0{,}71\ \text{A};\ \underline{I}_\gamma = -1{,}02 - j1{,}93\ \text{A};$$

$$\underline{U}_\alpha = 46{,}1 + j42{,}8\ \text{B};\ \underline{U}_{ab} = 80{,}8 - j42{,}8\ \text{B};\ \underline{U} = 127\ \text{V},$$

where the actual values of the quantities:

$I'_\alpha = 3{,}5$ A ; $I'_\beta = 4{,}51$ A ; $I'_\gamma = -1{,}02$ A ; $U'_\alpha = 46{,}1$ V ; $U'_{ab} = 80{,}8$ V ; $U' = 127$ V ,

imaginary values:

$I''_\alpha = -1{,}21$ A ; $I''_\beta = 0{,}71$ A ; $I''_\gamma = -1{,}93$ A ; $U''_\alpha = 42{,}8$ V ; $U''_{ab} = -42{,}8$ V ; $U'' = 0$ V .

Complex power of the energy source $\underline{S}_{\text{Supply}} = \underline{U} \cdot \overset{*}{\underline{I}}_\alpha = P_{\text{Supply}} + jQ_{\text{Supply}}$ is defined as the product of the complexes of voltage and conjugate current of the energy source, which are defined in an indicative (or algebraic) form.

Conjugate current complex $\overset{*}{\underline{I}}_\alpha = \underline{I}_\alpha \cdot e^{-j\psi_{i_\alpha}}$ determined from the source $\underline{I}_\alpha = \underline{I}_\alpha \cdot e^{j\psi_{i_\alpha}}$ by changing the sign in the exponent (or before the imaginary part, if the algebraic form of a complex number is used).

Substitute the complexes of voltage and conjugate complex current source and perform the transformation:

$$\underline{S}_{\text{Supply}} = \underline{U} \cdot \overset{*}{\underline{I}}_\alpha = 127 \cdot 3{,}7 \cdot e^{j19{,}09^\circ} = 469{,}9 \cdot e^{j19{,}09^\circ} =$$

$$= 469{,}9 \cdot (\cos 19{,}09^\circ + j\sin 19{,}09^\circ) = 444{,}1 + j153{,}7\ \text{VA},$$

where total power $\underline{S}_{\text{Supply}} = 469{,}9$ VA there is a module in the indicative form of complex power, and power: active $P_{\text{Supply}} = 444{,}1$ W and reactive $Q_{\text{Supply}} = 153{,}7$ VAr – real and imaginary parts of the algebraic form of a complex number.

Loads powers are similarly determined:

$$\underline{S}_\alpha = \underline{U}_\alpha \cdot \overset{*}{\underline{I}}_\alpha = 62{,}9 \cdot e^{j42{,}84^\circ} \cdot 3{,}7 \cdot e^{j19{,}09^\circ} = 232{,}7 \cdot e^{j61{,}93^\circ} = 109{,}5 + j205{,}3\ \text{VA};$$

$$\underline{S}_\beta = \underline{U}_{ab} \cdot \overset{*}{\underline{I}}_\beta = 91{,}4 \cdot e^{-j27{,}89^\circ} \cdot 4{,}57 \cdot e^{-j8{,}98^\circ} = 417{,}7 \cdot e^{-j36{,}87^\circ} = 334{,}2 - j250{,}6\,\text{VA};$$

$$\underline{S}_\gamma = \underline{U}_{ab} \cdot \overset{*}{\underline{I}}_\gamma = 91{,}4 \cdot e^{-j27{,}89^\circ} \cdot 2{,}18 \cdot e^{j117{,}89^\circ} = 199{,}3 \cdot e^{j90^\circ} = j199{,}3\ \text{VA},$$

where $\overset{*}{\underline{I}}_\alpha, \overset{*}{\underline{I}}_\beta, \overset{*}{\underline{I}}_\gamma$ – conjugate currents of loads;

active power of loads: $P_\alpha = 109{,}5$ W ; $P_\beta = 334{,}2$ W ; $P_\gamma = 0$,

reactive power of loads: $Q_\alpha = 205{,}3$ VAr ; $Q_\beta = -250{,}6$ VAr ; $Q_\gamma = 199{,}3$ VAr .

Equation of active powers balance:

$$P_{\text{Supply}} = P_\alpha + P_\beta + P_\gamma;\ 444{,}1\ \text{W} \approx 109{,}5 + 334{,}2 + 0 = 443{,}7\ \text{W}.$$

Equation of reactive power balance:

$$Q_{\text{Supply}} = Q_\alpha + Q_\beta + Q_\gamma;\quad 153{,}7\ \text{VAr} \approx 205{,}3 - 250{,}6 + 199{,}3 = 154\ \text{VAr}.$$

Example 4. In a three-phase circuit with a linear voltage $U_L = 380$ V symmetrical loading which is connected by "Y" is included. The load consists of two loads connected in series Z_α, Z_β in each phase with parameters $R_\alpha = 16$ Ohm; $X_{L\alpha} = 30$ Ohm; $R_\beta = 22{,}1$ Ohm; $X_{C\beta} = 8$ Ohm.

Calculation volume: a) draw the equivalent electric scheme of three-phase circuit with ideal elements instead of loads; b) determine phase and linear currents, as well as active and reactive power; c) construct a phasor diagram.

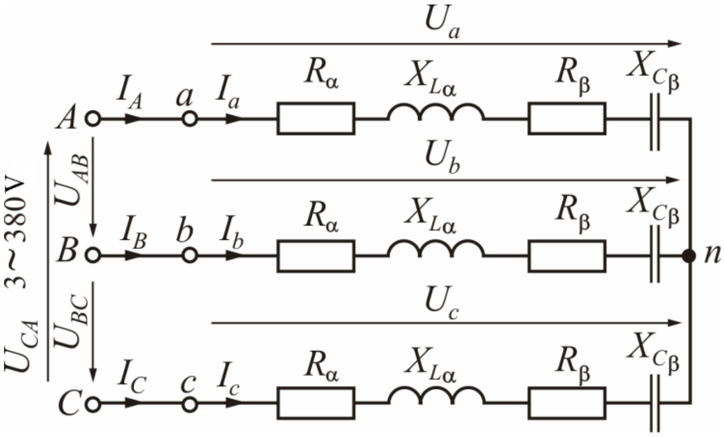

Figure 2.16

Solution. In Fig. 2.16 it is shown an equivalent electric scheme of a three-phase circuit when the load is connected in "Y" (star).

Phase voltages in the network $U_{Ph} = \dfrac{U_L}{\sqrt{3}} = \dfrac{380}{\sqrt{3}} = 220\,\text{V}$.

At symmetrical loading phase voltages on loads are equal to phase voltages of a circuit U_{Ph}, that is $U_a = U_b = U_c = U_{Ph} = 220\,\text{V}$.

Impedance of each phase:

$$Z_{Ph} = \sqrt{(R_\alpha + R_\beta)^2 + (X_{L\alpha} - X_{C\beta})^2} = \sqrt{(16 + 22{,}1)^2 + (30 - 8)^2} = 44 \text{ Ohm};$$

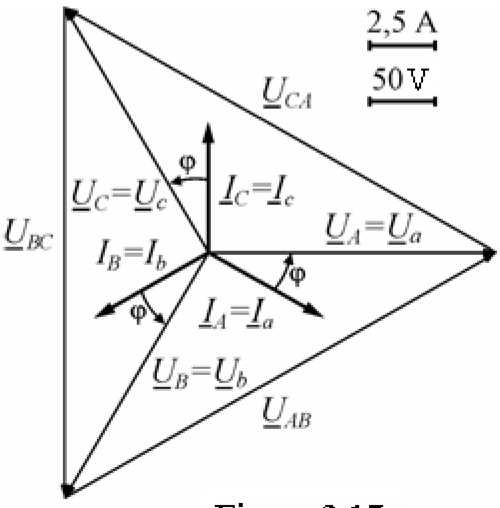

Figure 2.17

Phase shift between phase voltage and current in each phase:

$$\varphi = \text{arctg}\frac{X_{L\alpha} - X_{C\beta}}{R_\alpha + R_\beta} = \text{arctg}\frac{30 - 8}{16 + 22{,}1} = 30^\circ.$$

Phase and linear currents:

$$I_{Ph} = \frac{U_{Ph}}{Z_{Ph}} = \frac{220}{44} = 5 \text{ A};$$

$$I_a = I_b = I_c = I_{Ph} = 5\text{A};$$

$$I_A = I_B = I_C = I_L = I_{Ph} = 5 \text{ A}.$$

Active and reactive power of the whole circuit:

$$P_{Y} = 3P_{Ph} = 3U_{Ph}I_{Ph}\cos\varphi = = 3\cdot 220\cdot 5\cdot \cos 30^{\circ} = 2858 \text{ W};$$

$$Q_{Y} = 3Q_{Ph} = 3U_{Ph}I_{Ph}\sin\varphi = 3\cdot 220\cdot 5\cdot \sin 30^{\circ} = 1650 \text{ VAr}.$$

The phasor diagram is shown in Fig. 2.17. It is constructed this way. Choose the scale of currents and voltages and the initial phase $\Psi_{U_A} = 0^{\circ}$ voltage phasor $\underline{U}_A$ and draw this phasor along the horizontal axis. Phasors of phase voltages of loads $\underline{U}_a, \underline{U}_b, \underline{U}_c$ at symmetrical loading are equal to voltages of a circuit $\underline{U}_A, \underline{U}_B, \underline{U}_C$ and create a three-phase symmetric system, i.e. have the same effective values and phase-shifted relative to each other by an angle of 120°. Voltage phasor $\underline{U}_B$ lags behind the phasor $\underline{U}_A$ by an angle of 120°, so it returns to this angle clockwise, and the voltage phasor $\underline{U}_C$ ahead of the phasor $\underline{U}_A$ on an angle of 120°, so it returns to this angle counterclockwise.

Phasors of linear voltages $\underline{U}_{AB}$, $\underline{U}_{BC}$, $\underline{U}_{CA}$ we draw using Kirchhoff's voltage law according to formulas:

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B; \quad \underline{U}_{BC} = \underline{U}_B - \underline{U}_C; \quad \underline{U}_{CA} = \underline{U}_C - \underline{U}_A,$$

that is we will connect the ends of voltage phasors $\underline{U}_A, \underline{U}_B, \underline{U}_C$ among themselves and we obtain phasors of linear voltages, as shown in Fig. 2.17.

Each phase current phasor behind its phase voltage by an angle of $\varphi=30^{\circ}$, therefore, it rotates clockwise at this angle relative to its phase voltage

Example 5. In a three-phase network with a linear voltage U_L = 380 V symmetrical load is connected in "Δ" (delta). The load consists of two consumers connected in series Z_{α}, Z_{γ} in each phase with parameters R_{α} = 20 Ohm; R_{γ} = 10 Ohm; $X_{C\gamma}$ = 30 Ohm.

Calculation volume: a) draw the electrical scheme of replacement of three-phase circuit with ideal elements instead of loads; b) determine phase and linear currents, as well as active and reactive power; c) construct a phasor diagram.

Solution. Figure 2.18 shows an electrical scheme of the replacement of a three-phase circuit when connecting the load in "Δ".

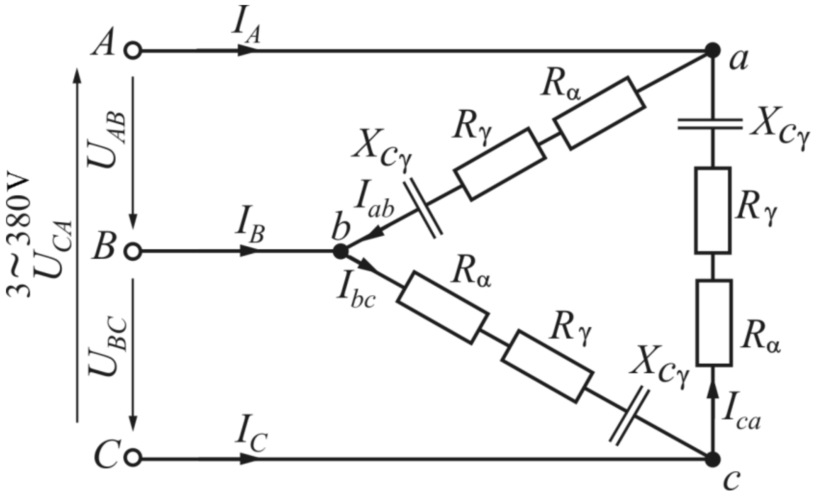

Figure 2.18

Impedance of each phase:

$$Z_{\text{Ph}} = \sqrt{(R_\alpha + R_\gamma)^2 + (-X_{C\gamma})^2} = \sqrt{(20+10)^2 + (-30)^2} = 42{,}43\ \text{Ohm}.$$

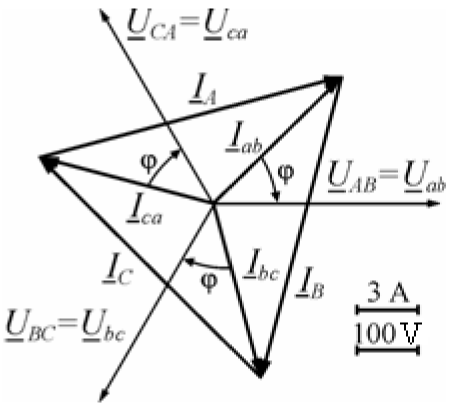

Figure 2.19

Phase shift between phase voltage and current in each phase:

$$\varphi = \text{arctg}\frac{-X_{C\gamma}}{R_\alpha + R_\gamma} = \text{arctg}\frac{-30}{20+10} = -45^{\text{o}}.$$

When connecting loads in "Δ" phase voltage U_{ab}, U_{bc}, U_{ca} coincide with the corresponding line voltages U_{AB}, U_{BC}, U_{CA}, therefore phase voltages of loads $U_{\text{Ph}} = U_{\text{L}} = 380$ V.

Phase currents:

$$I_{\text{Ph}} = \frac{U_{\text{Ph}}}{Z_{\text{Ph}}} = \frac{380}{42{,}43} = 8{,}96\ \text{A};$$

$$I_{ab} = I_{bc} = I_{ca} = I_{\text{Ph}} = 8{,}96\ \text{A}.$$

Line currents:

$$I_A = I_B = I_C = I_{\text{L}} = \sqrt{3}I_{\text{Ph}} = \sqrt{3}\cdot 8{,}96 = 15{,}52\ \text{A}.$$

Active and reactive power of the whole circuit:

$$P_\Delta = 3P_{\text{Ph}} = 3U_{\text{Ph}}I_{\text{Ph}}\cos\varphi = 3\cdot 380\cdot 8{,}96\cdot\cos(-45)^\circ = 7222\ \text{W};$$

$$Q_\Delta = 3Q_{\text{Ph}} = 3U_{\text{Ph}}I_{\text{Ph}}\sin\varphi = 3\cdot 380\cdot 8{,}96\cdot\sin(-45)^\circ = -7222\ \text{VAr}.$$

The phasor diagram is shown in Fig. 2.19. The diagram is drawn as follows. Determine the scale of currents and voltages and choose the initial phase $\Psi_{U_{AB}} = 0^\circ$ voltage phasor $\underline{U}_{AB}$ and draw this phasor along the horizontal axis. Network line voltage phasors $\underline{U}_{AB}, \underline{U}_{BC}, \underline{U}_{CA}$ (which are also phase voltages $\underline{U}_{ab}$, $\underline{U}_{bc}$, $\underline{U}_{ca}$ on loads) create a three-phase symmetric system, i.e. they have the same rms values and are shifted in phase relative to each other by an angle of 120°. Voltage phasor $\underline{U}_{BC}$ lags behind the phasor $\underline{U}_{AB}$ on the angle 120°, so it returns to this angle clockwise, and the voltage phasor $\underline{U}_{CA}$ ahead of the phasor $\underline{U}_{AB}$ on the angle 120°, so it returns to this angle counterclockwise.

Each phase current phasor $\underline{I}_{ab}, \underline{I}_{bc}, \underline{I}_{ca}$ ahead of its phase voltage phasor by an angle of $\varphi = -45^\circ$, therefore, it rotates counterclockwise at this angle relative to the phase voltage.

Phasors of linear currents $\underline{I}_A, \underline{I}_B, \underline{I}_C$ we build using Kirchhoff's current law according to formulas:

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca}, \quad \underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab}, \quad \underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc},$$

that is, we will connect the ends of phasors of currents $\underline{I}_{ab}, \underline{I}_{bc}, \underline{I}_{ca}$ among themselves and we obtain phasors of linear currents, as shown in Fig. 2.19.

Example 6. In a three-phase circuit with a linear voltage U_L = 380 V asymmetric load of phases is included Z_α, Z_β and Z_γ as asymmetric load of ideal elements: $R_\alpha = 12$ Ohm, $X_{L\alpha} = 16$ Ом, R_β=32 Ohm, $X_{C\beta} = 24$ Ом, $X_{L\gamma} = 20$ Ом.

<u>Calculation volume</u>: a) draw the equivalent electrical scheme of a three-phase circuit with ideal elements instead of the loads; b) determine the phase and line currents, as well as the current in the neutral wire; c) determine the active and reactive power; d) draw a phasor diagram.

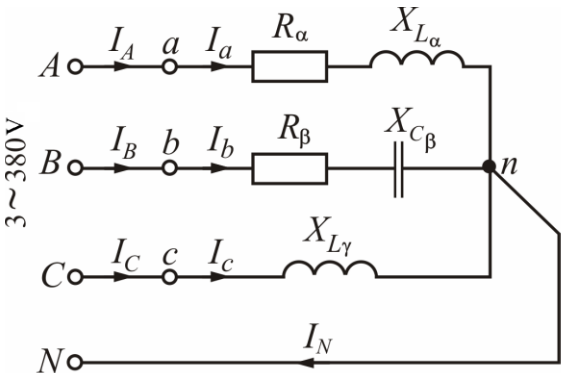

Figure 2.20

Solutions. Figure 2.20 shows the electrical circuit of a three-phase circuit.

Phase voltages

$$U_A = U_B = U_C = U_{Ph} = \frac{U_L}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220 \text{ V}.$$

In the presence of a neutral wire, the phase voltages of the network are equal to the phase voltages at the loads:

$$U_A = U_a; \; U_B = U_b; \; U_C = U_c.$$

$$U_a = U_b = U_c = U_{Ph} = 220 \text{ V}.$$

Impedances and shift angles of the phases loading:

$$Z_\alpha = \sqrt{R_\alpha^2 + X_{L\alpha}^2} = \sqrt{12^2 + 16^2} = 20 \text{ Ohm},$$

$$Z_\beta = \sqrt{R_\beta^2 + (-X_{C\beta})^2} = \sqrt{32^2 + (-24)^2} = 40 \text{ Ohm},$$

$$Z_\gamma = X_{L\gamma} = 20 \text{ Ohm},$$

$$\varphi_\alpha = \text{arctg}\frac{X_{L\alpha}}{R_\alpha} = \text{arctg}\frac{16}{12} = 53{,}13^\circ;$$

$\varphi_\beta = \text{arctg}\dfrac{-X_{C\beta}}{R_\beta} = \text{arctg}\dfrac{-24}{32} = -36{,}87^{\circ}$; $\varphi_\gamma = 90^{\circ}$.

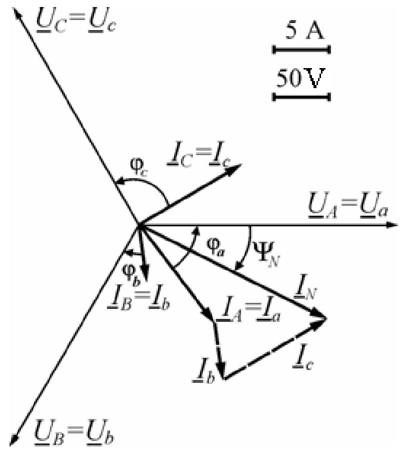

Figure 2.21

Current values of phase currents equal to linear:

$$I_a = I_A = \frac{U_a}{Z_\alpha} = \frac{220}{20} = 11 \text{ A},\ I_b = I_B = \frac{U_b}{Z_\beta} = \frac{220}{40} = 5{,}5 \text{ A},$$

$$I_c = I_C = \frac{U_c}{Z_\gamma} = \frac{220}{20} = 11 \text{ A}.$$

Let's draw a phasor diagram (Fig. 2.21). We accept the initial voltage phase for the phase *A* $\psi_{U_A} = 0^{\circ}$ and draw a phasor diagram of voltages and currents, as in example 4. Determine the rms value of the current in the neutral wire ($\underline{I}_N = \underline{I}_a + \underline{I}_b + \underline{I}_c$) direct measurement of the length of the phasor $\underline{I}_N$ and multiplying it by the current scale (I_N =19 A). The initial phase current of the neutral wire, which is determined by the protractor, is equal to $\psi_{i_N} = -27{,}5^{\circ}$. You can check the results by referring to the symbolic method.

Phase sinusoidal voltages form a three-phase symmetric system:

$$\underline{U}_A = \underline{U}_a = 220 \text{ V};\ \underline{U}_B = \underline{U}_b = 220\,e^{-j120^{\circ}} \text{ V};\ \underline{U}_C = \underline{U}_c = 220\,e^{j120^{\circ}} \text{ V}.$$

Complex impedances of phase loads:

$$\underline{Z}_\alpha = Z_\alpha\,e^{j\varphi_\alpha} = 20\,e^{j53{,}13^{\circ}} \text{ V},\quad \underline{Z}_\beta = Z_\beta\,e^{j\varphi_\beta} = 40\,e^{-j36{,}87^{\circ}} \text{ V},$$

$$\underline{Z}_\gamma = Z_\gamma\,e^{j\varphi_\gamma} = 20\,e^{j90^{\circ}} \text{ V}.$$

Linear and phase currents are:

$$\underline{I}_A = \underline{I}_a = \frac{\underline{U}_a}{\underline{Z}_\alpha} = \frac{220\,e^{j0^{\circ}}}{20\,e^{j53{,}13^{\circ}}} = 11e^{-j53{,}13^{\circ}} = 6{,}6 - j8{,}8 \text{ A};$$

$$\underline{I}_B = \underline{I}_b = \frac{\underline{U}_b}{\underline{Z}_\beta} = \frac{220e^{-j120^{\circ}}}{40e^{-j36{,}87^{\circ}}} = 5{,}5e^{-j83{,}13^{\circ}} = 0{,}66 - j5{,}46 \text{ A};$$

$$\underline{I}_C = \underline{I}_c = \frac{\underline{U}_c}{\underline{Z}_\gamma} = \frac{220e^{j120^{\circ}}}{20e^{j90^{\circ}}} = 11e^{j30^{\circ}} = 9{,}53 + j5{,}5 \text{ A},$$

where the rms values of phase currents equal to linear,

$$I_a = I_A = 11 \text{ A};\ I_b = I_B = 5{,}5 \text{ A};\ I_c = I_C = 11 \text{ A};$$

initial phases of currents:

$$\psi_{i_a} = -53{,}13^{\circ};\quad \psi_{i_b} = -83{,}13^{\circ};\quad \psi_{i_c} = 30^{\circ}.$$

The current in the neutral wire:

$$\underline{I}_N = \underline{I}_a + \underline{I}_b + \underline{I}_c = 6{,}6 - j8{,}8 + 0{,}66 - j5{,}46 + 9{,}53 + j5{,}5 = 16{,}79 - j8{,}76 =$$

$$=18{,}94\,e^{-j27{,}55^{\circ}} \text{ A},$$

where the rms value of the current in the neutral wire is $I_N = 18{,}94$ A, and its initial phase equals $\psi_{i_N} = -27{,}55^{\circ}$.

As can be seen, the calculated values of the current and its initial phase almost coincide with the values obtained from the phasor diagram.

Active and reactive power of the whole circuit:

$$P = P_{\alpha} + P_{\beta} + P_{\gamma} = U_a \cdot I_a \cdot \cos\varphi_{\alpha} + U_b \cdot I_b \cdot \cos\varphi_{\beta} + U_c \cdot I_c \cdot \cos\varphi_{\gamma} =$$

$$= 220 \cdot 11 \cdot \cos 53{,}13^{\circ} + 220 \cdot 5{,}5 \cdot \cos(-36{,}87^{\circ}) + 220 \cdot 11 \cdot \cos 90^{\circ} = 2420 \text{ W;}$$

$$Q = Q_{\alpha} + Q_{\beta} + Q_{\gamma} = U_a \cdot I_a \cdot \sin\varphi_{\alpha} + U_b \cdot I_b \cdot \sin\varphi_{\beta} + U_c \cdot I_c \cdot \sin\varphi_{\gamma} =$$

$$= 220 \cdot 11 \cdot \sin 53{,}13^{\circ} + 220 \cdot 5{,}5 \cdot \sin(-36{,}87^{\circ}) + 220 \cdot 11 \cdot \sin 90^{\circ} = 3630 \text{ VAr.}$$