

**MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE**

**NATIONAL TECHNICAL UNIVERSITY  
«KHARKIV POLYTECHNIC INSTITUTE»**



# **CALCULATION OF ELECTRIC CIRCUITS**

**METHODICAL INSTRUCTIONS FOR THE CALCULATION AND  
GRAPHIC WORK ON ELECTRICAL ENGINEERING**

**Kharkiv 2022**

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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This publication is devoted to the calculation and graphic work (individual work) on electrical engineering taught by the Department of Applied Electrical Engineering in NTU "KhPI" for full-time and part-time students of non-electrical specialties in the form of complex independent work. It contains the practice topic №1 "Calculation of DC circuits" and №2 "Calculation of sinusoidal current circuits" and is aimed at consolidating the theoretical knowledge of students on these topics and to practice practical calculation skills appropriate electrical circuits.

It has been designed for independent work of students in the preparation and implementation of calculation and graphic works, preparation for tests and exams.

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## INTRODUCTION

This publication is devoted to the calculation and graphic work (individual work) on electrical engineering. It contains the practice topic №1 "Calculation of DC circuits" and the practice topic №2 "Calculation of sinusoidal current circuits" and is aimed at consolidating the theoretical knowledge of students on these topics and to practice practical calculation skills appropriate electrical circuits.

Four control problems (tasks) have been formulated in the practice topic №1. In **Problem 1** (direct and inverse) electric circuits with a source of EMF and mixed connection of loads must be calculated. In **Problem 2** the calculation of an electrical circuit with also a source of EMF has to be performed using equivalent transformation of the circuit scheme with a star-connected load into a delta-connected load, or vice versa (Y- $\Delta$  transformation). In **Problem 3** the calculation of the electric circuit with two EMF sources has to be done using mesh currents. In **Problem 4** an electric current has to be calculated in one branch using the method of Thevenin's theorem.

Six control problems have been formulated in the practice topic №2. Initially calculations of single-phase electric circuits with a sinusoidal voltage source and series and parallel connection of loads must be carried out. Further a single-phase electric circuit with a voltage source and a mixed connection of loads using the symbolic method must be calculated. The last three problems are calculations of three-phase circuits with a symmetric and asymmetric load connected in star (Y) and in delta ( $\Delta$ ).

For the individual work of a student the variants of input data are given, and examples of solving typical problems samples are also presented. Specific scope of work specifies the lecturer who conducts a course of lectures on the specified discipline.

The report on the calculation and graphic work must be performed in accordance to a certain form. It begins with the title page, a sample of which is given in **Appendix**. At the report it is necessary to give completely statement of all problems and the corresponding input data according to the variant, the accompanying schemes, the order of calculations and their results, graphs and phasor diagrams.

Diagrams and phasor diagrams must be drawn using tools according to The State Ukrainian Standards. Dependence graphs must be performed with reference to the axes of the standard letter designations of quantities and units of measurement.

## **PRACTICE TOPIC № 1**

### **CALCULATION OF DIRECT CURRENT ELECTRIC CIRCUITS**

#### **Formulation of Problems**

To calculate the practice topic № 1, it is given a variant number for all problems.

**Problem 1. Calculation of an electric circuit with one source of electromotive force (EMF) with a series-parallel connected resistors.**

For a given variant from in Table 1.1:

- a) solve a direct Problem: draw a given electrical circuit scheme and evaluate the currents in all branches of this circuit. Use the reducing and transformations and verify the calculation by checking power balance of the sources and loads;
- b) solve an inverse Problem: evaluate EMF of the source in the circuit if you know the current  $I$ , and all of the resistances.

**Problem 2. Calculation of an electric circuit with one source of EMF by the method of a delta-wye ( $\Delta$ -Y) transformation.**

In the given variant in Table 1.2, draw a given electrical circuit with the EMF in the branch, which is specified in the table (last column). The polarity of the EMF can be random selected. Then you need to evaluate the currents in all the branches by the method of a delta-wye equivalent transformations (or Y- $\Delta$  transformation).

**Problem 3. Calculation of an electric circuit by the mesh-current method.**

In the given variant in Table 1.3, draw a given multi-loop circuit and evaluate the currents in all branches by the mesh-current method. Then you need to verify the calculation by checking power balance of the sources and load.

**Problem 4. Calculation of an electric circuit by the method of Thevenin's theorem.**

According to the given variant, which is given in Table 1.3, draw a electrical circuit scheme and determine the current in the branch with resistance  $R_1$  by the Thevenin's theorem method.

Table 1.1 – Input data to **Problem 1**

Variant	Scheme number from Fig. 1.1	Resistance, Ohm						Direct problem	Inverse problem
		$R_0$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$E, V$	$I_1, A$
1	1	1	5	10	20	15	4	120	12
2	2	2	40	20	10	40	20	130	10
3	3	1	10	15	14	7	30	200	1,8
4	4	0,5	15	30	15	17	25	180	1,5
5	5	0,5	20	20	10	36	10	90	1,6
6	6	1	4	30	70	9	30	100	10
7	7	2	20	10	10	45	20	240	1,8
8	8	1	6	45	30	24	12	140	2,5
9	9	1	4	60	15	60	40	130	10
10	10	1	6	20	10	15	16	120	6
11	11	2,5	10	30	12	60	5	270	4
12	12	2,5	4	50	30	20	15	150	3
13	13	2,5	15	8	13	10	9	90	2
14	14	1	25	15	8	5	8	135	6
15	15	2	7	18	5	10	15	90	12
16	16	1,5	7	4	3	6	5	220	3
17	17	2,5	20	35	25	5	60	140	9
18	18	1	10	6	4	12	8	100	6
19	19	2,5	24	15	10	30	6	210	3
20	20	1,2	15	9	4	21	6	135	8
21	21	1	20	10	20	8	12	110	10
22	22	2,5	20	20	24	10	25	240	1,5
23	23	1	20	15	60	20	7	100	6
24	24	2	12	30	20	30	120	200	2,5
25	25	2	15	10	17	15	7	180	5
26	26	1	15	10	40	10	50	80	9
27	27	2,5	20	20	20	5	5	150	4
28	28	2	11	5	9	8	24	150	2
29	29	1	40	15	20	10	60	220	1
30	30	1	22	10	45	40	18	200	3
31	1	0,9	6	3	4	3	7	160	5
32	2	2	6	10	5	20	90	240	2,5
33	3	1	5	20	14	3	12	140	8
34	4	2,5	10	30	60	5	50	165	8
35	5	2	20	30	25	200	50	210	1
36	6	0,5	4	10	30	5	30	240	6
37	7	2	40	20	20	100	5	110	9

Variant	Scheme number from Fig. 1.1	Resistance, Ohm						Direct problem	Inverse problem
		$R_0$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$E, V$	$I_1, A$
38	8	2	9	24	24	70	30	100	8
39	9	0,5	7	30	60	30	70	190	6
40	10	2	16	12	20	5	18	70	9
41	11	1	5	6	30	6	7	180	3
42	12	1	8	10	14	12	48	130	15
43	13	2	60	10	7	15	7	150	4
44	14	2,5	50	90	21	60	21	125	1,8
45	15	2	12	60	13	80	80	220	4
46	16	0,5	30	10	20	30	5	250	4,8
47	17	1	10	5	10	6	4	80	6
48	18	1	3	4	5	10	10	130	10
49	19	1	5	19	70	30	5	90	4
50	20	0,5	20	40	10	10	20	100	6
51	21	1	12	3	4	5	7	90	4
52	22	2,5	30	40	30	19	40	45	1,2
53	23	1	30	6	30	120	4	200	2,5
54	24	1	6	15	10	15	60	100	10
55	25	2,5	20	40	6	10	6	75	6
56	26	2	40	40	30	60	30	240	1,5
57	27	1	40	30	70	14	5	210	2,5
58	28	2	30	40	30	16	40	220	1
59	29	2,5	5	12	40	8	15	200	4
60	30	1	8	20	100	30	25	110	10
61	1	1	4	5	7	12	4	90	3
62	2	0,5	7	10	5	6	15	100	10
63	3	1,5	48	16	3	6	15	120	2
64	4	1	10	100	25	7	20	150	4
65	5	1	40	80	52	25	80	105	0,8
66	6	1	4	3	6	8	10	120	6
67	7	2,5	12	16	20	15	6	200	9
68	8	1	10	15	10	20	80	180	5
69	9	1	10	10	15	80	20	90	10
70	10	2,5	5	10	30	30	10	150	8
71	11	2	20	45	30	90	10	85	2,4
72	12	4	50	45	15	60	20	120	3
73	13	3	10	10	7	40	7	60	1,6
74	14	2,5	50	90	21	60	21	125	1,8
75	15	1	14	25	16	40	40	230	3

Variant	Scheme number from Fig. 1.1	Resistance, Ohm						Direct problem	Inverse problem
		$R_0$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$E, V$	$I_1, A$
76	16	0,5	20	60	30	20	5	130	4,8
77	17	1	15	20	10	4	14	160	2
78	18	2,5	4	10	6	20	12	220	4
79	19	2	10	38	140	60	10	180	4
80	20	1	5	10	10	15	10	110	16
81	21	2	24	6	8	10	14	180	2
82	22	1	30	10	10	10	18	100	2,5
83	23	2	60	14	20	80	18	300	2
84	24	2,5	20	30	30	15	15	100	1,5
85	25	2,5	20	40	6	10	6	75	6
86	26	1	20	20	15	30	15	150	3
87	27	2,5	35	30	70	5	9	160	2
88	28	1	10	30	10	8	20	110	6
89	29	2,5	20	10	20	50	10	75	4
90	30	2	16	40	200	60	50	220	2,5
91	1	1	3	40	50	10	6	190	5
92	2	2	5	12	3	36	6	220	7,5
93	3	2	15	35	4,5	4	60	225	14
94	4	2	8	30	20	7	80	150	6
95	5	2,5	10	8	4	30	9	120	2,5
96	6	1,5	6	20	5	16	60	180	4
97	7	1	4	5	7	30	17	260	4,5
98	8	1	2	20	80	90	10	120	10
99	9	2	6	45	30	10	15	170	7,5
100	10	2	11,5	15	10	40	24	270	6



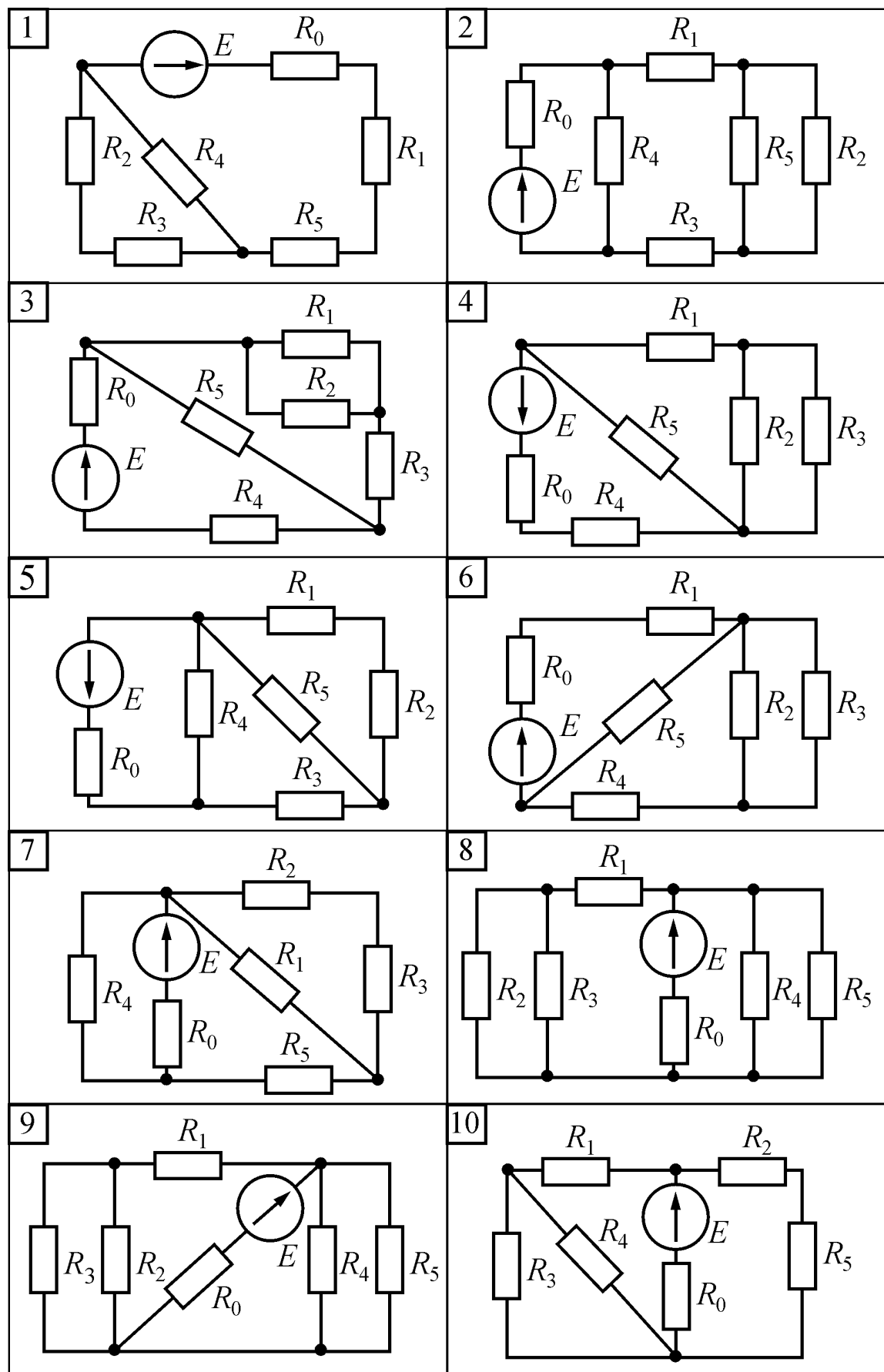


Figure 1.1

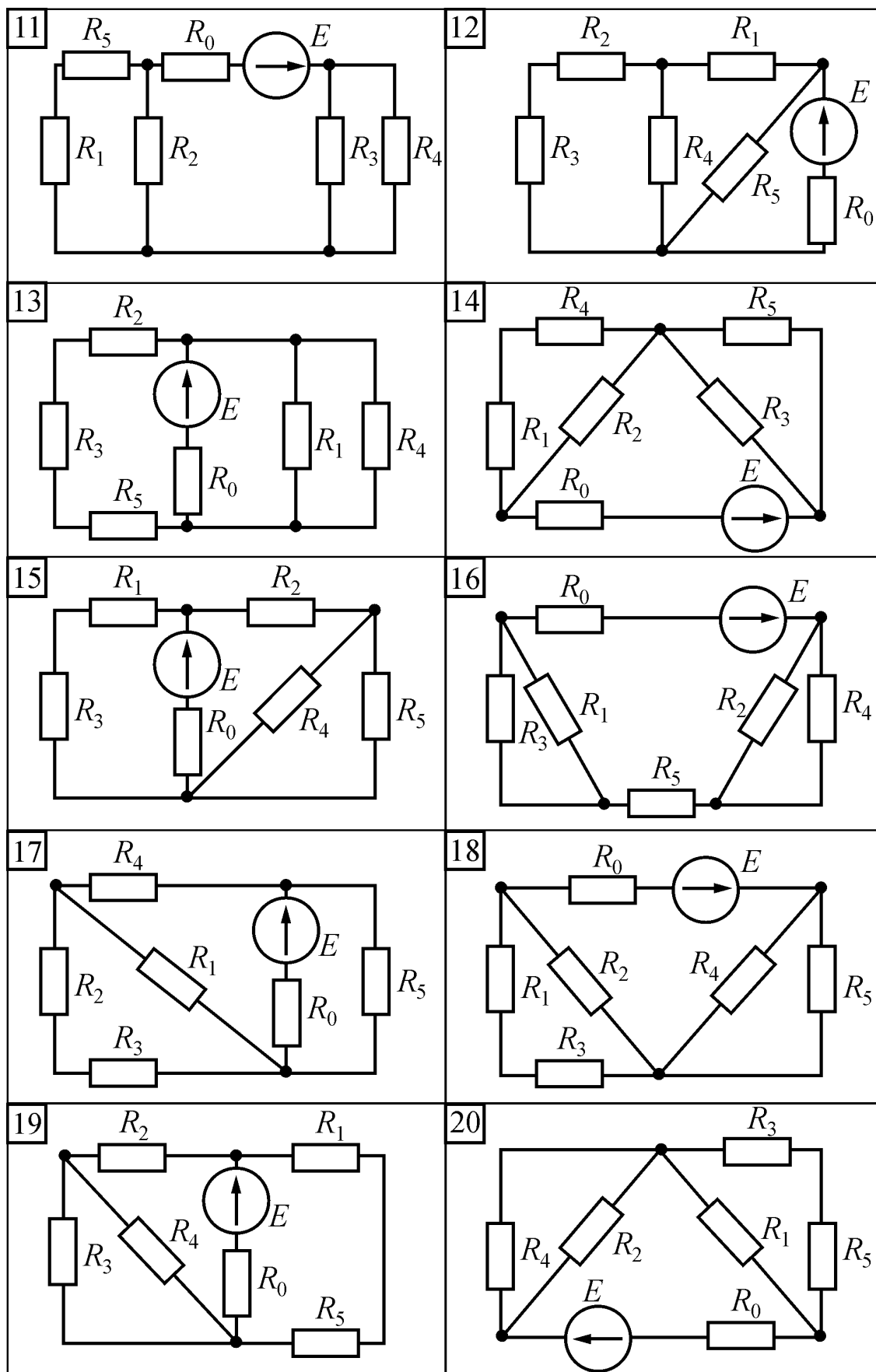


Figure 1.1 (continued)

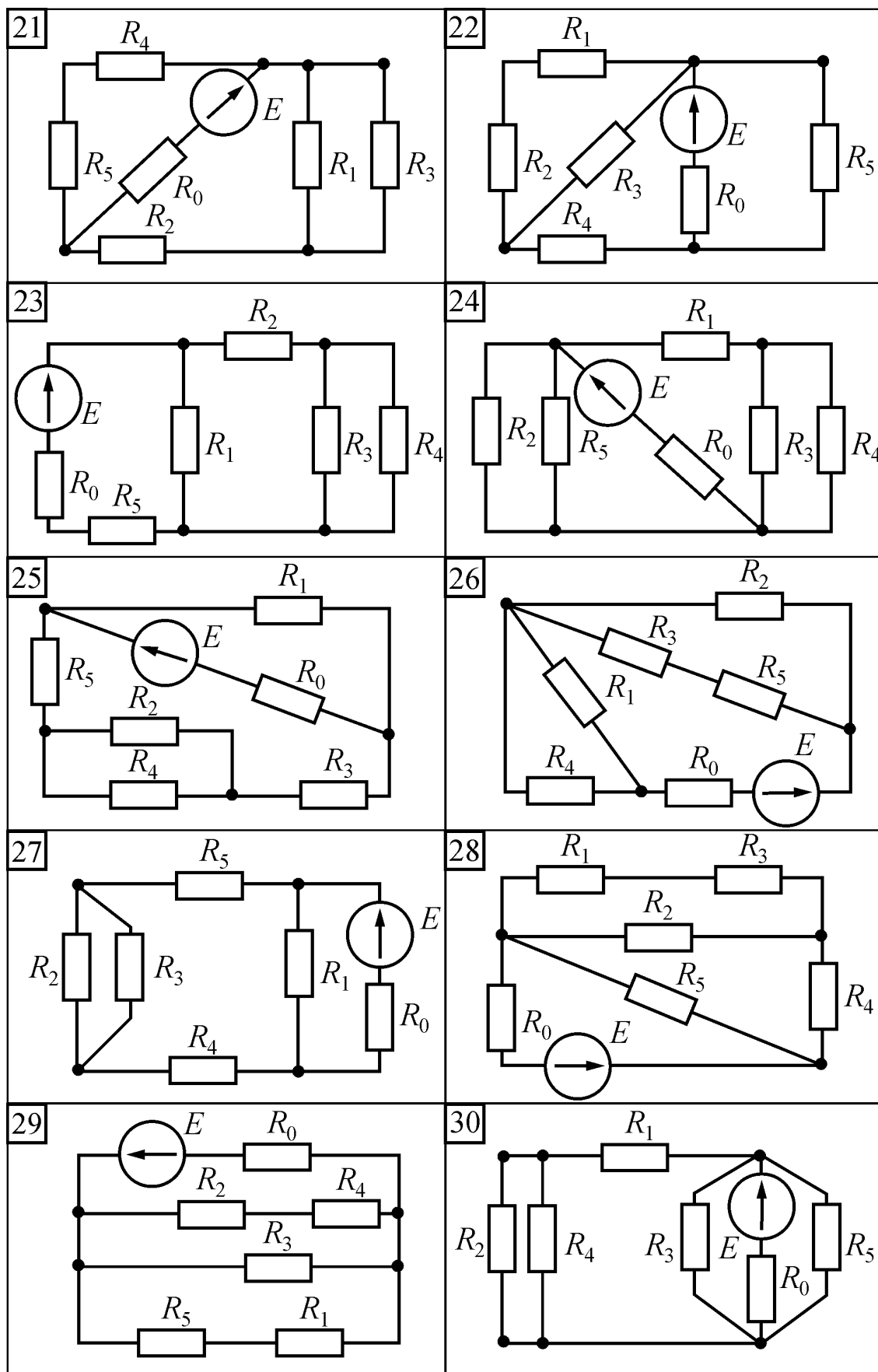


Figure 1.1 (continued)

Table 1.2 – Input data to **Problem 2**

Variant	Scheme number from Fig. 1.2	Resistance, Ohm						EMF	
		$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$E, V$	Branch
1	1	10	40	20	8	20	20	200	$bc$
2	2	30	14	18	30	7	15	250	$ac$
3	3	30	30	14	30	70	10	160	$bc$
4	4	12	24	36	7	74	16	210	$cd$
5	5	8	10	30	30	30	20	180	$bd$
6	6	12	5	40	40	20	14	250	$ad$
7	7	20	5	30	30	30	5	150	$cb$
8	8	14	40	7	80	54	80	180	$ac$
9	9	10	20	30	65	8	50	150	$bd$
10	10	4	7	18	12	6	8	90	$ac$
11	11	20	10	10	6	85	5	175	$cd$
12	12	3	9	33	33	33	69	165	$ad$
13	13	8	36	24	11	12	24	270	$ed$
14	14	18	4	48	48	48	64	200	$ab$
15	15	25	5	13	20	5	15	300	$ed$
16	16	5	10	8	2	10	10	150	$ec$
17	17	8	20	10	20	12	5	170	$ed$
18	18	14	20	30	50	25	30	200	$cd$
19	19	3	9	6	17	4	4	285	$ac$
20	20	8	2	6	6	6	13	135	$cd$
21	21	10	4	6	5	3	10	90	$ad$
22	22	12	24	12	7	17	24	150	$cd$
23	23	19	9	33	33	33	4	135	$ab$
24	24	10	6	50	40	25	10	250	$ed$
25	25	4	28	60	14	60	60	270	$ec$
26	26	9	6	3	2,5	37	9	180	$ad$
27	27	3	5	3	2	2	3	90	$ab$
28	28	6	2	7	20	10	20	160	$ad$
29	29	2	24	24	22	24	7	180	$ac$
30	30	11	30	3	8	30	40	300	$ec$
31	1	4	5	20	30	50	9	150	$ab$
32	2	8	7	6	12	18	4	180	$ec$
33	3	4	40	30	60	90	15	210	$ac$
34	4	18	10	12	12	4,6	3	180	$ed$
35	5	12	12	24	4,5	9	32	300	$ac$

Variant	Scheme number from Fig. 1.2	Resistance, Ohm						EMF	
		$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$E, V$	Branch
36	6	75	5	75	4	45	75	200	<i>bd</i>
37	7	18	18	18	4	9	6	180	<i>ad</i>
38	8	14	5	28	3	8	14	145	<i>bd</i>
39	9	10	10	15	5	12	25	120	<i>cd</i>
40	10	14	6	12	36	24	8	240	<i>ad</i>
41	11	9	9	9	37	7	4	90	<i>bd</i>
42	12	8	18	36	36	36	12	240	<i>cd</i>
43	13	12	28	8	62	13	11	250	<i>bc</i>
44	14	10	20	30	30	30	3	300	<i>ad</i>
45	15	50	40	26	10	5	10	250	<i>bc</i>
46	16	51	51	13	51	43	13	225	<i>ab</i>
47	17	6	7	4	18	12	8	180	<i>bc</i>
48	18	14	20	8	50	15	30	210	<i>ac</i>
49	19	12	3	13	7	6	18	270	<i>ab</i>
50	20	1	9	6	6	24	6	120	<i>ac</i>
51	21	32	2	24	24	24	4	200	<i>bc</i>
52	22	12	3	12	4	7	24	140	<i>ac</i>
53	23	68	8	36	36	36	7	175	<i>bc</i>
54	24	5	15	10	9	15	15	300	<i>ad</i>
55	25	30	10	70	30	4	30	240	<i>ab</i>
56	26	11	27	5,5	17	18	9	240	<i>bd</i>
57	27	6	3	6	4	18	6	100	<i>bc</i>
58	28	10	20	8	10	2	5	210	<i>bd</i>
59	29	20	7	4	12	40	40	150	<i>ab</i>
60	30	9	13	27	9	9	5	300	<i>ac</i>
61	1	20	6	32	20	8	10	300	<i>bd</i>
62	2	25	15	10	5	15	15	90	<i>cd</i>
63	3	10	10	17,5	60	7,5	5	240	<i>bd</i>
64	4	2	5	10	20	10	35	200	<i>ab</i>
65	5	20	5	30	30	30	5	150	<i>cb</i>
66	6	12	5	40	40	20	14	250	<i>ad</i>
67	7	45	5	10	20	30	15	300	<i>ac</i>
68	8	15	30	75	6	4,5	60	240	<i>ad</i>
69	9	6	42	24	24	24	5	220	<i>ac</i>
70	10	3	6	4	18	6	6	150	<i>ed</i>
71	11	18	5	12	3	12	42	215	<i>ec</i>

Variant	Scheme number from Fig. 1.2	Resistance, Ohm						EMF	
		$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$E, V$	Branch
72	12	5	9	45	45	45	65	200	$ed$
73	13	9	18	10	8	6	12	190	$ad$
74	14	6	24	3	4	36	12	240	$ec$
75	15	10	2	6	8	10	55	180	$ab$
76	16	2	13	10	2	8	2	140	$cd$
77	17	51	11	57	10	57	57	150	$ac$
78	18	35	5	15	30	15	7	200	$ed$
79	19	12	4	8	2	6	18	120	$ec$
80	20	3	3	3	3	2	5	270	$ed$
81	21	20	5	3	5	13	25	260	$ab$
82	22	4	60	9	30	22	10	200	$ed$
83	23	5	20	25	7	2	10	155	$cd$
84	24	34	12	4	8	18	6	180	$ac$
85	25	7	12	6	12	12	86	300	$cd$
86	26	12	12	12	20	3	2	295	$ac$
87	27	20	40	15	10	20	5	200	$cd$
88	28	4	6	36	20	20	10	300	$ab$
89	29	7	24	24	7	24	22	225	$bc$
90	30	5	18	84	4	18	18	300	$bd$
91	1	1	20	2	6	20	10	225	$ac$
92	2	15	2,5	15	30	5	45	240	$ab$
93	3	65	5	45	45	45	5	170	$ad$
94	4	12	9	3	6	24	12	180	$bc$
95	5	10	7	8	4	4	11	180	$ab$
96	6	2	16	5	10	10	3	120	$ac$
97	7	15	20	15	9	2	4	125	$bd$
98	8	5	25	15	15	15	2,5	90	$bc$
99	9	26	1	10	30	10	2	235	$ab$
100	10	20	30	60	9	30	30	120	$bc$

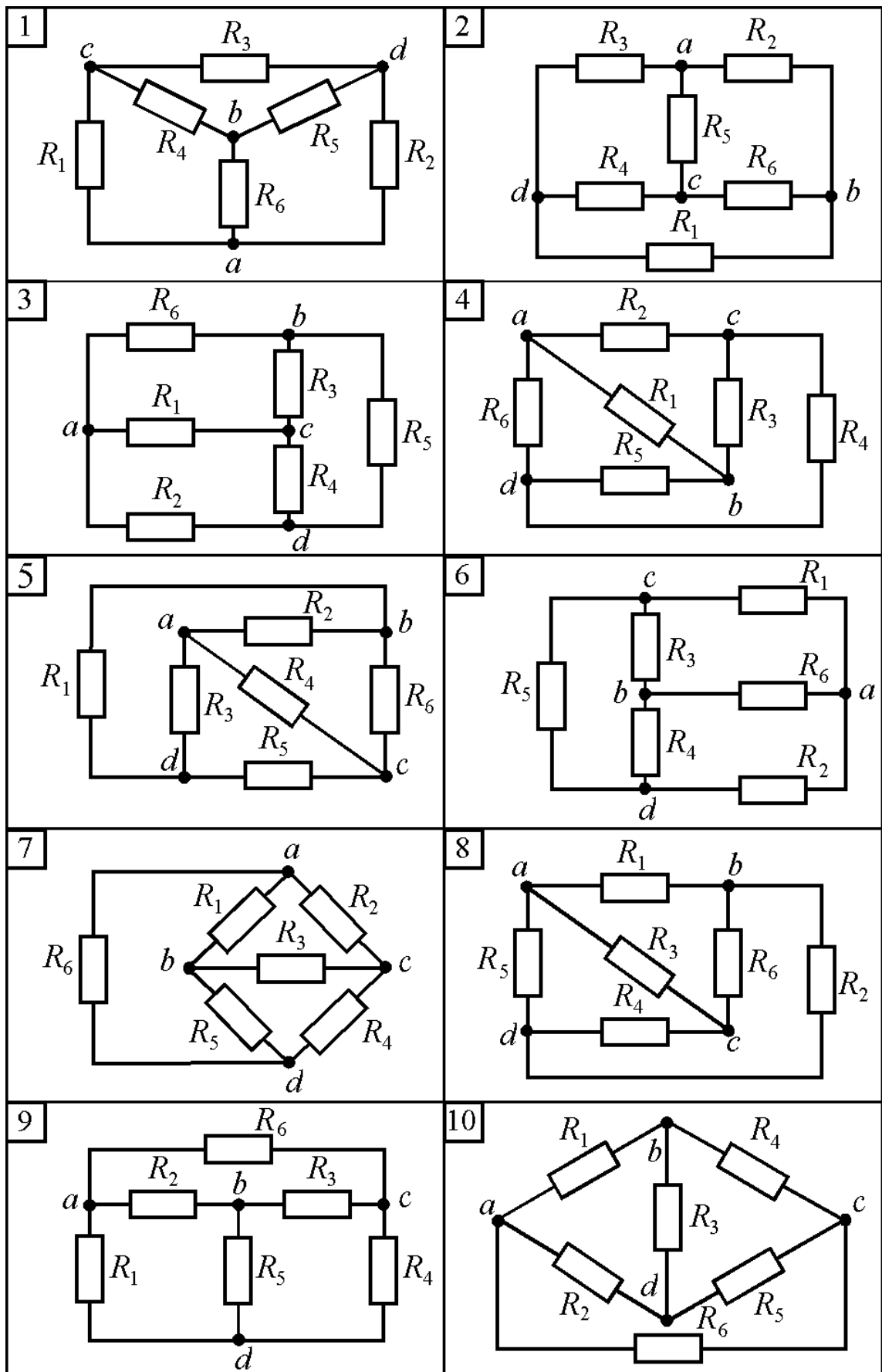


Figure 1.2

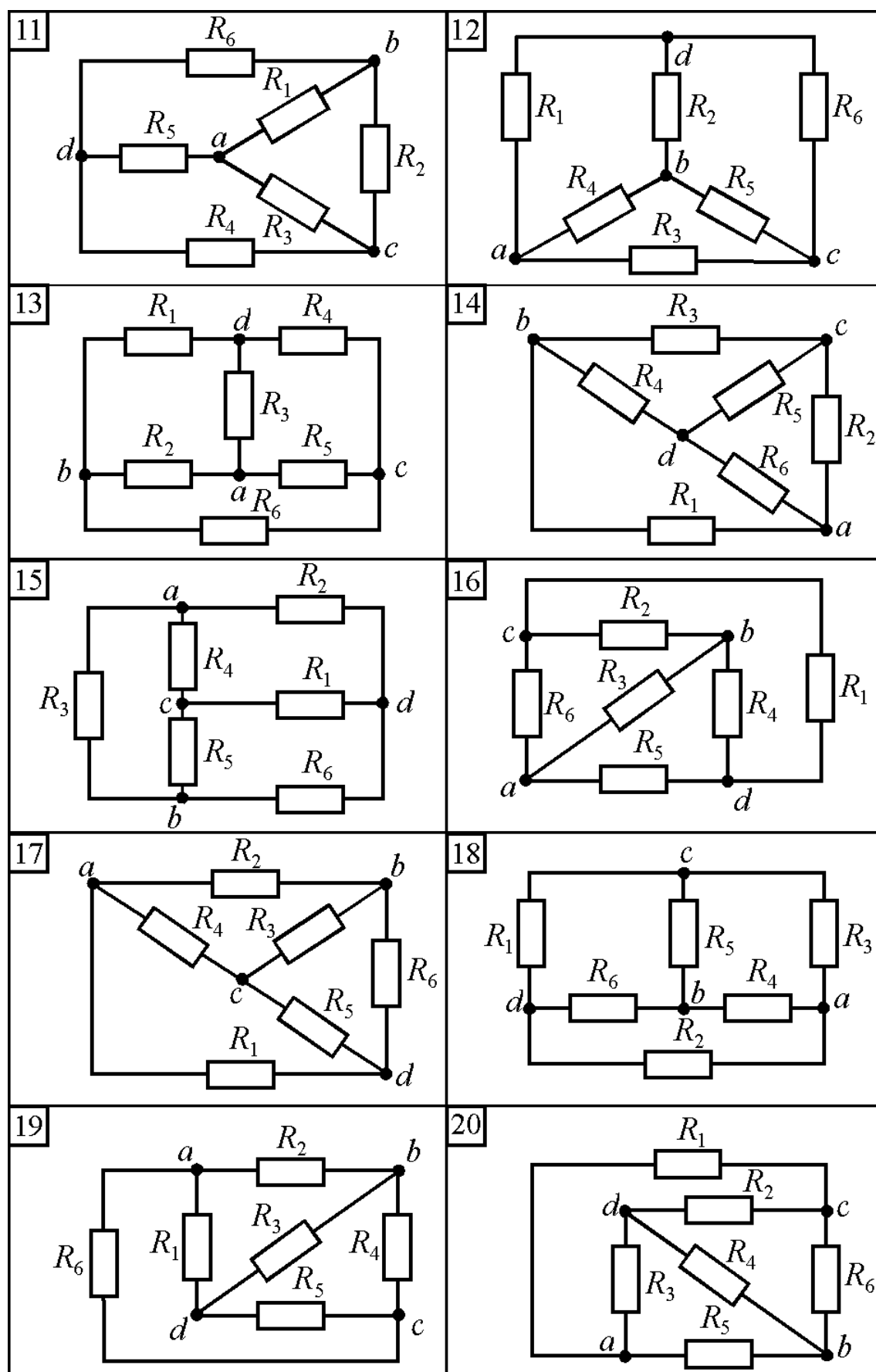


Figure 1.2 (continued)



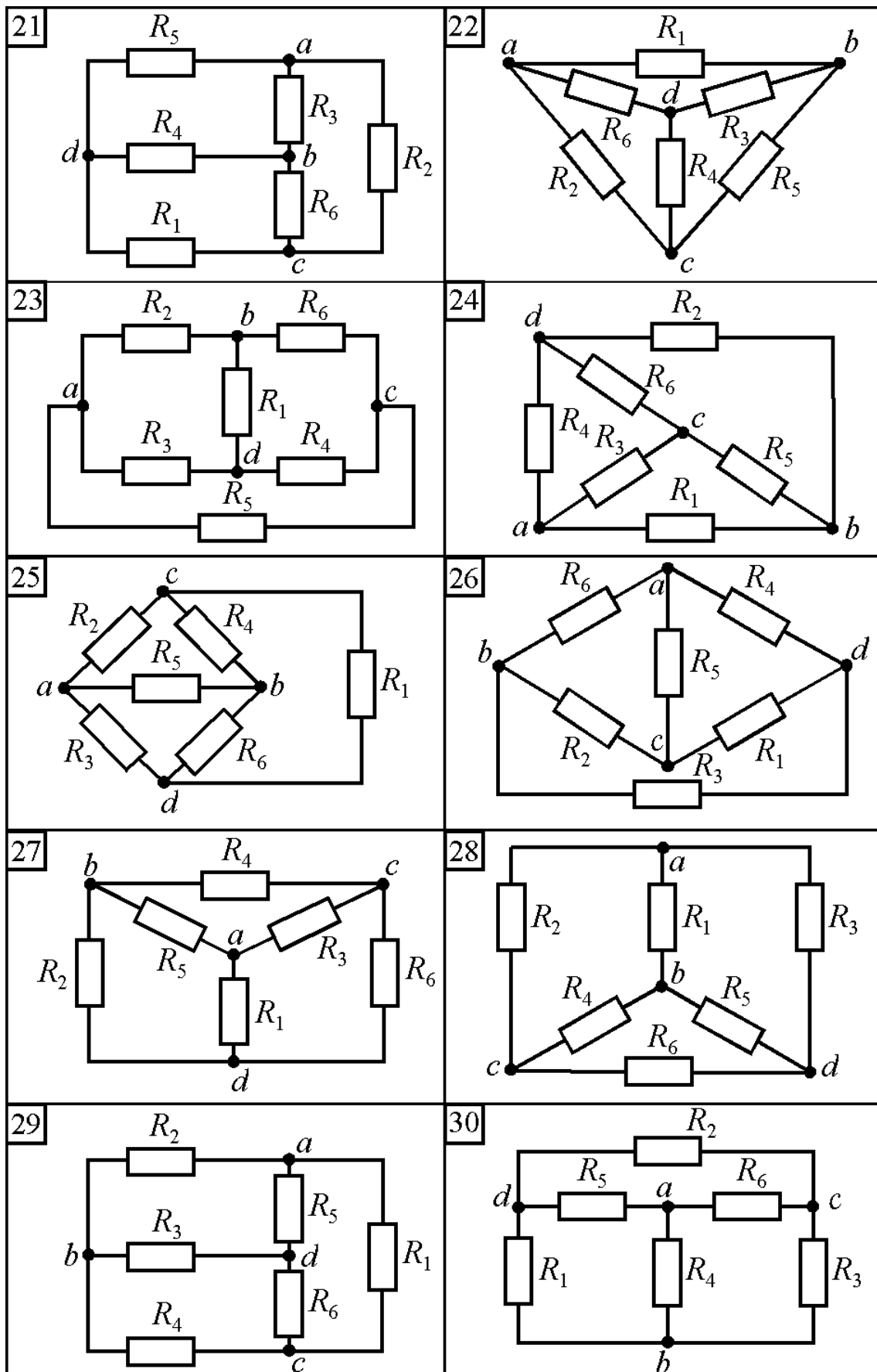


Figure 1.2 (continued)

Table 1.3 – Input data to **Problem 3** and **Problem 4**

Variant	Scheme number for Fig. 1.3	EMF, V		Resistance, Ohm					
		$E_1$ , V	$E_2$ , V	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$
1	1	90	70	6	15	6	8	5	12
2	2	100	30	6	1	8	6	7	20
3	3	170	95	4	5	18	6	10	6
4	4	130	220	10	17	14	8	7	20
5	5	150	85	13	6	9	2	10	7
6	6	90	100	6	3	7	12	4	8
7	7	125	155	4	2	21	3	4	11
8	8	150	50	5	4	6	14	9	16
9	9	120	40	8	10	12	13	6	30
10	10	90	80	2	4	4	7	10	5
11	11	130	160	4	2	10	4	20	18
12	12	60	30	33	4	12	5	1	5
13	13	145	160	10	5	15	10	25	10
14	14	100	60	5	21	10	6	9	2
15	15	110	75	12	7	15	4	3	10
16	16	270	120	9	9	7	9	2	17
17	17	240	100	1,5	5	14	8	10	12
18	18	110	55	10	10	15	20	15	15
19	19	195	480	5	35	15	20	15	15
20	20	100	100	7,5	15	15	4	45	15
21	21	90	75	12	6	3	15	4	3
22	22	35	40	7	10	4	2	3,5	8
23	23	70	55	60	10	10	5	4	15
24	24	100	80	7	5	8	8	12	7
25	25	70	110	20	14	8	2	25	6
26	26	150	55	4	5	4,5	7	2	16
27	27	190	135	26	6	10	24	5	15
28	28	75	70	5	3	10	15	10	2
29	29	135	90	2	11	10	0,5	5	0,5
30	30	120	50	11	4	2	4	7	6
31	1	270	210	12	30	12	16	10	24
32	2	150	135	10	12	10	30	11	56
33	3	80	25	1	3	5,5	34	3	1,5
34	4	195	330	20	34	28	16	14	40

Variant	Scheme number for Fig. 1.3	EMF, V		Resistance, Ohm					
		$E_1$ , V	$E_2$ , V	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$
35	5	130	110	9	4	20	35	4	2
36	6	140	105	12	4	5	4	15	17
37	7	55	150	4	3	10	5	15	4
38	8	110	70	5	6	40	2	1	6
39	9	140	160	10	3	6	8	16	8
40	10	40	35	1	6	5	2,5	1	12
41	11	120	40	1	6	4	11	2	1,5
42	12	40	110	14	4	15	6	3	2
43	13	125	35	12	2	25	4	3	7
44	14	185	210	6	11	5	10	9	30
45	15	90	90	6	6	6	1	4	16
46	16	125	145	6	9	5	15	2	3
47	17	110	30	5	24	6	6	6	4
48	18	150	75	1	14	5	5	1	9
49	19	130	20	8	10	6	5	15	3
50	20	70	130	10	15	16	10	17	6
51	21	120	50	10	2	4	4	6	3
52	22	105	195	15	5	5	10	20	28
53	23	90	70	60	10	10	5	9	20
54	24	100	120	3	9	8	8	12	12
55	25	80	220	15	14	8	2	25	6
56	26	300	110	8	10	9	14	4	32
57	27	200	135	20	7	10	30	5	15
58	28	150	140	10	6	20	30	20	4
59	29	270	180	4	22	20	1	10	1
60	30	240	100	22	8	4	8	14	12
61	1	180	140	3	7,5	3	4	2,5	6
62	2	50	45	5	6	5	15	5,5	28
63	3	240	75	2	6	11	68	6	3
64	4	130	220	5	8,5	7	4	3,5	10
65	5	80	80	3	21	2	6	5	11
66	6	120	150	4	4	8	10	3	5
67	7	360	300	16	8	26	14	30	27
68	8	110	70	10	4	5	7	3	7
69	9	90	95	15	5	10	5	20	30

Variant	Scheme number for Fig. 1.3	EMF, V		Resistance, Ohm					
		$E_1$ , V	$E_2$ , V	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$
70	10	160	140	2	12	10	5	2	24
71	11	240	80	2	12	8	22	4	3
72	12	40	40	3	8	18	4	5	14
73	13	60	120	4	2	10	2	5	2
74	14	120	150	7	5	2	9	3	15
75	15	210	170	6	4	3	5	4	10
76	16	55	65	2	3	10	30	4	16
77	17	100	200	4	5	6	11	12	8
78	18	120	60	1	12	6	10	20	10
79	19	240	90	1	6	12	16	12	12
80	20	100	50	20	24	2	8	21	10
81	21	50	20	10	2	5	6	2,5	20
82	22	140	160	14	20	8	4	7	16
83	23	140	110	30	5	5	2,5	2	7,5
84	24	230	160	11	6	8	8	15	7
85	25	40	155	15	14	8	7	25	6
86	26	130	85	3	5	5	6	2	15
87	27	150	80	9	10	5	14	8	8
88	28	90	75	6	9	10	9	3	5
89	29	155	170	15	5	10	27	20	18
90	30	65	25	5,5	2	1	2	3,5	3
91	1	110	230	30	4	20	12	6	16
92	2	40	90	6	2	4	3	6	4
93	3	130	220	9	10	3	9	8	3
94	4	145	70	2	10	10	2	3	6
95	5	105	15	11	10	16	1	5	12
96	6	160	100	10	4	8	7	2	30
97	7	120	100	8	4	13	7	15	13,5
98	8	95	130	5	9	8	2	10	18
99	9	180	210	15	5	20	5	20	40
100	10	50	95	2	4	5	5	25	2

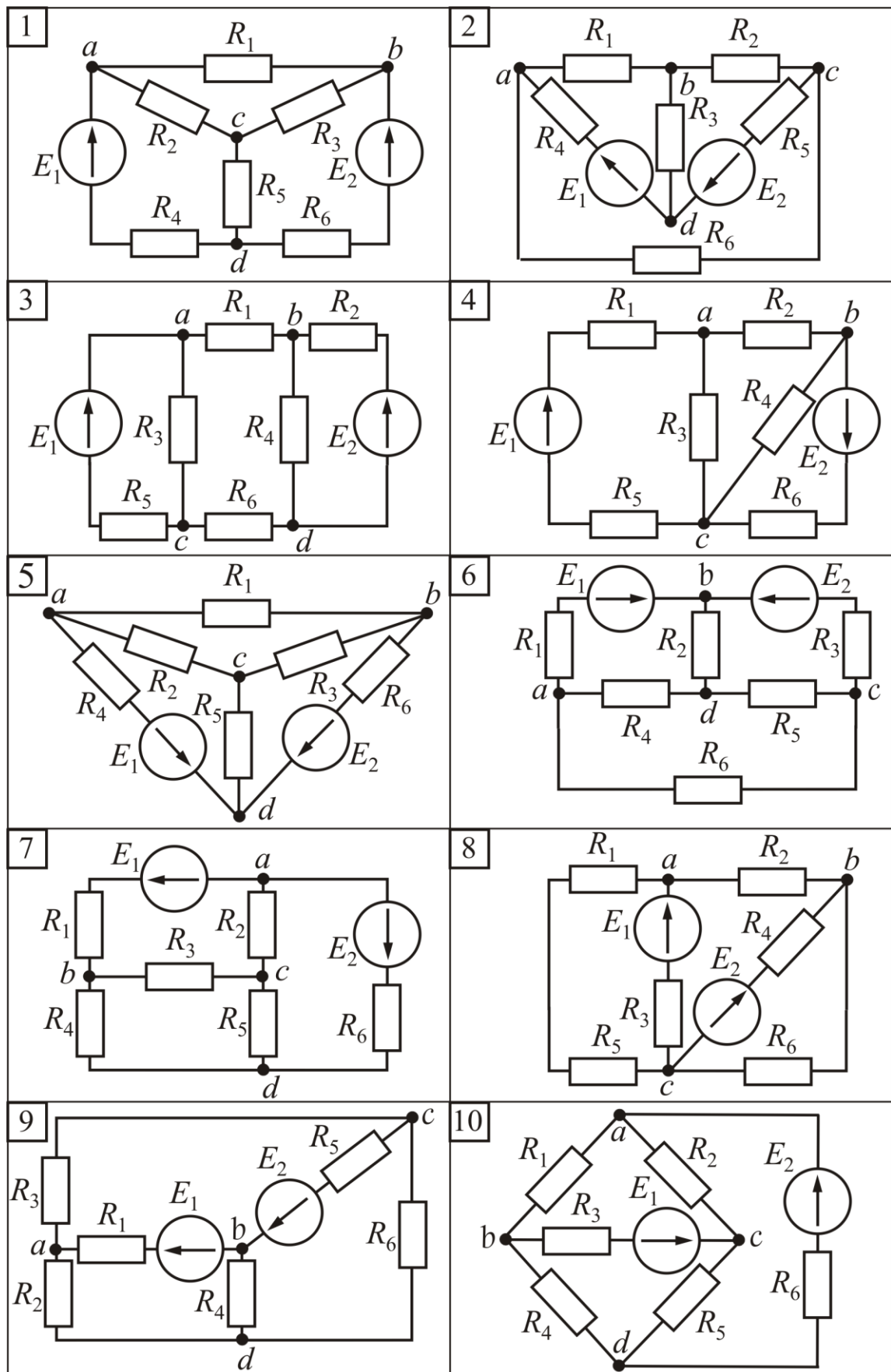


Figure 1.3

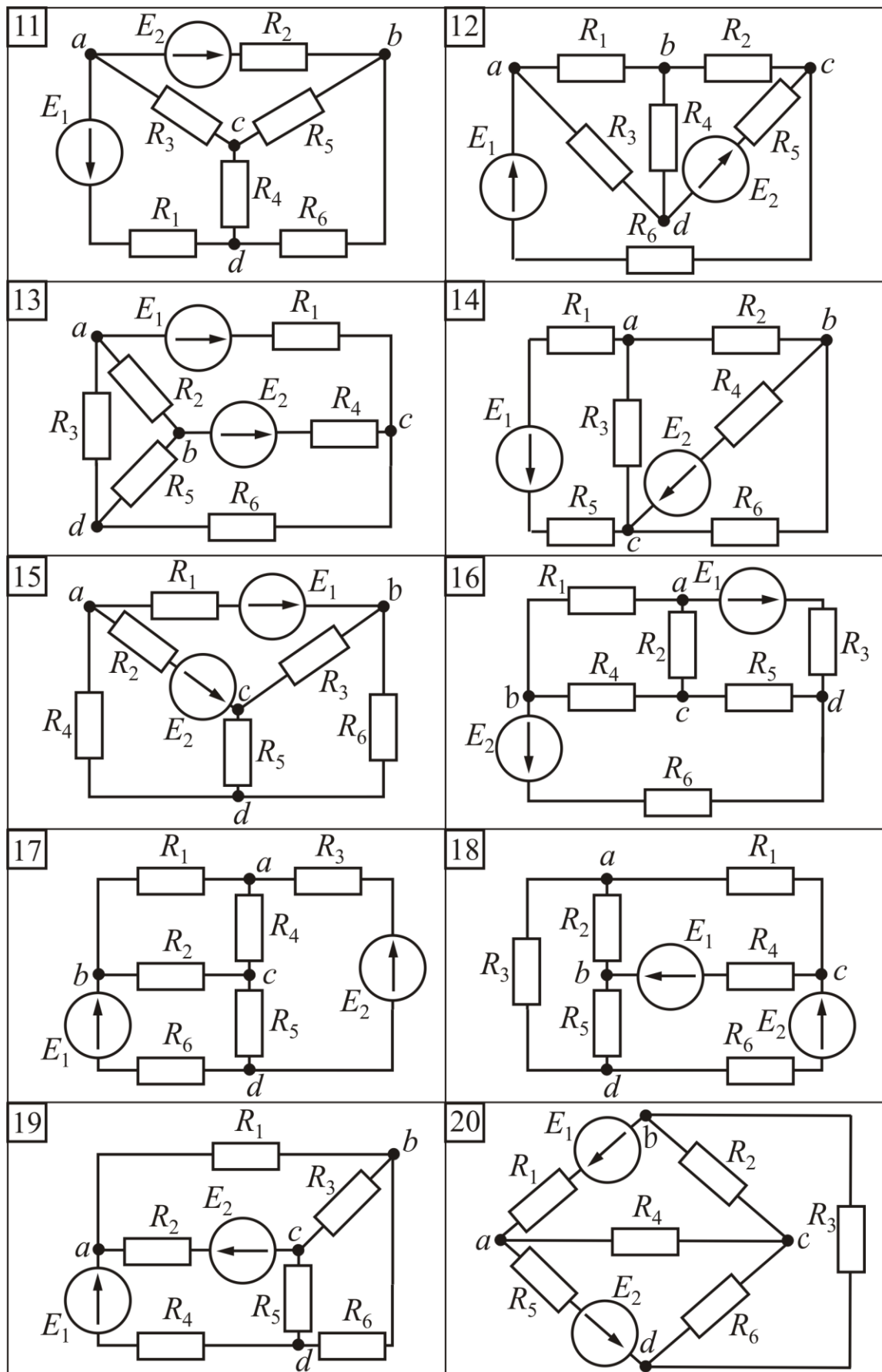


Figure 1.3 (continued)

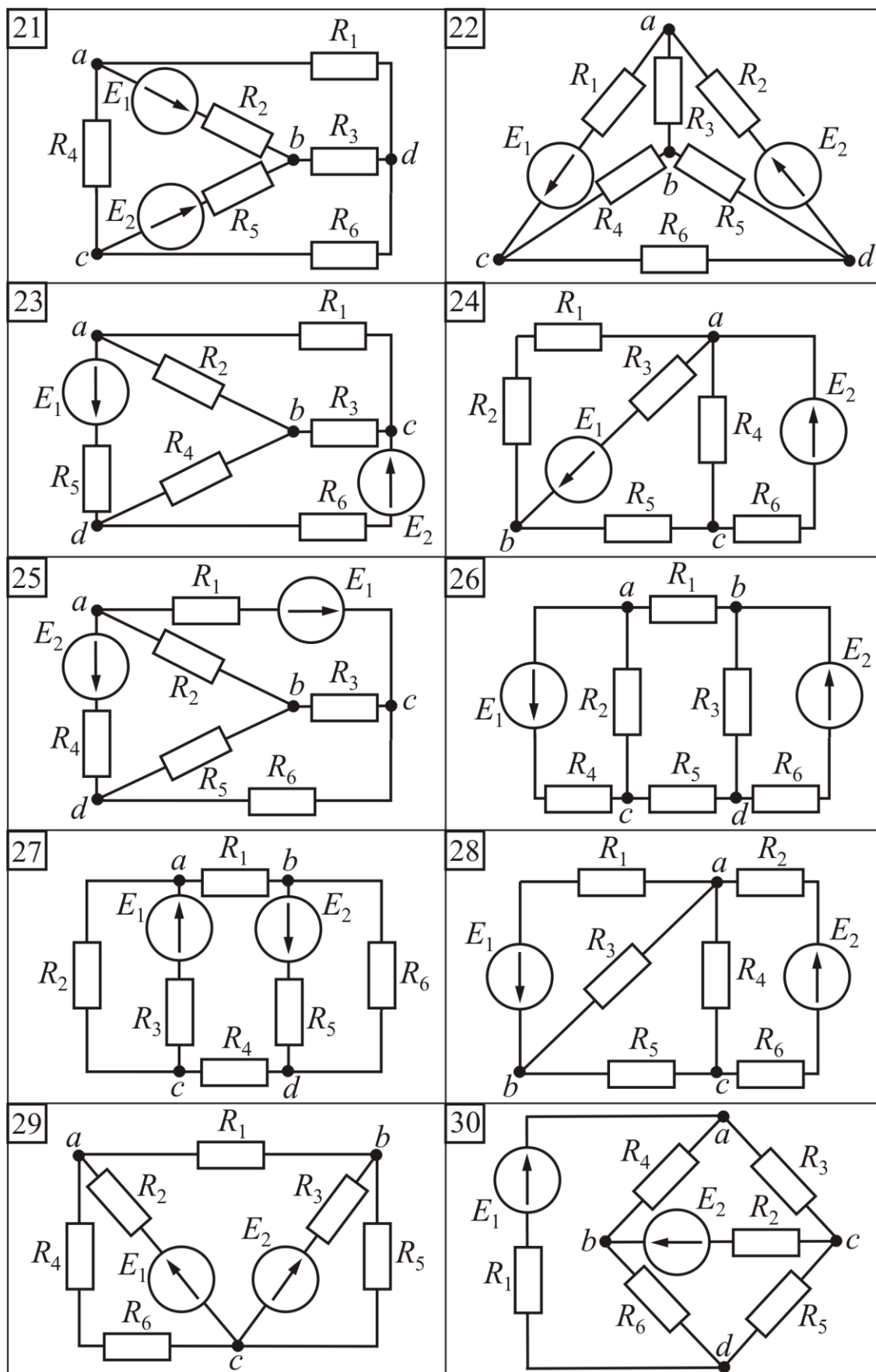


Figure 1.3 (continued)

## Examples of the Problems solution

**Example 1. Calculation of an electric circuit with one source of EMF at the series-parallel (mixed) connected loads.**

a) determine the currents in the branches of an electric circuit, the scheme of which is shown in Fig. 1.4, and it is given EMF  $E = 120$  V; resistances  $R_0 = 2$  Ohm;  $R_1 = 3$  Ohm;  $R_2 = 25$  Ohm;  $R_3 = 5$  Ohm;  $R_4 = 20$  Ohm;  $R_5 = 30$  Ohm. Write the equation of the power balance of the circuit and calculate it.

b) determine the EMF of energy sources  $E$  and currents in the branches of an electric circuit, the scheme of which is shown in Fig. 1.4, if the values of resistance of resistors and the current  $I_1 = 4$  A.

### Solution.

a) direct Problem

We choose the positive directions of the currents in the all branches of the electric circuit, according to the direction of the EMF, shown on the scheme. We use the method of consistent reducing of the initial scheme into non-branched by the method of equivalent replacement of some parts of the scheme by another, as shown in Fig. 1.4, *b, c, d*.

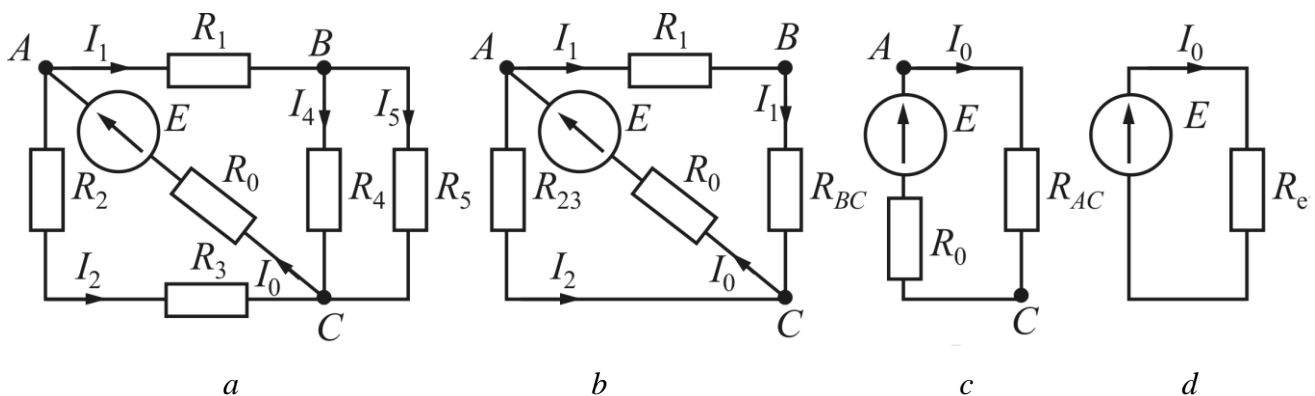


Figure 1.4



We determine the equivalent resistances of the parts and of the whole circuit after reducing:

$$R_{23} = R_2 + R_3 = 25 + 5 = 30 \text{ Ohm}; \quad R_{BC} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{20 \cdot 30}{20 + 30} = 12 \text{ Ohm};$$

$$R_{AC} = \frac{(R_1 + R_{BC}) \cdot R_{23}}{R_1 + R_{BC} + R_{23}} = \frac{(3 + 12) \cdot 30}{3 + 12 + 30} = 10 \text{ Ohm}; \quad R_e = R_0 + R_{AC} = 2 + 10 = 12 \text{ Ohm}.$$

Having received  $R_e$ , we can determine the current  $I_0$  through the EMF source (Fig. 1.4, *d*)  $I_0 = \frac{E}{R_{\text{eKB}}} = \frac{120}{12} = 10 \text{ A}$ , and then the voltage in the part  $AC$  (Fig. 1.4, *c*)

$U_{AC} = R_{AC} \cdot I_0 = 10 \cdot 10 = 100 \text{ V}$  and currents in parallel branches in the same part (Fig. 1.4, *b*) are equal to:

$$I_1 = \frac{U_{AC}}{R_1 + R_{BC}} = \frac{100}{3 + 12} = 6,67 \text{ A}; \quad I_2 = \frac{U_{AC}}{R_{23}} = \frac{100}{30} = 3,33 \text{ A}.$$

Using current  $I_1$ , we can determine the voltage in the part  $BC$  (Fig. 1.4, *b*)  $U_{BC} = R_{BC} \cdot I_1 = 12 \cdot 6,67 = 80 \text{ V}$  and currents in its parallel branches (Fig. 1.4, *a*):

$$I_4 = \frac{U_{BC}}{R_4} = \frac{80}{20} = 4 \text{ A}; \quad I_5 = \frac{U_{BC}}{R_5} = \frac{80}{30} = 2,67 \text{ A}.$$

The equation of the power balance of the circuit in Fig. 1.4, *a* is:

$$P_{\text{source}} = \sum_{k=1}^6 P_{l,k},$$

where the power of the EMF source  $P_{\text{source}} = E \cdot I_0 = 120 \cdot 10 = 1200 \text{ W}$ .

Arithmetic sum of loads power:

$$\begin{aligned} \sum_{k=1}^6 P_{\text{load},k} &= R_0 I_0^2 + R_1 I_1^2 + R_2 I_2^2 + R_3 I_2^2 + R_4 I_4^2 + R_5 I_5^2 = \\ &= 2 \cdot 10^2 + 3 \cdot 6,67^2 + 25 \cdot 3,33^2 + 5 \cdot 3,33^2 + 20 \cdot 4^2 + 30 \cdot 2,67^2 = \\ &= 200 + 133,47 + 277,22 + 55,44 + 320 + 213,87 = 1200 \text{ W}. \end{aligned}$$

If the powers values are not equal, then we calculate the relative error, which should be less than 5 percent

$$\delta = \left| \frac{P_{\text{source}} - \sum_{k=1}^n P_{\text{load},k}}{P_{\text{source}}} \right| \cdot 100\% < 5\%$$

If this condition is not met, the problem is not solved correctly.

b) inverse Problem

Voltage in the part BC (Fig. 1.4, *b*) is equal to:

$$U_{BC} = R_{BC} \cdot I_1 = 12 \cdot 4 = 48 \text{ V.}$$

Currents in parallel branches in this part (Fig. 1.4, *b*) are equal to:

$$I_4 = \frac{U_{BC}}{R_4} = \frac{48}{20} = 2,4 \text{ A}; \quad I_5 = \frac{U_{BC}}{R_5} = \frac{48}{30} = 1,6 \text{ A.}$$

Voltage in the part AC (Fig. 1.4, *b*) is:

$$U_{AC} = I_1 \cdot R_1 + U_{BC} = 4 \cdot 3 + 48 = 60 \text{ V.}$$

The current in the branch with resistance  $R_{23}$  (Fig. 1.4, *b*) is:

$$I_2 = \frac{U_{AC}}{R_{23}} = \frac{60}{30} = 2 \text{ A.}$$

According to Kirchhoff's current law (Kirchhoff's first law), we can determine the source current for node *A* (Fig. 1.4, *b*)

$$I_0 = I_1 + I_2 = 4 + 2 = 6 \text{ A.}$$

We can determine the voltage of the source EMF (Fig. 1.4, *d*)

$$E = R_e \cdot I_0 = 12 \cdot 6 = 72 \text{ V.}$$

**Example 2. Calculation of an electric circuit with one EMF source by the method of a wye-delta (Y-Δ) transformation.**

EMF source  $E$  is connected to the branch  $D-A$  in the electric circuit shown in Fig. 1.5. The direction of the EMF is pointed from node  $D$  to node  $A$ . Parameters of the elements of the circuit:  $E = 60 \text{ V}$ ;  $R_1 = 8 \text{ Ohm}$ ;  $R_2 = 10 \text{ Ohm}$ ;  $R_3 = 20 \text{ Ohm}$ ;  $R_4 = 15 \text{ Ohm}$ ;  $R_5 = 7 \text{ Ohm}$ ;  $R_6 = 9 \text{ Ohm}$ .

Determine the currents in the branches of an electric circuit using the method of converting the delta-connected resistances into an equivalent wye (star) connection and vice versa. Write the equation of the power balance equation of the circuit and calculate it.

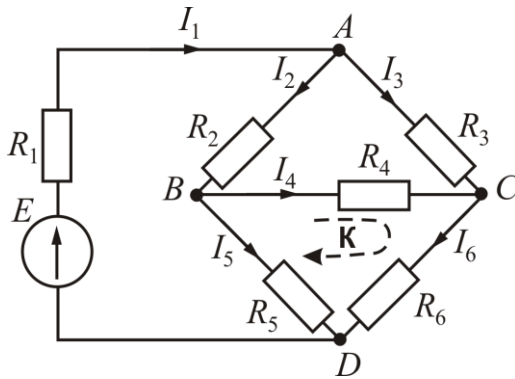


Figure 1.5

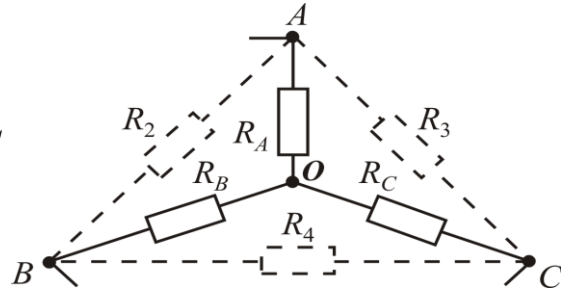


Figure 1.6

### Solution

Previously we must choose the positive directions of currents in the branches of the circuit, as it is shown in Fig. 1.5. If the current direction will be different from the one we have chosen, then in the calculation of the problem there will be a sign “–”.

To solve the problem we use transformations, for example, of three resistances  $R_2$ ,  $R_3$  and  $R_4$ , which are delta connected between nodes  $A$ ,  $B$  and  $C$  (Fig. 1.5), in an equivalent Y-connected with resistances  $R_A$ ,  $R_B$  and  $R_C$  and the central node  $O$  (Fig. 1.6).

Determine the equivalent of the Y-connection resistances:

$$R_A = \frac{R_2 \cdot R_3}{R_2 + R_3 + R_4} = \frac{10 \cdot 20}{10 + 20 + 15} = 4,44 \text{ Ohm};$$

$$R_B = \frac{R_2 \cdot R_4}{R_2 + R_3 + R_4} = \frac{10 \cdot 15}{10 + 20 + 15} = 3,33 \text{ Ohm};$$

$$R_C = \frac{R_3 \cdot R_4}{R_2 + R_3 + R_4} = \frac{20 \cdot 15}{10 + 20 + 15} = 6,67 \text{ Ohm}.$$

Equivalent electrical circuit scheme of the replacement is given on the Fig. 1.7,

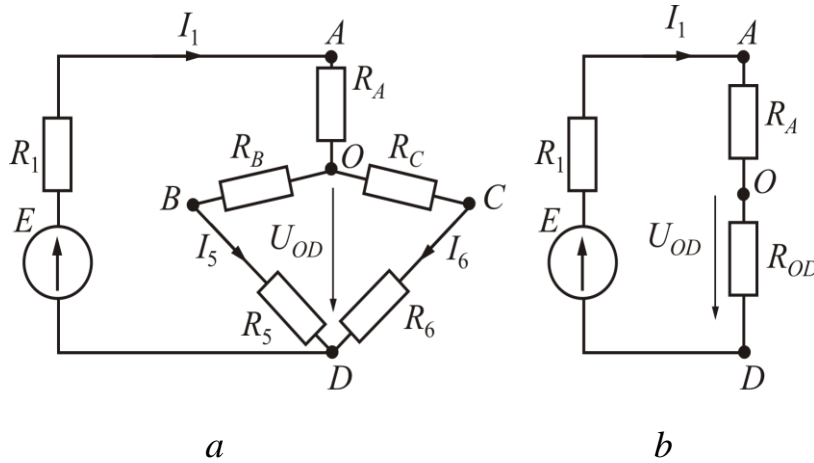


Figure 1.7

where it's shown the directions of the currents through the resistances that are not taken to the transformation.

Determine the equivalent resistance between nodes  $O$  and  $D$  and transform the electric circuit in Fig. 1.7,  $a$  to

the equivalent electric circuit shown in Fig. 1.7,  $b$

$$R_{OD} = \frac{(R_B + R_5) \cdot (R_C + R_6)}{R_B + R_5 + R_C + R_6} = \frac{(3,33 + 7) \cdot (6,67 + 9)}{3,33 + 7 + 6,67 + 9} = 6,23 \text{ Ohm.}$$

Equivalent resistance of the whole circuit (Fig. 1.7,  $b$ ) is

$$R_e = R_1 + R_A + R_{OD} = 8 + 4,44 + 6,23 = 18,67 \text{ Ohm.}$$

Current through the EMF source is equal to

$$I_1 = \frac{E}{R_e} = \frac{60}{18,67} = 3,21 \text{ A.}$$

Voltage between nodes  $OD$  (Fig. 1.8,  $b$ ) is

$$U_{OD} = R_{OD} \cdot I_1 = 6,23 \cdot 3,21 = 20 \text{ V.}$$

We can determine the currents  $I_5$  and  $I_6$  in the parallel branches (Fig. 1.7,  $a$ ):

$$I_5 = \frac{U_{OD}}{R_B + R_5} = \frac{20}{3,33 + 7} = 1,94 \text{ A; } I_6 = \frac{U_{OD}}{R_C + R_6} = \frac{20}{6,67 + 9} = 1,28 \text{ A.}$$

According to Kirchhoff's voltage law, for the loop  $K$  (Fig. 1.5), when bypassing it clockwise, we receive the equation

$$R_4 \cdot I_4 + R_6 \cdot I_6 - R_5 \cdot I_5 = 0;$$

from this equation we can determine the current

$$I_4 = \frac{R_5 \cdot I_5 - R_6 \cdot I_6}{R_4} = \frac{7 \cdot 1,94 - 9 \cdot 1,28}{15} = 0,14 \text{ A.}$$

According to Kirchhoff's current law for currents in branches which are connected in node *B* (Fig. 1.5), we have the following equation  $I_2 - I_4 - I_5 = 0$ ; from this equation we determine the current  $I_2 = I_4 + I_5 = 0,14 + 1,94 = 2,08 \text{ A}$ .

According to Kirchhoff's current law for currents of branches which are connected in node *C*, we have the equation  $I_3 + I_4 - I_6 = 0$ ; from this equation we determine the current  $I_3 = I_6 - I_4 = 1,28 - 0,14 = 1,14 \text{ A}$ .

The equation of the power balance for the circuit in Fig. 1.5

$$P_{\text{source}} = \sum_{k=1}^6 P_{l,k},$$

where the power of the EMF source  $P_{\text{source}} = E \cdot I_1 = 60 \cdot 3,21 = 192,6 \text{ W}$ ; sum of load powers:

$$\begin{aligned} \sum_{k=1}^6 P_{l,k} &= R_1 \cdot I_1^2 + R_2 \cdot I_2^2 + R_3 \cdot I_3^2 + R_4 \cdot I_4^2 + R_5 \cdot I_5^2 + R_6 \cdot I_6^2 = \\ &= 8 \cdot 3,21^2 + 10 \cdot 2,08^2 + 20 \cdot 1,14^2 + 15 \cdot 0,14^2 + 7 \cdot 1,94^2 + 9 \cdot 1,28^2 = \\ &= 82,43 + 43,26 + 25,99 + 0,29 + 26,34 + 14,75 = 193,06 \text{ W.} \end{aligned}$$

Calculating percentage error, we have

$$\delta = \left| \frac{P_{\text{source}} - \sum_{k=1}^6 P_{l,k}}{P_{\text{source}}} \cdot 100\% \right| = \left| \frac{192,6 - 193,06}{192,6} \cdot 100\% \right| = 0,24\%$$

would be considered as acceptable.

### **Example 3. Calculation of an electrical circuit by the mesh-current method.**

The scheme for an electric circuit is shown in Fig.1.8, determine the currents in all branches, using the mesh current method of calculating loop currents, if  $E_1 = 100 \text{ V}$ ;  $E_2 = 50 \text{ V}$ ;  $R_1 = R_5 = 8 \text{ Ohm}$ ;  $R_2 = 10 \text{ Ohm}$ ;  $R_3 = 7 \text{ Ohm}$ ;  $R_4 = 9 \text{ Ohm}$ ;  $R_6 = 13 \text{ Ohm}$ .

## Solution

Selection of the independent loops I, II, III, designation of the circuit currents direction  $I_{11}$ ,  $I_{22}$ ,  $I_{33}$  and the direction of bypassing the loops are shown in Fig. 1.8.

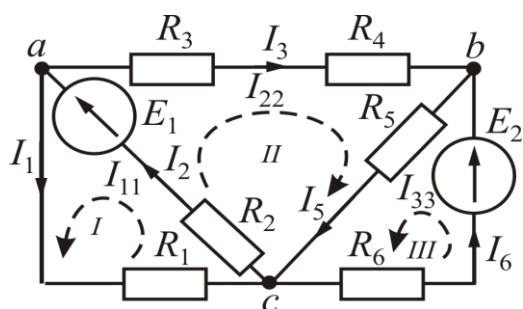


Figure 1.8

The system of equations for loops I, II and III is as follows:

$$\begin{cases} R_{11} \cdot I_{11} + R_{12} \cdot I_{22} + R_{13} \cdot I_{33} = E_{11}; \\ R_{21} \cdot I_{11} + R_{22} \cdot I_{22} + R_{23} \cdot I_{33} = E_{22}; \\ R_{31} \cdot I_{11} + R_{32} \cdot I_{22} + R_{33} \cdot I_{33} = E_{33}, \end{cases} \quad (1)$$

where the loop EMF:  $E_{11} = E_1 = 100$  V;  $E_{22} = E_1 = 100$  V;  $E_{33} = E_2 = 50$  V,

total loop resistances of the loops:

$$R_{11} = R_1 + R_2 = 8 + 10 = 18 \text{ Ohm}; \quad R_{22} = R_2 + R_3 + R_4 + R_5 = 10 + 7 + 9 + 8 = 34 \text{ Ohm};$$

$$R_{33} = R_5 + R_6 = 8 + 13 = 21 \text{ Ohm},$$

mutual resistance of loops:  $R_{12} = R_{21} = R_2 = 10$  Ohm;  $R_{23} = R_{32} = R_5 = 8$  Ohm;

$R_{13} = R_{31} = 0$  (loops I and III are not connected). Resistances  $R_{12} = R_{21}$ ,  $R_{23} = R_{32}$

written with a “+” sign because the loop currents in them coincide.

After substituting the numerical values, the system of equations (1) has the form:

$$\begin{cases} 18 \cdot I_{11} + 10 \cdot I_{22} + 0 \cdot I_{33} = 100; \\ 10 \cdot I_{11} + 34 \cdot I_{22} + 8 \cdot I_{33} = 100; \\ 0 \cdot I_{11} + 8 \cdot I_{22} + 21 \cdot I_{33} = 50, \end{cases} \quad (2)$$

the common determinant of which  $\Delta = \begin{vmatrix} 18 & 10 & 0 \\ 10 & 34 & 8 \\ 0 & 8 & 21 \end{vmatrix}$ .

Solving it by the elements of the first line, we find

$$\begin{aligned} \Delta &= \begin{vmatrix} 18 & 10 & 0 \\ 10 & 34 & 8 \\ 0 & 8 & 21 \end{vmatrix} = (-1)^{1+1} \cdot 18 \cdot \begin{vmatrix} 34 & 8 \\ 8 & 21 \end{vmatrix} + (-1)^{1+2} \cdot 10 \cdot \begin{vmatrix} 10 & 8 \\ 0 & 21 \end{vmatrix} + (-1)^{1+3} \cdot 0 \cdot \begin{vmatrix} 10 & 34 \\ 0 & 8 \end{vmatrix} = \\ &= 18 \cdot (34 \cdot 21 - 8 \cdot 8) - 10 \cdot (10 \cdot 21 - 0 \cdot 8) + 0 = 11700 - 2100 = 9600. \end{aligned}$$

Auxiliary determinants of the system are obtained from the general determinant by replacing columns 1, 2 and 3 with the EMF column of the system of equations (2), namely:

$$\Delta_{11} = \begin{vmatrix} 100 & 10 & 0 \\ 100 & 34 & 8 \\ 50 & 8 & 21 \end{vmatrix} = (-1)^{1+1} \cdot 100 \cdot \begin{vmatrix} 34 & 8 \\ 8 & 21 \end{vmatrix} + (-1)^{1+2} \cdot 10 \cdot \begin{vmatrix} 100 & 8 \\ 50 & 21 \end{vmatrix} =$$

$$= 100 \cdot (34 \cdot 21 - 8 \cdot 8) - 10 \cdot (100 \cdot 21 - 50 \cdot 8) = 65000 - 17000 = 48000.$$

$$\Delta_{22} = \begin{vmatrix} 18 & 100 & 0 \\ 10 & 100 & 8 \\ 0 & 50 & 21 \end{vmatrix} = (-1)^{1+1} \cdot 18 \cdot \begin{vmatrix} 100 & 8 \\ 50 & 21 \end{vmatrix} + (-1)^{1+2} \cdot 100 \cdot \begin{vmatrix} 10 & 8 \\ 0 & 21 \end{vmatrix} =$$

$$= 18 \cdot (100 \cdot 21 - 50 \cdot 8) - 100 \cdot (10 \cdot 21 - 0 \cdot 8) = 30600 - 21000 = 9600.$$

$$\Delta_{33} = \begin{vmatrix} 18 & 10 & 100 \\ 10 & 34 & 100 \\ 0 & 8 & 50 \end{vmatrix} = (-1)^{1+1} \cdot 18 \cdot \begin{vmatrix} 34 & 100 \\ 8 & 50 \end{vmatrix} + (-1)^{1+2} \cdot 10 \cdot \begin{vmatrix} 10 & 100 \\ 0 & 50 \end{vmatrix} +$$

$$+ (-1)^{1+3} \cdot 100 \cdot \begin{vmatrix} 10 & 34 \\ 0 & 8 \end{vmatrix} = 18 \cdot (34 \cdot 50 - 100 \cdot 8) - 10 \cdot (10 \cdot 50 - 0 \cdot 100) +$$

$$+ 100 \cdot (10 \cdot 8 - 0 \cdot 34) = 16200 - 5000 + 8000 = 19200.$$

Loop currents are equal to:

$$I_{11} = \frac{\Delta_{11}}{\Delta} = \frac{48000}{9600} = 5 \text{ A}; \quad I_{22} = \frac{\Delta_{22}}{\Delta} = \frac{9600}{9600} = 1 \text{ A}; \quad I_{33} = \frac{\Delta_{33}}{\Delta} = \frac{19200}{9600} = 2 \text{ A}.$$

They determine the currents in the branches as follows:

$$I_1 = I_{11} = 5 \text{ A}; \quad I_2 = I_{11} + I_{22} = 5 + 1 = 6 \text{ A};$$

$$I_3 = I_{22} = 1 \text{ A}; \quad I_5 = I_{22} + I_{33} = 1 + 2 = 3 \text{ A}; \quad I_6 = I_{33} = 2 \text{ A}.$$

Equation of the power balance

$$E_1 \cdot I_2 + E_2 \cdot I_6 = R_1 \cdot I_1^2 + R_2 \cdot I_2^2 + R_3 \cdot I_3^2 + R_4 \cdot I_3^2 + R_5 \cdot I_5^2 + R_6 \cdot I_6^2;$$

$$100 \cdot 6 + 50 \cdot 2 = 8 \cdot 5^2 + 10 \cdot 6^2 + 7 \cdot 1^2 + 9 \cdot 1^2 + 8 \cdot 3^2 + 13 \cdot 2^2 ;$$

$$700 \text{ W} = 700 \text{ W}.$$

**Example 4. Calculation of an electrical circuit by the Thevenin's theorem method.**

Determine the current  $I_1$  in a branch with resistance  $R_1$  in the electric circuit, which is shown in Fig. 1.9, if  $E_1 = 120$  V;  $E_2 = 60$  V;  $R_1 = 8$  Ohm;  $R_2 = 5$  Ohm;  $R_3 = 4$  Ohm;  $R_4 = 6$  Ohm;  $R_5 = 10$  Ohm;  $R_6 = 15$  Ohm.

**Solution**

We replace the electric circuit without a section of the circuit with resistance  $R_1$  with Thevenin's generator (in Fig. 1.9 it is circled by a dashed line). Thevenin's generator (Fig. 1.10) is characterized by the value of EMF (EMF of Thevenin's generator  $E_{eg}$ ) and internal (equivalent resistance)  $R_{eg}$ .

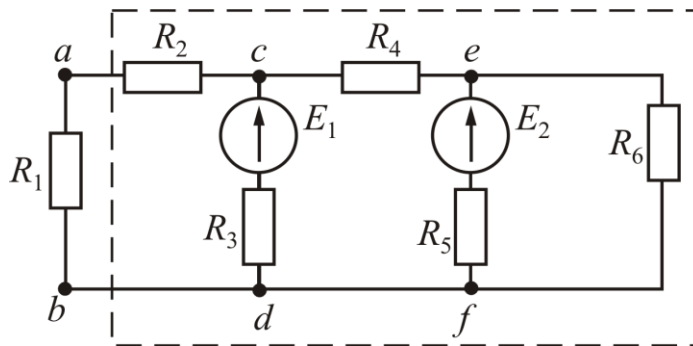


Figure 1.9

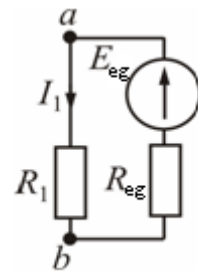


Figure 1.10

The EMF is equal to the no-load voltage  $U_{abnl}$  across the open terminals  $a-b$  of the circuit (Fig. 1.11),  $E_{eg} = U_{abnl}$ . The total equivalent resistance  $R_{eg}$  is equal to the input resistance of the circuit according to Fig. 1.9 between the terminals  $a-b$ , when the circuit becomes passive (Fig. 1.12), i.e. all EMF are equal zero, and in the branches

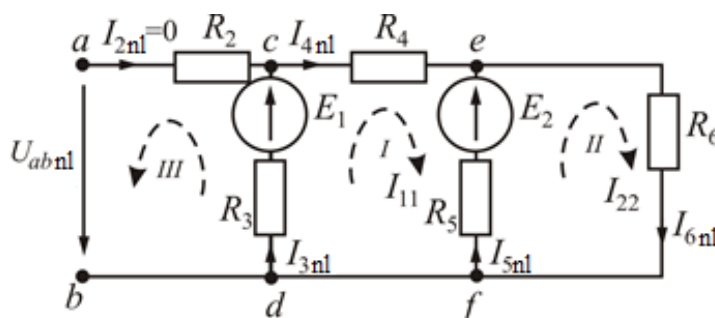


Figure 1.11

instead of real EMF their internal resistances are stored.



Current  $I_1$  in the given circuit (Fig. 1.16) is determined by the formula

$$I_1 = \frac{E_{eg}}{R_1 + R_{eg}} = \frac{U_{abnl}}{R_1 + R_{eg}}. \quad (1)$$

For the circuit shown in Fig. 1.11, we determine the voltage  $U_{abnl}$  according to Kirchhoff's voltage law for the loop  $a-c-d-b-a$  (circuit III) when bypassing it counterclockwise

$$E_1 = I_{3nl} \cdot R_3 + U_{abnl}, \text{ where}$$

$$U_{abnl} = E_1 - I_{3nl} \cdot R_3. \quad (2)$$

To determine the current,  $I_{3nl}$  we use the mesh current method.

For two independent loops I and II, by bypassing them clockwise, we can write the equation according to Kirchhoff's voltage law in general:

$$\begin{cases} R_{11} \cdot I_{11} + R_{12} \cdot I_{22} = E_{11}; \\ R_{21} \cdot I_{11} + R_{22} \cdot I_{22} = E_{22}, \end{cases} \quad (3)$$

where the loop EMF:

$$E_{11} = E_1 - E_2 = 120 - 60 = 60 \text{ V};$$

$$E_{22} = E_2 = 60 \text{ V},$$

total (own) resistances of loops:

$$R_{11} = R_3 + R_4 + R_5 = 4 + 6 + 10 = 20 \text{ Ohm};$$

$$R_{22} = R_5 + R_6 = 10 + 15 = 25 \text{ Ohm},$$

mutual resistance of loops is  $R_{12} = R_{21} = -R_5 = -10 \text{ Ohm}$ .

After substituting the numerical values, the system of equations (3) has the form:

$$\begin{cases} 20 \cdot I_{11} - 10 \cdot I_{22} = 60; \\ -10 \cdot I_{11} + 25 \cdot I_{22} = 60. \end{cases}$$

The general determinant of the system is

$$\Delta = \begin{vmatrix} 20 & -10 \\ -10 & 25 \end{vmatrix} = 20 \cdot 25 - (-10) \cdot (-10) = 400.$$

The determinant of the system needed to find the current  $I_{11} = I_{3nl}$  is

$$\Delta_1 = \begin{vmatrix} 60 & -10 \\ 60 & 25 \end{vmatrix} = 60 \cdot 25 - 60 \cdot (-10) = 2100.$$

$$\text{Then } I_{11} = I_{3nl} = \frac{\Delta_1}{\Delta} = \frac{2100}{400} = 5,25 \text{ A.}$$

After substituting the numerical values in equation (2)

$$U_{abnl} = E_1 - I_{3nl} \cdot R_3 = 120 - 5,25 \cdot 4 = 99 \text{ V.}$$

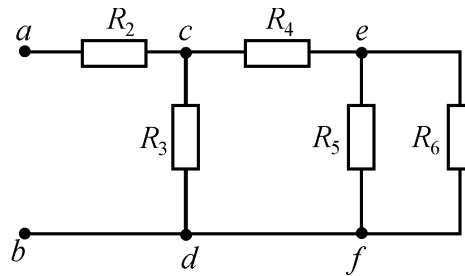


Figure 1.12

We determine the equivalent resistance  $R_{eg}$  for Fig. 1.12 as:

$$R_{56} = \frac{R_5 \cdot R_6}{R_5 + R_6} = \frac{10 \cdot 15}{10 + 15} = 6 \text{ Ohm};$$

$$R_{456} = R_4 + R_{56} = 6 + 6 = 12 \text{ Ohm};$$

$$R_{eg} = R_2 + \frac{R_3 \cdot R_{456}}{R_3 + R_{456}} = 5 + \frac{4 \cdot 12}{4 + 12} = 8 \text{ Ohm.}$$

We determine the current  $I_1$  as

$$I_1 = \frac{U_{abnl}}{R_1 + R_{eg}} = \frac{99}{10 + 8} = 5,5 \text{ A.}$$

## PRACTICE TOPIC № 2

### CALCULATION OF ELECTRIC CIRCUITS OF SINUSOID CURRENT

#### Formulation of Problems

The formulation of practice topic 2 consists of the next Problems.

The content of the assignment, i.e. the total number of Problems and their specific numbers, may be varied by the teacher depending on the scope of the training course.

Each student is given a variant of input for three loads ( $Z_\alpha, Z_\beta, Z_\gamma$ ), used in all Problems (Table 2.1).

#### Problem 5. Calculation of a single-phase electric circuit with a sinusoidal voltage source and a series connection of loads.

To calculate the electrical circuit in Fig. 2.1 with a series connection of loads  $Z_\alpha, Z_\beta, Z_\gamma$ , parameters which must be taken from the Table 2.1. The energy source voltage is  $U = 220$  V and its frequency is equal to 50 Hz.

**Calculation volume:** a) draw the electrical scheme in Fig. 2.1 with the ideal elements of substitution of each load; b) determine the current in the circuit, the voltages across each load, active and reactive power of the source and loads; c) build a phasor diagram of the current and voltages of the source and all loads; d) check the solution of the Problem with the help of the phasor diagram and balance of active and reactive power; e) build the sinusoidal time functions of the current and voltage of the energy source.

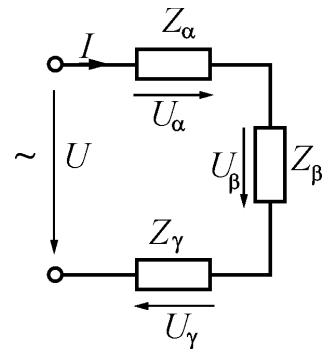


Figure 2.1

#### Problem 6. Calculation of a single-phase electric circuit with a sinusoidal voltage source of electrical energy and a parallel connection of loads.

Calculate the electrical circuit in Fig. 2.2 with parallel connection of loads  $Z_\alpha, Z_\beta, Z_\gamma$ , the parameters of which are defined in **Problem 5**. The voltage source is  $U = 127$  V and its frequency is equal to 50 Hz.

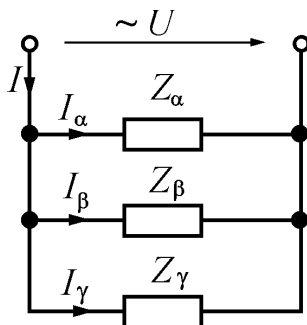


Figure 2.2

**Calculation volume:** a) draw the electrical circuit in Fig. 2.2 with the ideal elements of replacement for each load; b) determine all currents in the circuit, active and reactive power of the source and loads; c) build a phasor diagram of voltage and currents of all loads; d) check the solution of the Problem by means of the phasor diagram and balance of active and reactive power.

**Problem 7. Calculation of a single-phase electric circuit of sinusoidal voltage source and a mixed loads connection using the symbolic method.**

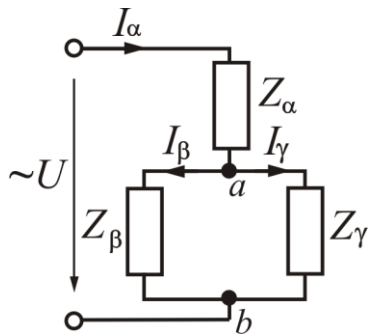


Figure 2.3

Carry out the calculation of the electrical circuit in Fig. 2.3 with a mixed connection of loads  $Z_\alpha, Z_\beta, Z_\gamma$ , which parameters are defined in **Problem 5**. The voltage source is  $U = 220 \text{ V}$  and its frequency  $f$  is equal to  $50 \text{ Hz}$ .

**Calculation volume:** a) draw the equivalent electrical scheme in Fig. 2.3 with ideal elements instead of each load; b) determine the currents in the circuit, active and reactive power of the energy source and loads; c) build a phasor diagram of voltages and currents; d) check the solution of the Problem using a phasor diagram and the balance of active and reactive powers.

**Problem 8. Calculation of a three-phase electric circuit with a balanced load connected in wye.**

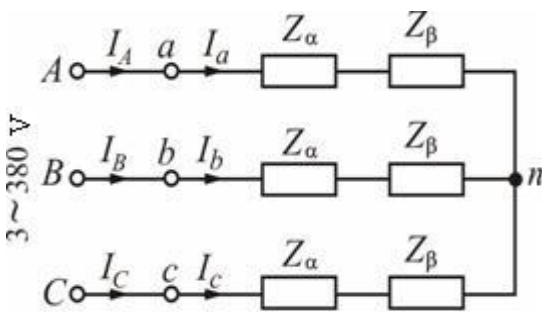


Figure 2.4

In a three-phase circuit with line voltage  $U_L = 380 \text{ V}$  a balanced load is switched on as a series connection of the loads  $Z_\alpha, Z_\beta$  in each phase (Fig. 2.4). The loads  $Z_\alpha, Z_\beta$  are the same as in from **Problem 5**.

**Calculation volume:** a) draw the electrical scheme of replacement of three-phase circuit with ideal elements instead of loads; b) determine phase and line currents, as well as active and reactive power; c) build a phasor diagram.

**Problem 9. Calculation of a three-phase electric circuit with a balanced load connected in delta.**

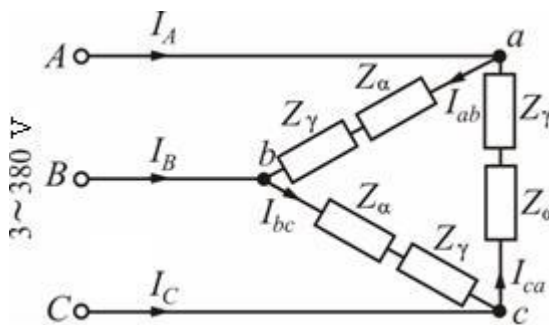
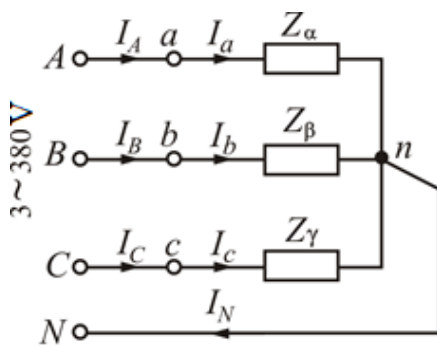


Figure 2.5

In a three-phase circuit with line voltage  $U_L = 380 \text{ V}$  a balanced load is switched on as series connection of the loads  $Z_\alpha, Z_\gamma$  in each phase (Fig. 2.5). The loads  $Z_\alpha, Z_\gamma$  take from **Problem 5**.

**Calculation volume:** a) draw the electric equivalent scheme of three-phase electric circuit with ideal elements instead of the loads; b) determine phase and line currents, as well as active and reactive power; c) build a phasor diagram.

**Problem 10. Calculation of a three-phase electric circuit with an unbalanced load connected in wye with the neutral wire.**



**Figure 2.6**

In a three-phase network with line voltage  $U_L = 380 \text{ V}$  the loads  $Z_\alpha$ ,  $Z_\beta$ ,  $Z_\gamma$  are switched on in the phases (Fig. 2.6), which due to their inequality leads to an unbalanced circuit load. The loads parameters  $Z_\alpha$ ,  $Z_\beta$ ,  $Z_\gamma$  are taken from **Problem 5**.

**Calculation volume:** a) draw a three-phase electric circuit with ideal elements instead of the loads  $Z_\alpha$ ,  $Z_\beta$ ,  $Z_\gamma$ ; b) determine the phase and line currents, as

well as the current in the neutral wire; c) determine the active and reactive power; d) build a phasor diagram.

**Table 2.1 – Input data of AC load parameters**

Variant	$Z_\alpha$			$Z_\beta$			$Z_\gamma$		
	$R_\alpha$ , Ohm	$L_\alpha$ , mH	$C_\alpha$ , uF	$R_\beta$ , Ohm	$L_\beta$ , mH	$C_\beta$ , uF	$R_\gamma$ , Ohm	$L_\gamma$ , mH	$C_\gamma$ , uF
1	12	50,96	-	12	-	199,04	-	76,43	-
2	7	76,43	-	5	-	265,39	20	-	-
3	20	-	212,31	10	95,54	-	-	-	127,39
4	25	-	-	12	50,96	-	10	-	90,99
5	8	19,11	-	-	-	176,93	10	63,69	-
6	-	79,62	-	20	47,77	-	-	-	127,4
7	12	50,96	-	12	-	79,62	-	152,87	-
8	5	-	265,39	15	63,69	-	25	-	-
9	16	38,22	-	7	76,43	-	-	-	63,69
10	12	50,96	-	12	-	199,04	-	-	133
11	-	-	144,76	24	101,91	-	12	15,92	-
12	7	76,43	-	12	-	636,94	-	-	83,81
13	-	63,69	-	24	22,29	-	16	-	265,39
14	10	-	-	12	-	199,04	7	76,43	-
15	12	50,96	-	8	-	530,79	-	-	106,16
16	24	-	176,93	6	25,48	-	-	101,91	-
17	20	-	-	15	63,69	-	5	-	265,39
18	-	63,69	-	10	-	318,47	10	31,85	-
19	18	76,43	-	-	-	79,62	12	-	199,04
20	17	25,48	-	5	38,22	-	-	-	199,52
21	-	-	96,51	30	41,4	-	15	63,69	-
22	15	25,48	-	5	38,22	-	-	-	159,24

Variant	$Z_{\alpha}$			$Z_{\beta}$			$Z_{\gamma}$		
	$R_{\alpha}$ , Ohm	$L_{\alpha}$ , mH	$C_{\alpha}$ , uF	$R_{\beta}$ , Ohm	$L_{\beta}$ , mH	$C_{\beta}$ , uF	$R_{\gamma}$ , Ohm	$L_{\gamma}$ , mH	$C_{\gamma}$ , uF
23	27	114,65	-	-	-	63,69	13	44,59	-
24	9	41,4	-	13	28,66	-	-	-	106,16
25	7	-	132,7	7	76,43	-	25	-	-
26	-	105,1	-	20	-	212,31	24	-	454,96
27	12	-	199,04	12	50,96	-	20	-	-
28	-	-	90,99	20	47,77	-	20	-	212,31
29	16	38,22	-	12	50,96	-	-	-	199,04
30	-	-	79,62	20	63,69	-	20	63,69	-
31	20	-	-	7	-	132,7	5	38,22	-
32	12		199,04	12	50,96	-	22	-	-
33	15	63,69	-	20	47,77	-	-	-	144,76
34	30	-	79,62	30	127,39	-	-	159,24	-
35	-	79,62	-	10	-	212,31	15	-	318,47
36	6	25,48	-	16	38,22	-	-	-	72,38
37	-	-	95,51	15	41,4	-	15	63,69	-
38	24	57,32	-	15	31,85	-	-	-	113,74
39	18	-	132,7	10	-	212,31	-	124,2	-
40	11	-	167,17	11	60,67	-	-	121,34	-
41	24	57,32	-	24	-	176,93	-	-	114,65
42	18	-	132,7	18	76,43	-	14	-	-
43	-	127,39	-	30	-	79,62	15	-	159,24
44	5	-	265,39	15	63,69	-	10	-	-
45	12	50,96	-	8	-	530,78	-	-	106,16
46	6	25,48	-	16	38,22	-	-	-	159,24
47	7	76,43	-	-	-	66,35	12	-	636,94
48	-	-	106,16	20	47,77	-	10	143,31	-
49	8	19,11	-	12	50,96	-	-	-	144,76
50	14	-	66,35	14	152,87	-	50	-	-
51	20	95,54	-	10	31,85	-	-	-	159,24
52	-	-	127,39	20	-	212,31	20	127,39	-
53	24	-	99,52	24	101,92	-	40	-	-
54	-	105,1	-	30	-	244,98	20	-	159,24
55	30	-	212,31	10	47,77	-	-	79,62	-
56	-	63,69	-	20	63,69	-	20	-	212,31
57	16	-	132,7	-	-	132,7	24	76,43	-
58	18	82,8	-	26	57,32	-	-	-	53,08
59	20	-	-	18	-	132,7	18	76,43	-
60	-	-	117,96	10	117,83	-	12	-	318,47
61	40	95,54	-	7	76,43	-	-	-	318,47

Variant	$Z_{\alpha}$			$Z_{\beta}$			$Z_{\gamma}$		
	$R_{\alpha}$ , Ohm	$L_{\alpha}$ , mH	$C_{\alpha}$ , uF	$R_{\beta}$ , Ohm	$L_{\beta}$ , mH	$C_{\beta}$ , uF	$R_{\gamma}$ , Ohm	$L_{\gamma}$ , mH	$C_{\gamma}$ , uF
62	20	-	-	12	-	199,04	12	50,96	-
63	25	111,46	-	20	-	212,31	22	-	-
64	22	-	-	5	38,22	-	5	-	265,39
65	-	159,42	-	40	95,54	-	14	-	66,35
66	18	76,43	-	18	-	-	9	-	79,62
67	-	-	159,24	30	111,46	-	20	-	212,31
68	18	-	132,7	18	-	-	9	127,39	-
69	20	-	212,31	30	111,46	-	-	-	159,24
70	40	-	106,16	7	-	132,7	-	31,85	-
71	20	-	-	12	50,96	-	12	-	199,04
72	-	127,39	-	20	-	159,24	20	63,69	-
73	15	-	66,35	15	146,5	-	50	-	-
74	-	114,65	-	20	-	212,31	20	-	88,46
75	20	63,69	-	25	-	159,24	-	114,65	-
76	8	19,11	-	8	-	530,79	20	-	-
77	-	-	127,39	25	15,92	-	25	-	636,94
78	40	-	79,62	-	127,39	-	20	63,69	-
79	17	31,85	-	12	15,92	-	-	-	127,39
80	-	-	117,95	27	104,14	-	15	63,69	-
81	-	159,24	-	24	-	132,7	20	-	79,62
82	-	-	95,64	20	58,28	-	20	-	174,03
83	10	-	183,87	10	55,16	-	30	-	-
84	12	38,6	-	18	-	-	30	-	79,62
85	-	-	83,81	19	60,51	-	11	63,69	-
86	12	-	199,04	25	-	-	12	50,96	-
87	25	-	-	12	50,96	-	9	-	79,62
88	22	-	-	22	-	83,59	22	121,34	-
89	-	89,17	-	28	-	113,74	12	50,96	-
90	35	-	90,99	30	-	-	10	31,85	-
91	12	50,96	-	20	-	212,31	-	-	144,76
92	18	-	132,7	-	63,69	-	20	47,77	-
93	-	70,06	-	24	-	454,96	25	-	127,39
94	-	-	144,76	27	88,46	-	15	-	398,09
95	16	28,66	-	-	-	144,76	12	-	636,94
96	-	-	127,39	20	47,77	-	15	-	159,24
97	-	79,62	-	15	-	159,24	7	76,43	-
98	-	-	144,76	18	76,43	-	12	50,96	-
99	20	-	212,31	-	70,06	-	18	-	132,76
100	15	63,69	-	22	-	83,59	25	-	-

## Examples of the Problems solution

**Example 5.** Calculate an electrical circuit with a series connection of loads  $Z_\alpha$ ,  $Z_\beta$  and  $Z_\gamma$ , the scheme of which is shown in Fig. 2.1. The current voltage value  $U = 200$  V, its frequency  $f = 50$  Hz. Parameters of ideal elements of the loads:  $R_\alpha = 20$  Ohm;  $L_\alpha = 47,77$  mH;  $R_\beta = 23,3$  Ohm;  $C_\gamma = 79,62$   $\mu$ F.

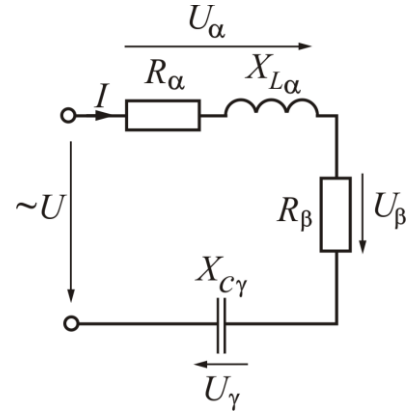


Figure 2.7

**Calculation volume:** a) draw the electrical scheme in Fig. 2.1 with the ideal elements of substitution of each load; b) determine current in the circuit, voltages across each load, active and reactive power of the source and loads; c) build a phasor diagram of the current and voltages of the source and all loads; d) check the solution of the Problem with the help of phasor diagram and balance of active and reactive power; e) build the sinusoidal time functions of the current and voltage of the energy source and draw these functions.

Angular frequency of current and voltage is

$$\omega = 2\pi \cdot f = 2 \cdot 3,14 \cdot 50 = 314 \text{ s}^{-1}.$$

Inductive reactance of the load  $Z_\alpha$  is

$$X_{L\alpha} = \omega \cdot L_\alpha = 314 \cdot 47,77 \cdot 10^{-3} = 15 \text{ Ohm}.$$

Capacitive reactance of the load  $Z_\gamma$  is

$$X_{C\gamma} = \frac{1}{\omega \cdot C_\gamma} = \frac{1}{314 \cdot 79,62 \cdot 10^{-6}} = 40 \text{ Ohm}.$$

Resistance, reactance and impedance of the circuit accordingly are:

$$R = R_\alpha + R_\beta = 20 + 23,3 = 43,3 \text{ Ohm}; \quad X = X_{L\alpha} - X_{C\gamma} = 15 - 40 = -25 \text{ Ohm};$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{43,3^2 + (-25)^2} = 50 \text{ Ohm}.$$

The RMS (root mean square) current value in the electrical circuit we define as

$$I = \frac{U}{Z} = \frac{200}{50} = 4 \text{ A}.$$

The phase shift angle between the current phasors and the source voltage is

$$\varphi = \arctg \frac{X}{R} = \arctg \frac{-25}{43,3} = -30^\circ.$$



Impedances of the loads are:

$$Z_{\alpha} = \sqrt{R_{\alpha}^2 + X_{L\alpha}^2} = \sqrt{20^2 + 15^2} = 25 \text{ Ohm};$$

$$Z_{\beta} = R_{\beta} = 23,3 \text{ Ohm}; \quad Z_{\gamma} = X_{C\gamma} = 40 \text{ Ohm}.$$

The voltages RMS values across the loads are:

$$U_{\alpha} = I Z_{\alpha} = 4 \cdot 25 = 100 \text{ V}; \quad U_{\beta} = I Z_{\beta} = 4 \cdot 23,3 = 93,2 \text{ V};$$

$$U_{\gamma} = I Z_{\gamma} = 4 \cdot 40 = 160 \text{ V}.$$

Phase difference angles between the current phasor  $\underline{I}$  and voltages phasors  $\underline{U}_{\alpha}$ ,  $\underline{U}_{\beta}$ ,  $\underline{U}_{\gamma}$  across the loads are:

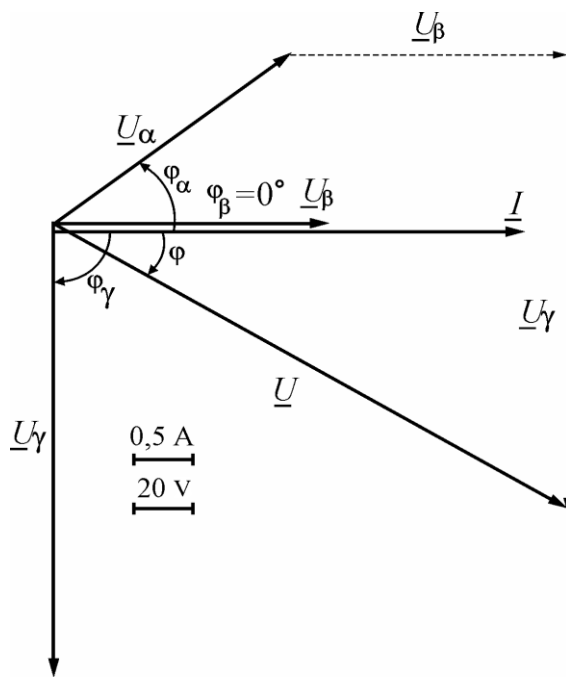


Figure 2.8

$$\varphi_{\alpha} = \arctg \frac{X_{L\alpha}}{R_{\alpha}} = \arctg \frac{15}{20} = 36,87^{\circ};$$

$$\varphi_{\beta} = \arctg \frac{0}{R_{\beta}} = \arctg \frac{0}{23,3} = 0^{\circ};$$

$$\varphi_{\gamma} = \arctg \frac{-X_{C\gamma}}{0} = \arctg \frac{-40}{0} = -90^{\circ}.$$

Figure 2.8 shows a phasor diagram. Let's explain the order of solving.

For the circuit in Fig. 2.7 common is the current for which we take the initial phase  $\psi_i = 0^{\circ}$ . Choose the current scale and draw a phasor  $\underline{I}$  along the horizontal axis. Next, choose the voltage scale and draw the voltage phasor of the source at an angle  $\varphi$  clockwise relative to the current phasor  $\underline{I}$ , as well as voltage phasors

across the loads  $\underline{U}_{\alpha}$ ,  $\underline{U}_{\beta}$ ,  $\underline{U}_{\gamma}$  at the appropriate phase shift angles  $\varphi_{\alpha}$ ,  $\varphi_{\beta}$ ,  $\varphi_{\gamma}$  relative to the phasor  $\underline{I}$ .

Using the phasor diagram, we check the solution based on Kirchhoff's voltage law for a loop in Fig. 2.7. The phasor sum of load's voltages gives the voltage phasor of the energy source, i.e.  $\underline{U} = \underline{U}_{\alpha} + \underline{U}_{\beta} + \underline{U}_{\gamma}$  (Fig. 2.8), which on the accepted scale must be equal to 200 V.

Total, active and reactive powers of the energy source respectively are:

$$S = U \cdot I = 200 \cdot 4 = 800 \text{ VA};$$

$$P = U \cdot I \cdot \cos \varphi = 200 \cdot 4 \cdot \cos(-30^{\circ}) = 692,8 \text{ W};$$

$$Q = U \cdot I \cdot \sin \varphi = 200 \cdot 4 \cdot \sin(-30^{\circ}) = -400 \text{ VAr}.$$

The ratio of these powers is shown in Fig. 2.9 using their triangle.

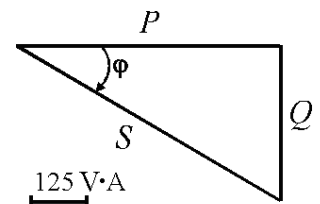


Figure 2.9

According to the balance of active powers

$$P = R_{\alpha} \cdot I^2 + R_{\beta} \cdot I^2 = 20 \cdot 4^2 + 23,3 \cdot 4^2 = 692,8 \text{ W},$$

which coincides with the active power of the source.

According to the balance of reactive powers

$$Q = X_{L\alpha} \cdot I^2 - X_{C\gamma} \cdot I^2 = 15 \cdot 4^2 - 40 \cdot 4^2 = -400 \text{ var},$$

which coincides with the reactive power of the source.

The amplitude values of current and voltage of the energy source are:

$$I_m = \sqrt{2} \cdot I = \sqrt{2} \cdot 4 = 5,66 \text{ A}; \quad U_m = \sqrt{2} \cdot U = \sqrt{2} \cdot 200 = 282 \text{ V}.$$

Sinusoidal time functions of current and voltage of the energy source are:

$$i = I_m \cdot \sin(\omega t + \psi_i) = 5,66 \cdot \sin 314 t \text{ A};$$

$$u = U_m \sin(\omega t + \psi_u) = 282,8 \cdot \sin(314 t - 30^\circ) \text{ V},$$

where the initial phase of the current is already accepted ( $\psi_i = 0^\circ$ ), and the initial phase  $\psi_u$  find, remembering that the angle  $\varphi = \psi_u - \psi_i$ :  $\psi_u = \varphi + \psi_i = -30^\circ + 0^\circ = -30^\circ$ .

Graphs of current and voltage sine curves are drawn when using no time scale  $t$ , and a multiple of its scale  $\omega t$ , which is more convenient when building these graphs. In the Table 2.2 are shown the calculations of current and voltage for different values  $\omega t$  and in Fig. 2.10 are shown the graphs of time functions  $i(t)$  and  $u(t)$ .

Table 2.2 – The value of current and voltage of the energy source depending on time

$\omega t, \text{ rad}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	$2\pi$
$\omega t, \text{ grad}$	0	30	60	90	120	150	180	210	240	270	300	330	360
$i, \text{ A}$	0	2,83	4,9	5,66	4,9	2,83	0	-2,83	-4,9	-5,66	-4,9	-2,83	0
$u, \text{ V}$	-141	0	141	245	283	245	141	0	-141	-245	-283	-245	-141

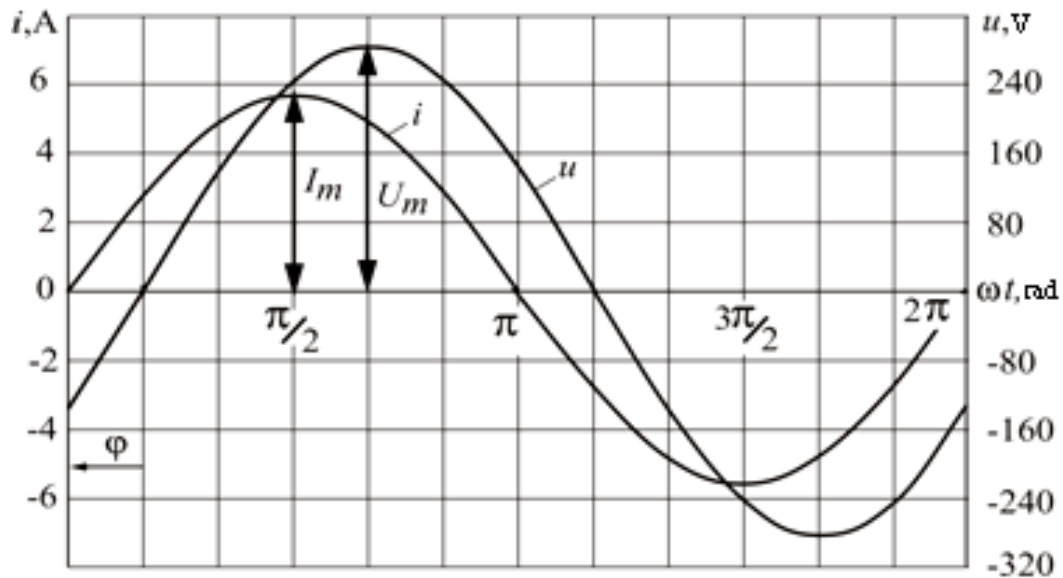


Figure 2.10

**Example 6.** Calculate the electrical circuit with a parallel connection of the loads  $Z_\alpha$ ,  $Z_\beta$  and  $Z_\gamma$ , the electrical scheme of which is shown in Fig. 2.2 (parameters of which are defined in **Problem 5**). The voltage value  $U = 220$  V and its frequency  $f = 50$  Hz. Parameters of ideal elements of the loads:  $R_\alpha = 24$  Ohm;  $X_{C\alpha} = 32$  Ohm;  $R_\beta = 20$  Ohm;  $X_{L\beta} = 20$  Ohm;  $X_{L\gamma} = 80$  Ohm.

Calculate the electrical circuit in Fig. 2.2 with parallel connection of loads  $Z_\alpha$ ,  $Z_\beta$ ,  $Z_\gamma$ , the parameters of which are defined in Problem 1. The source voltage  $U = 220$  V at a frequency of  $f = 50$  Hz.

**Calculation volume:** a) draw the electrical scheme in Fig. 2.2 with the ideal elements of replacement for each load; b) determine currents in the circuit, active and reactive power of the source and loads; c) draw a phasor diagram of voltage and currents of all loads; d) check the solution of the problem by means of a phasor diagram and balance of active and reactive power.

### Solution

In the Fig. 2.11 is shown the electrical scheme of the substitution of the electrical circuit in Fig. 2.2 with ideal elements instead  $Z_\alpha$ ,  $Z_\beta$ ,  $Z_\gamma$ .

The impedances values of the parallel branches are equal to:

$$Z_\alpha = \sqrt{R_\alpha^2 + (-X_{C\alpha})^2} = \sqrt{24^2 + (-32)^2} = 40 \text{ Ohm};$$

$$Z_\beta = \sqrt{R_\beta^2 + X_{L\beta}^2} = \sqrt{20^2 + 20^2} = 28,28 \text{ Ohm};$$

$$Z_\gamma = X_{L\gamma} = 80 \text{ Ohm}.$$

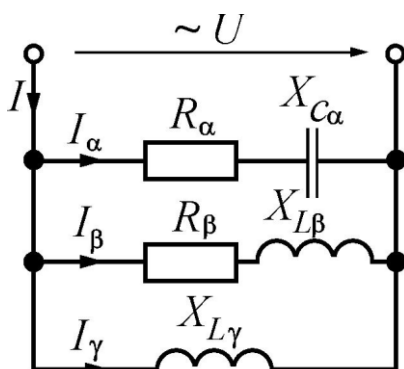


Figure 2.11

The RMS values of currents in the parallel branches are:

$$I_\alpha = \frac{U}{Z_\alpha} = \frac{220}{40} = 5,5 \text{ A}; \quad I_\beta = \frac{U}{Z_\beta} = \frac{220}{28,28} = 7,78 \text{ A}; \quad I_\gamma = \frac{U}{Z_\gamma} = \frac{220}{80} = 2,75 \text{ A}.$$

Phase difference angles between the current phasors  $\underline{I}_\alpha, \underline{I}_\beta, \underline{I}_\gamma$  and voltage phasor  $\underline{U}$ :

$$\varphi_\alpha = \arctg \frac{-X_{C\alpha}}{R_\alpha} = \arctg \frac{-32}{24} = -53,13^\circ;$$

$$\varphi_\beta = \arctg \frac{X_{L\beta}}{R_\beta} = \arctg \frac{20}{20} = 45^\circ;$$

$$\varphi_\gamma = 90^\circ \text{ (inductive element).}$$

To determine the source current, we use a phasor diagram (Fig. 2.12), where the phasors are built at these scales. We are the first to build a voltage phasor  $\underline{U}$ . Its initial phase is accepted arbitrarily ( $\psi_u = 0^\circ$ ). Current phasors  $\underline{I}_\alpha, \underline{I}_\beta$  and  $\underline{I}_\gamma$  we construct using the initial phases. Because  $\psi_u = 0^\circ$ , the initial phases are determined by formulas:

$$\psi_{i_\alpha} = -\varphi_\alpha = 53,13^\circ; \quad \psi_{i_\beta} = -\varphi_\beta = -45^\circ; \quad \psi_{i_\gamma} = -\varphi_\gamma = -90^\circ.$$

According to Kirchhoff's current law (Fig. 2.11), the phasor of the source current is the phasor sum of the currents of the parallel branches, i.e.  $\underline{I} = \underline{I}_\alpha + \underline{I}_\beta + \underline{I}_\gamma$  (Fig. 2.12). The geometric addition of these phasors gives the value of the source current on the basis of measurements  $I \approx 9,7\text{ A}$  and the phase shift angle  $\varphi \approx 24^\circ$ .

More accurate results can be obtained directly by calculations, using the decomposition of current phasors into active and reactive components: the first – parallel to the phasor  $\underline{U}$ , and the second is perpendicular to it. The diagram of phasors on components is given on the phasor diagram (Fig. 2.12).

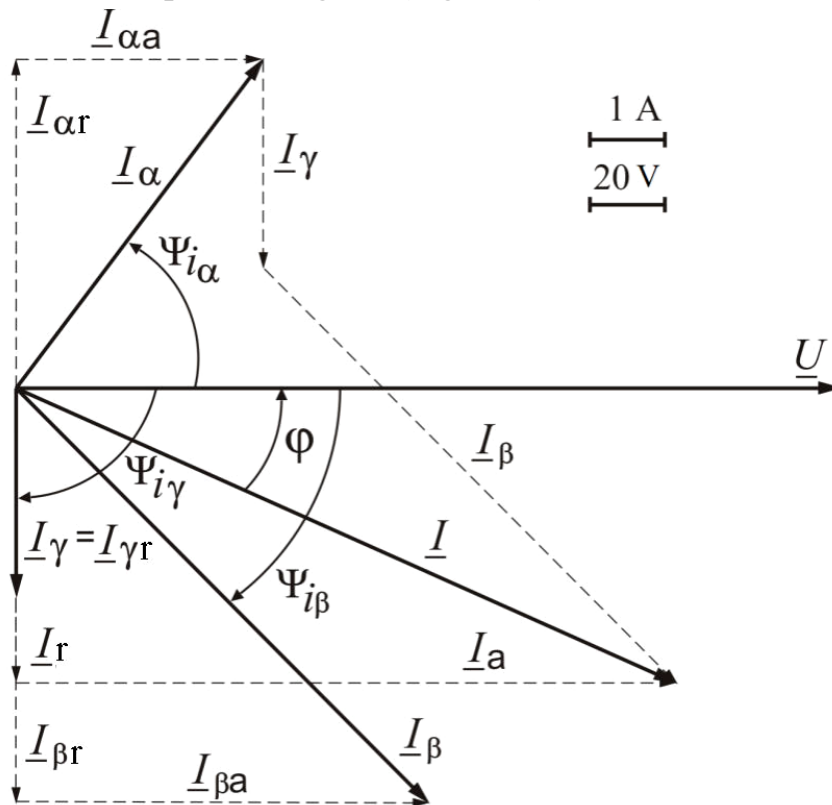


Figure 2.12

Active and reactive components of the load currents:

$$I_{\alpha a} = I_{\alpha} \cos \psi_{i_{\alpha}} = 5,5 \cos 53,13^{\circ} = 3,3 \text{ A};$$

$$I_{\beta a} = I_{\beta} \cos \psi_{i_{\beta}} = 7,78 \cos(-45^{\circ}) = 5,5 \text{ A};$$

$$I_{\gamma a} = I_{\gamma} \cos \psi_{i_{\gamma}} = 2,75 \cos(-90^{\circ}) = 0 \text{ A};$$

$$I_{\alpha r} = I_{\alpha} \sin \psi_{i_{\alpha}} = 5,5 \sin 53,13^{\circ} = 4,4 \text{ A};$$

$$I_{\beta r} = I_{\beta} \sin \psi_{i_{\beta}} = 7,78 \sin(-45^{\circ}) = -5,5 \text{ A};$$

$$I_{\gamma r} = I_{\gamma} \sin \psi_{i_{\gamma}} = 2,75 \sin(-90^{\circ}) = -2,75 \text{ A}.$$

Active and reactive components of the source current and its RMS (root-mean-square) value:

$$I_a = I_{\alpha a} + I_{\beta a} + I_{\gamma a} = 3,3 + 5,5 + 0 = 8,8 \text{ A};$$

$$I_r = I_{\alpha r} + I_{\beta r} + I_{\gamma r} = 4,4 - 5,5 - 2,75 = -3,85 \text{ A};$$

$$I = \sqrt{I_a^2 + I_r^2} = \sqrt{8,8^2 + (-3,85)^2} = 9,61 \text{ A}.$$

The phase difference angle of the current phasor  $\underline{I}$  is relatively to the voltage phasor  $\underline{U}$ :

$$\varphi = -\psi_i = -\arctg \frac{I_r}{I_a} = -\arctg \frac{-3,85}{8,8} = 23,63^{\circ}.$$

Apparent, active and reactive source powers are:

$$S = U \cdot I = 220 \cdot 9,61 = 2114,2 \text{ VA}; \quad P = S \cos \varphi = 2114,2 \cdot \cos 23,63^{\circ} = 1936,9 \text{ W};$$

$$Q = S \sin \varphi = 2114,2 \cdot \sin 23,63^{\circ} = 847,4 \text{ var}.$$

According to the balance of active and reactive powers:

$$P = R_{\alpha} \cdot I_{\alpha}^2 + R_{\beta} \cdot I_{\beta}^2 = 24 \cdot 5,5^2 + 20 \cdot 7,78^2 = 1936,6 \text{ W},$$

$$Q = -X_{C\alpha} \cdot I_{\alpha}^2 + X_{L\beta} \cdot I_{\beta}^2 + X_{L\gamma} \cdot I_{\gamma}^2 = -32 \cdot 5,5^2 + 20 \cdot 7,78^2 + 80 \cdot 2,75^2 = 847,6 \text{ var}.$$

**Example 7.** Calculate the electrical circuit in Fig. 2.3 at the mixed connection of the loads  $Z_{\alpha}, Z_{\beta}$  and  $Z_{\gamma}$  by a symbolic method, i.e. using complex numbers. The energy source voltage  $U = 127 \text{ V}$ , its frequency  $f = 50 \text{ Hz}$ . Parameters of the ideal elements of loads:  $R_{\alpha} = 8 \text{ Ohm}$ ;  $X_{L\alpha} = 15 \text{ Ohm}$ ;  $R_{\beta} = 16 \text{ Ohm}$ ;  $X_{C\beta} = 12 \text{ Ohm}$ ;  $X_{L\gamma} = 42 \text{ Ohm}$ .

In a three-phase circuit with line voltage  $U_L = 380 \text{ V}$  balanced load is activated when connected in series with the loads  $Z_{\alpha}, Z_{\beta}$  in each phase (Fig. 2.3). The loads  $Z_{\alpha}, Z_{\beta}$  will be taken from **Problem 5**.

**Calculation volume:** a) draw the electrical scheme of replacement of three-phase circuit with ideal elements instead of the loads  $Z_\alpha, Z_\beta, Z_\gamma$ ; b) determine phase and line currents, as well as active and reactive powers; c) build a phasor diagram.

### Solution

The electrical scheme of substitution of the electrical circuit in Fig. 2.3 with ideal elements is shown in Fig. 2.13. Currents, voltages and also loads will be defined by a symbolic method therefore ideal elements which are connected in series, unite in total complex impedances. From the initial electrical scheme according to Fig. 2.13 we come to the electrical scheme with complex impedances of branches (Fig. 2.14, a) where currents and voltages are also represented in a complex form.

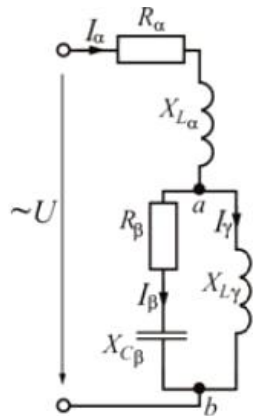


Figure 2.13

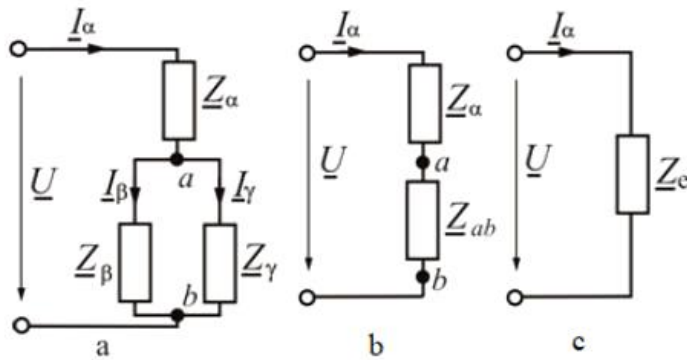


Figure 2.14

The values of complex impedances of branches in algebraic and exponential forms are:

$$\underline{Z}_\alpha = R_\alpha + jX_{L\alpha} = 8 + j15 = \sqrt{8^2 + 15^2} \cdot e^{j \arctg \frac{15}{8}} = 17 \cdot e^{j61,93^\circ} \text{ Ohm};$$

$$\underline{Z}_\beta = R_\beta - jX_{C\beta} = 16 - j12 = \sqrt{16^2 + (-12)^2} \cdot e^{j \arctg \frac{-12}{16}} = 20 \cdot e^{-j36,87^\circ} \text{ Ohm};$$

$$\underline{Z}_\gamma = jX_{L\gamma} = j42 = 42 \cdot e^{j \arctg \frac{42}{0}} = 42 \cdot e^{j90^\circ} \text{ Ohm}.$$

A part of electrical circuit with a parallel connection  $\underline{Z}_\beta$  and  $\underline{Z}_\gamma$  replace with equivalent complex impedance

$$\begin{aligned} \underline{Z}_{ab} &= \frac{\underline{Z}_\beta \cdot \underline{Z}_\gamma}{\underline{Z}_\beta + \underline{Z}_\gamma} = \frac{20 \cdot e^{-j36,87^\circ} \cdot 42 \cdot e^{j90^\circ}}{16 - j12 + j42} = \frac{840 \cdot e^{j53,13^\circ}}{16 + j30} = \frac{840 \cdot e^{j53,13^\circ}}{\sqrt{16^2 + 30^2} \cdot e^{j \arctg \frac{30}{16}}} = \\ &= \frac{840 \cdot e^{j53,13^\circ}}{34 \cdot e^{j61,93^\circ}} = 24,71 \cdot e^{-j8,8^\circ} = 24,71 [\cos(-8,8^\circ) + j \sin(-8,8^\circ)] = 24,42 - j3,78 \text{ Ohm} \end{aligned}$$

and we obtain a simpler scheme for Fig. 2.14, b.

Two loads connected in series  $\underline{Z}_\alpha$  and  $\underline{Z}_{ab}$  for Fig. 2.14,  $b$  is replaced by one impedance, which is equivalent to the impedance of the whole circuit (Fig. 2.14,  $c$ )

$$\begin{aligned}\underline{Z}_e &= \underline{Z}_\alpha + \underline{Z}_{ab} = 8 + j15 + 24,42 - j3,78 = 32,42 + j11,22 = \\ &= \sqrt{32,42^2 + 11,22^2} \cdot e^{j \arctg \frac{11,22}{32,42}} = 34,3 \cdot e^{j19,09^\circ} \text{ Ohm.}\end{aligned}$$

The complex value of source voltage  $\underline{U} = U \cdot e^{j\psi_u} = 127 \cdot e^{j0^\circ} = 127 \text{ V}$ , if the initial phase of this voltage is accepted  $\psi_u = 0^\circ$ .

Current source according to Ohm's law (Fig. 2.14,  $c$ )

$$\underline{I}_\alpha = \frac{\underline{U}}{\underline{Z}_e} = \frac{127 \cdot e^{j0^\circ}}{34,3 \cdot e^{j19,09^\circ}} = 3,7 \cdot e^{-j19,09^\circ} = 3,5 - j1,21 \text{ A.}$$

The voltage across the load  $\underline{Z}_\alpha$  (Fig. 2.14,  $b$ )

$$\underline{U}_\alpha = \underline{Z}_\alpha \cdot \underline{I}_\alpha = 17 \cdot e^{j61,93^\circ} \cdot 3,7 \cdot e^{-j19,09^\circ} = 62,9 \cdot e^{j42,84^\circ} = 46,1 + j42,8 \text{ V.}$$

The voltage value on the terminals  $a-b$  (Fig. 2.14,  $b$ ) and, accordingly, on the loads  $\underline{Z}_\beta$  and  $\underline{Z}_\gamma$  (Fig. 2.14,  $a$ )

$$\underline{U}_{ab} = \underline{Z}_{ab} \cdot \underline{I}_\alpha = 24,71 \cdot e^{-j8,8^\circ} \cdot 3,7 \cdot e^{-j19,09^\circ} = 91,4 \cdot e^{-j27,89^\circ} = 80,8 - j42,8 \text{ V.}$$

The currents in the parallel branches (Fig. 2.14,  $a$ ):

$$\begin{aligned}\underline{I}_\beta &= \frac{\underline{U}_{ab}}{\underline{Z}_\beta} = \frac{91,4 \cdot e^{-j27,89^\circ}}{20 \cdot e^{-j36,87^\circ}} = 4,57 \cdot e^{j8,98^\circ} = 4,51 + j0,71 \text{ A;} \\ \underline{I}_\gamma &= \frac{\underline{U}_{ab}}{\underline{Z}_\gamma} = \frac{91,4 \cdot e^{-j27,89^\circ}}{42 \cdot e^{j90^\circ}} = 2,18 \cdot e^{-j117,89^\circ} = -1,02 - j1,93 \text{ A.}\end{aligned}$$

Checking currents according to Kirchhoff's current law (Fig. 2.14,  $a$ ):

$$\underline{I}_\beta + \underline{I}_\gamma = \underline{I}_\alpha ;$$

$$4,51 + j0,71 - 1,02 - j1,93 = 3,49 - j1,22 \text{ A (found earlier } \underline{I}_\alpha = 3,5 - j1,21 \text{ A).}$$

Checking voltages according to Kirchhoff's voltage law (Fig. 2.14,  $a$ ):

$$\underline{U}_\alpha + \underline{U}_{ab} = \underline{U};$$

$$46,1 + j42,8 + 80,8 - j42,8 = 126,9 \text{ V (accepted } U = 127 \text{ V).}$$

A small difference in values is due to truncating in the calculations.

Voltage ratio  $\underline{U} = \underline{U}_\alpha + \underline{U}_{ab}$  and currents  $\underline{I}_\alpha = \underline{I}_\beta + \underline{I}_\gamma$  shown in Fig. 2.15 using a phasor diagram at the specified scale. The structure of the diagram is performed according to the RMS values of currents and voltages and their initial phases. The RMS value of voltage and current is defined as a module, and the initial phase is an argument in the form of each complex number that determines the current or voltage.

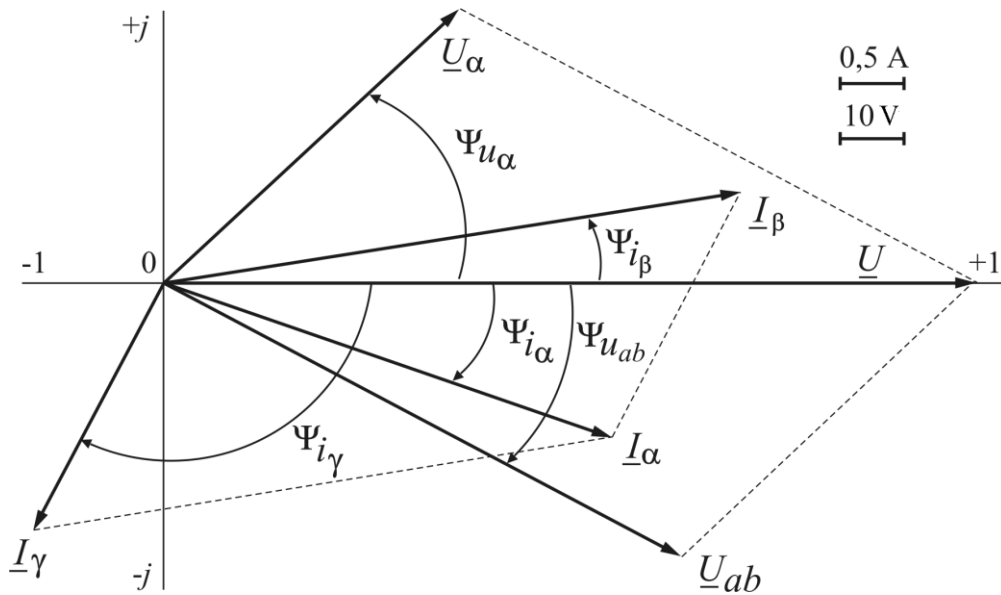


Figure 2.15

The RMS values of currents and voltages and their initial phases:  $I_\alpha = 3,7$  A;  $\psi_{i_\alpha} = -19,09^\circ$ ;  $I_\beta = 4,57$  A;  $\psi_{i_\beta} = 8,98^\circ$ ;  $I_\gamma = 2,18$  A;  $\psi_{i_\gamma} = -117,89^\circ$ ;  $U_\alpha = 62,9$  V;  $\psi_{u_\alpha} = 42,84^\circ$ ;  $U_{ab} = 91,4$  V;  $\psi_{u_{ab}} = -27,89^\circ$ ;  $U = 127$  V;  $\psi_u = 0^\circ$ .

The phasor diagram can also be build from real and imaginary parts of complex values of currents and voltages:

$$\underline{I}_\alpha = 3,5 - j1,21 \text{ A}; \underline{I}_\beta = 4,51 + j0,71 \text{ A}; \underline{I}_\gamma = -1,02 - j1,93 \text{ A};$$

$$\underline{U}_\alpha = 46,1 + j42,8 \text{ V}; \underline{U}_{ab} = 80,8 - j42,8 \text{ V}; \underline{U} = 127 \text{ V},$$

where the real values of the quantities:

$$I'_\alpha = 3,5 \text{ A}; I'_\beta = 4,51 \text{ A}; I'_\gamma = -1,02 \text{ A}; U'_\alpha = 46,1 \text{ V}; U'_{ab} = 80,8 \text{ V}; U' = 127 \text{ V},$$

imaginary values:

$$I''_\alpha = -1,21 \text{ A}; I''_\beta = 0,71 \text{ A}; I''_\gamma = -1,93 \text{ A}; U''_\alpha = 42,8 \text{ V}; U''_{ab} = -42,8 \text{ V}; U'' = 0 \text{ V}.$$

Complex power of the energy source  $\underline{S}_{\text{source}} = \underline{U} \cdot \underline{I}_\alpha^* = P_{\text{source}} + jQ_{\text{source}}$  is defined as the product of the complexes of voltage and conjugate current of the energy source, which are defined in an indicative (or algebraic) form.

Conjugate current complex  $\underline{I}_\alpha^* = \underline{I}_\alpha \cdot e^{-j\psi_{i_\alpha}}$  determined from the source  $\underline{I}_\alpha = \underline{I}_\alpha \cdot e^{j\psi_{i_\alpha}}$  by changing the sign in the exponent (or before the imaginary part, if the algebraic form of a complex number is used).

Substitute the complexes of voltage and conjugate complex current source and perform the transformation:

$$\begin{aligned} \underline{S}_{\text{source}} &= \underline{U} \cdot \underline{I}_\alpha^* = 127 \cdot 3,7 \cdot e^{j19,09^\circ} = 469,9 \cdot e^{j19,09^\circ} = \\ &= 469,9 \cdot (\cos 19,09^\circ + j \sin 19,09^\circ) = 444,1 + j153,7 \text{ VA}, \end{aligned}$$



where total power  $\underline{S}_{\text{source}} = 469,9 \text{ VA}$  there is a module in the indicative form of complex power, and power: active  $P_{\text{source}} = 444,1 \text{ W}$  and reactive  $Q_{\text{source}} = 153,7 \text{ var}$  – real and imaginary parts of the algebraic form of a complex number.

The loads powers are similarly determined:

$$\begin{aligned}\underline{S}_{\alpha} &= \underline{U}_{\alpha} \cdot \underline{I}_{\alpha}^* = 62,9 \cdot e^{j42,84^{\circ}} \cdot 3,7 \cdot e^{j19,09^{\circ}} = 232,7 \cdot e^{j61,93^{\circ}} = 109,5 + j205,3 \text{ VA}; \\ \underline{S}_{\beta} &= \underline{U}_{ab} \cdot \underline{I}_{\beta}^* = 91,4 \cdot e^{-j27,89^{\circ}} \cdot 4,57 \cdot e^{-j8,98^{\circ}} = 417,7 \cdot e^{-j36,87^{\circ}} = 334,2 - j250,6 \text{ VA}; \\ \underline{S}_{\gamma} &= \underline{U}_{ab} \cdot \underline{I}_{\gamma}^* = 91,4 \cdot e^{-j27,89^{\circ}} \cdot 2,18 \cdot e^{j117,89^{\circ}} = 199,3 \cdot e^{j90^{\circ}} = j199,3 \text{ VA},\end{aligned}$$

where  $\underline{I}_{\alpha}^*, \underline{I}_{\beta}^*, \underline{I}_{\gamma}^*$  – conjugate currents of loads;

active power of loads:  $P_{\alpha} = 109,5 \text{ W}$ ;  $P_{\beta} = 334,2 \text{ W}$ ;  $P_{\gamma} = 0$ ,

reactive power of loads:  $Q_{\alpha} = 205,3 \text{ VAr}$ ;  $Q_{\beta} = -250,6 \text{ VAr}$ ;  $Q_{\gamma} = 199,3 \text{ var}$ .

Equation of active powers balance:

$$P_{\text{source}} = P_{\alpha} + P_{\beta} + P_{\gamma}; 444,1 \text{ W} \approx 109,5 + 334,2 + 0 = 443,7 \text{ W}.$$

Equation of reactive power balance:

$$Q_{\text{source}} = Q_{\alpha} + Q_{\beta} + Q_{\gamma}; 153,7 \text{ VAr} \approx 205,3 - 250,6 + 199,3 = 154 \text{ var}.$$

**Example 8.** In a three-phase circuit with a line voltage  $U_L = 380 \text{ V}$  there is a balanced “Y” connected load. It consists of  $Z_{\alpha}, Z_{\beta}$  connected in series in each phase with parameters  $R_{\alpha} = 16 \text{ Ohm}$ ;  $X_{L\alpha} = 30 \text{ Ohm}$ ;  $R_{\beta} = 22,1 \text{ Ohm}$ ;  $X_{C\beta} = 8 \text{ Ohm}$ .

**Calculation volume:** a) draw the equivalent electric scheme of three-phase circuit with ideal elements instead of the loads  $Z_{\alpha}, Z_{\beta}$ ; b) determine phase and line currents, as well as active and reactive power; c) build a phasor diagram.

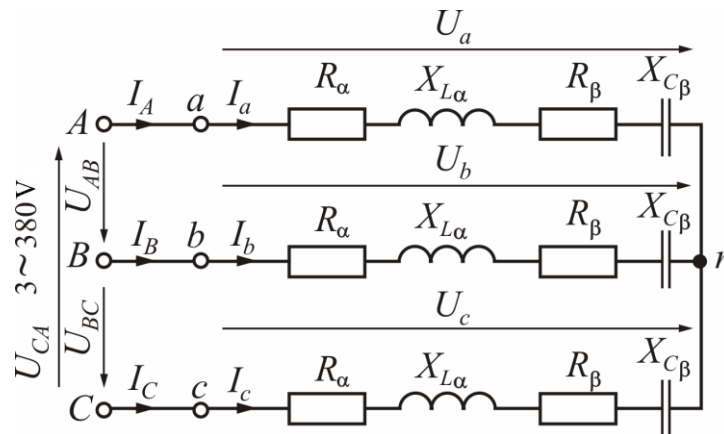


Figure 2.16

## Solution

In Fig. 2.16 it is shown an equivalent electric scheme of a three-phase circuit when the load is connected in star “Y” (wye).

$$\text{Phase voltages in the network } U_{\text{Ph}} = \frac{U_L}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220 \text{ V}.$$

At a balanced load the phase voltages across the loads are equal to phase voltages of a circuit  $U_{\text{Ph}}$ , that is  $U_a = U_b = U_c = U_{\text{Ph}} = 220 \text{ V}$ .

Impedance of each phase is

$$Z_{\text{Ph}} = \sqrt{(R_\alpha + R_\beta)^2 + (X_{L\alpha} - X_{C\beta})^2} = \sqrt{(16 + 22,1)^2 + (30 - 8)^2} = 44 \text{ Ohm};$$

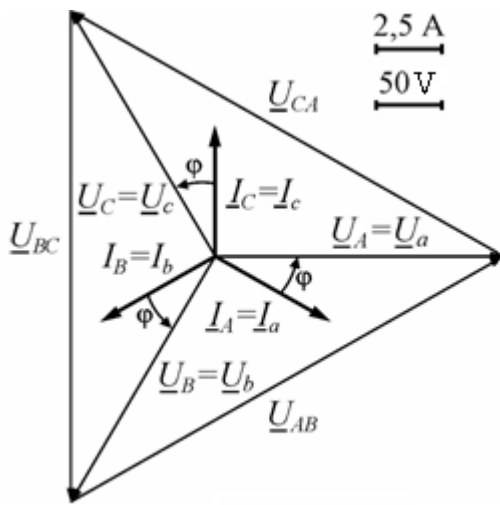


Figure 2.17

The phase difference angle between a phase voltage and current in each phase is

$$\varphi = \arctg \frac{X_{L\alpha} - X_{C\beta}}{R_\alpha + R_\beta} = \arctg \frac{30 - 8}{16 + 22,1} = 30^\circ.$$

The RMS phase and line currents values are:

$$I_{\text{Ph}} = \frac{U_{\text{Ph}}}{Z_{\text{Ph}}} = \frac{220}{44} = 5 \text{ A};$$

$$I_a = I_b = I_c = I_{\text{Ph}} = 5 \text{ A};$$

$$I_A = I_B = I_C = I_L = I_{\text{Ph}} = 5 \text{ A}.$$

Active and reactive power of the whole circuit are:

$$P_Y = 3P_{\text{Ph}} = 3U_{\text{Ph}}I_{\text{Ph}}\cos\varphi = 3 \cdot 220 \cdot 5 \cdot \cos 30^\circ = 2858 \text{ W};$$

$$Q_Y = 3Q_{\text{Ph}} = 3U_{\text{Ph}}I_{\text{Ph}}\sin\varphi = 3 \cdot 220 \cdot 5 \cdot \sin 30^\circ = 1650 \text{ var}.$$

The phasor diagram is shown in Fig. 2.17. It is built this way. Choose the scale of currents and voltages and the initial phase  $\Psi_{U_A} = 0^\circ$  voltage phasor  $\underline{U}_A$  and draw this phasor along the horizontal axis. Phasors of phase voltages of loads  $\underline{U}_a, \underline{U}_b, \underline{U}_c$  at a balanced load are equal to voltages of a source  $\underline{U}_A, \underline{U}_B, \underline{U}_C$  and create a three-phase symmetric system, i.e. have the same RMS values and phase-shifted relative to each other by an angle of  $120^\circ$ . The voltage phasor  $\underline{U}_B$  lags behind the phasor  $\underline{U}_A$  by an angle of  $120^\circ$ , so it returns to this angle clockwise, and the voltage phasor  $\underline{U}_C$  leads the phasor  $\underline{U}_A$  by an angle of  $120^\circ$ , so it returns to this angle counterclockwise.

Phasors of line voltages  $\underline{U}_{AB}, \underline{U}_{BC}, \underline{U}_{CA}$  we draw using Kirchhoff's voltage law according to formulas:

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B; \quad \underline{U}_{BC} = \underline{U}_B - \underline{U}_C; \quad \underline{U}_{CA} = \underline{U}_C - \underline{U}_A,$$

that is we will connect the ends of voltage phasors  $\underline{U}_A, \underline{U}_B, \underline{U}_C$  among themselves and we obtain phasors of line voltages, as shown in Fig. 2.17.

Each phase current phasor lags its phase voltage by an angle of  $\varphi=30^\circ$ , therefore, it rotates clockwise at this angle relative to its phase voltage.

**Example 9.** In a three-phase network with a line voltage  $U_L = 380 \text{ V}$  a balanced load is connected in delta. It consists of  $Z_\alpha, Z_\gamma$  connected in series in each phase with parameters  $R_\alpha = 20 \text{ Ohm}$ ;  $R_\gamma = 10 \text{ Ohm}$ ;  $X_{C\gamma} = 30 \text{ Ohm}$ .

**Calculation volume:** a) draw the electrical scheme of replacement of three-phase circuit with ideal elements instead of loads  $Z_\alpha, Z_\gamma$ ; b) determine phase and line currents, as well as active and reactive power; c) build a phasor diagram.

### Solution

Figure 2.18 shows an electrical scheme of the replacement of a three-phase circuit when connecting the loads in delta.

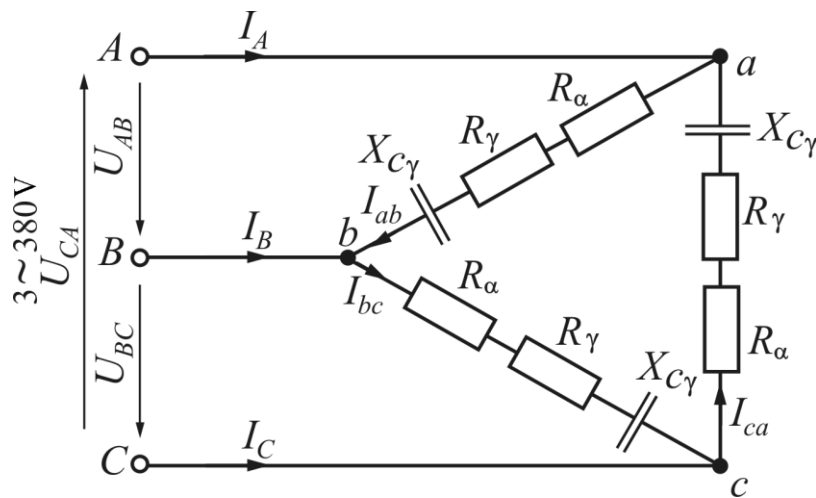


Figure 2.18

The impedance of each phase is

$$Z_{Ph} = \sqrt{(R_\alpha + R_\gamma)^2 + (-X_{C\gamma})^2} = \sqrt{(20 + 10)^2 + (-30)^2} = 42,43 \text{ Ohm}.$$

The phase difference angle  $\varphi$  between a phase voltage and a current in each phase is

$$\varphi = \arctg \frac{-X_{C\gamma}}{R_\alpha + R_\gamma} = \arctg \frac{-30}{20 + 10} = -45^\circ.$$

When connecting the loads in delta phase voltage  $U_{ab}$ ,  $U_{bc}$ ,  $U_{ca}$  coincide with the corresponding line voltages  $U_{AB}$ ,  $U_{BC}$ ,  $U_{CA}$ , therefore the phase voltages of loads  $U_{Ph} = U_L = 380$  V.

The RMS phase currents are:

$$I_{Ph} = \frac{U_{Ph}}{Z_{Ph}} = \frac{380}{42,43} = 8,96 \text{ A};$$

$$I_{ab} = I_{bc} = I_{ca} = I_{Ph} = 8,96 \text{ A}.$$

The RMS line currents are:

$$I_A = I_B = I_C = I_L = \sqrt{3} I_{Ph} = \sqrt{3} \cdot 8,96 = 15,52 \text{ A}.$$

Active and reactive power of the whole circuit:

$$P_{\Delta} = 3P_{Ph} = 3U_{Ph}I_{Ph}\cos\varphi = 3 \cdot 380 \cdot 8,96 \cdot \cos(-45)^{\circ} = 7222 \text{ W};$$

$$Q_{\Delta} = 3Q_{Ph} = 3U_{Ph}I_{Ph}\sin\varphi = 3 \cdot 380 \cdot 8,96 \cdot \sin(-45)^{\circ} = -7222 \text{ var}.$$

The phasor diagram is shown in Fig. 2.19. The diagram is drawn as follows. Determine the scale of currents and voltages and choose the initial phase  $\Psi_{U_{AB}} = 0^{\circ}$  voltage phasor  $\underline{U}_{AB}$  and draw this phasor along the horizontal axis. Network line voltage phasors  $\underline{U}_{AB}$ ,  $\underline{U}_{BC}$ ,  $\underline{U}_{CA}$  (which are also phase voltages  $\underline{U}_{ab}$ ,  $\underline{U}_{bc}$ ,  $\underline{U}_{ca}$  across the loads) create a three-phase symmetric system, i.e. they have the same RMS values and are shifted in phase relative to each other by an angle of  $120^{\circ}$ . Voltage phasor  $\underline{U}_{BC}$  lags the phasor  $\underline{U}_{AB}$

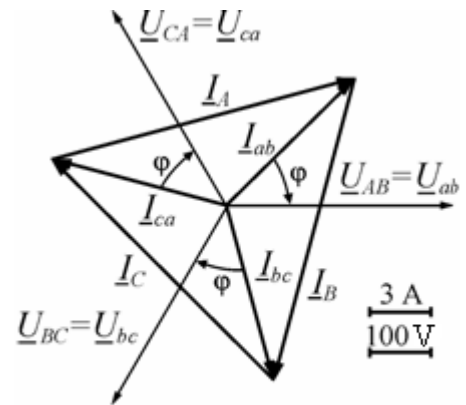


Figure 2.19

by an angle of  $120^{\circ}$ , so it returns to this angle clockwise, and the voltage phasor  $\underline{U}_{CA}$  ahead of the phasor  $\underline{U}_{AB}$  on the angle  $120^{\circ}$ , so it returns to this angle counterclockwise.

Each phase current phasor  $\underline{I}_{ab}$ ,  $\underline{I}_{bc}$ ,  $\underline{I}_{ca}$  ahead of its phase voltage phasor by an angle of  $\varphi = -45^{\circ}$ , therefore, it rotates counterclockwise at this angle relative to the phase voltage.

Phasors of line currents  $\underline{I}_A$ ,  $\underline{I}_B$ ,  $\underline{I}_C$  we build using Kirchhoff's current law according to formulas:

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca}, \quad \underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab}, \quad \underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc},$$

that is, we will connect the ends of phasors of currents  $\underline{I}_{ab}$ ,  $\underline{I}_{bc}$ ,  $\underline{I}_{ca}$  among themselves and we obtain phasors of line currents, as shown in Fig. 2.19.

**Example 10.** In a three-phase circuit with a line voltage  $U_L = 380 \text{ V}$  unbalanced load of phases is included  $Z_\alpha$ ,  $Z_\beta$  and  $Z_\gamma$  as asymmetric load of ideal elements:  $R_\alpha = 12 \text{ Ohm}$ ,  $X_{L\alpha} = 16 \text{ Ohm}$ ,  $R_\beta = 32 \text{ Ohm}$ ,  $X_{C\beta} = 24 \text{ Ohm}$ ,  $X_{L\gamma} = 20 \text{ Ohm}$ .

**Calculation volume:** a) draw the equivalent electrical scheme of a three-phase circuit with ideal elements instead of the loads; b) determine the phase and line currents, as well as the current in the neutral wire; c) determine the active and reactive power; d) build a phasor diagram.

### Solutions

Figure 2.20 shows the electrical circuit of a three-phase circuit.

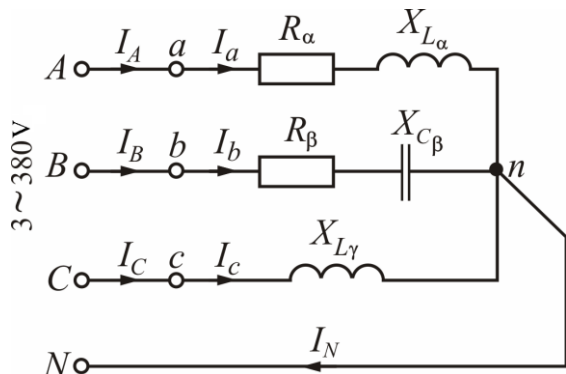


Figure 2.20

The RMS phase voltages values are:

$$U_A = U_B = U_C = U_{Ph} = \frac{U_L}{\sqrt{3}} = \frac{380}{\sqrt{3}} = 220 \text{ V}.$$

In the presence of a neutral wire, the phase voltages of the network are equal to the RMS phase voltages values across the loads:

$$U_A = U_a; U_B = U_b; U_C = U_c.$$

$$U_a = U_b = U_c = U_{Ph} = 220 \text{ V}.$$

The impedances values and phase

difference angles of the phases loads are:

$$Z_\alpha = \sqrt{R_\alpha^2 + X_{L\alpha}^2} = \sqrt{12^2 + 16^2} = 20 \text{ Ohm},$$

$$Z_\beta = \sqrt{R_\beta^2 + (-X_{C\beta})^2} = \sqrt{32^2 + (-24)^2} = 40 \text{ Ohm},$$

$$Z_\gamma = X_{L\gamma} = 20 \text{ Ohm},$$

$$\varphi_\alpha = \arctg \frac{X_{L\alpha}}{R_\alpha} = \arctg \frac{16}{12} = 53,13^\circ;$$

$$\varphi_\beta = \arctg \frac{-X_{C\beta}}{R_\beta} = \arctg \frac{-24}{32} = -36,87^\circ; \varphi_\gamma = 90^\circ.$$

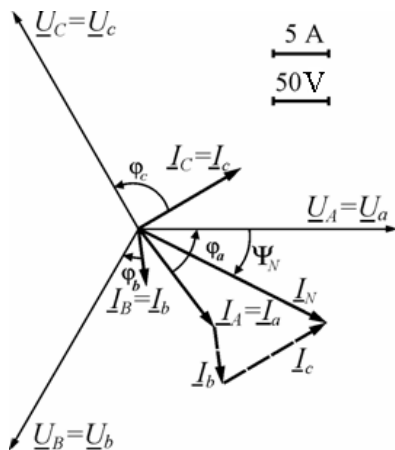


Figure 2.21

The RMS phase currents values are equal to the line ones:

$$I_a = I_A = \frac{U_a}{Z_\alpha} = \frac{220}{20} = 11 \text{ A}, I_b = I_B = \frac{U_b}{Z_\beta} = \frac{220}{40} = 5,5 \text{ A},$$

$$I_c = I_C = \frac{U_c}{Z_\gamma} = \frac{220}{20} = 11 \text{ A}.$$

Let's build a phasor diagram (Fig. 2.21). We accept the initial voltage phase for the phase A  $\psi_{U_A} = 0^\circ$  and draw a phasor diagram of voltages and currents, as in example 8. Determine the RMS value of the current in the neutral wire ( $I_N = I_a + I_b + I_c$ ) direct measurement of the

length of the phasor  $\underline{I}_N$  and multiplying it by the current scale ( $I_N = 19$  A). The initial phase current of the neutral wire, which is determined by the protractor, is equal to  $\psi_{i_N} = -27,5^\circ$ . You can check the results by referring to the symbolic method.

The phase sinusoidal voltages form a three-phase symmetric system:

$$\underline{U}_A = \underline{U}_a = 220 \text{ V}; \quad \underline{U}_B = \underline{U}_b = 220 e^{-j120^\circ} \text{ V}; \quad \underline{U}_C = \underline{U}_c = 220 e^{j120^\circ} \text{ V}.$$

The complex impedances of phase loads are:

$$\underline{Z}_\alpha = Z_\alpha e^{j\varphi_\alpha} = 20 e^{j53,13^\circ} \text{ V}, \quad \underline{Z}_\beta = Z_\beta e^{j\varphi_\beta} = 40 e^{-j36,87^\circ} \text{ V},$$

$$\underline{Z}_\gamma = Z_\gamma e^{j\varphi_\gamma} = 20 e^{j90^\circ} \text{ V}.$$

The line and phase currents are:

$$\underline{I}_A = \underline{I}_a = \frac{\underline{U}_a}{\underline{Z}_\alpha} = \frac{220 e^{j0^\circ}}{20 e^{j53,13^\circ}} = 11 e^{-j53,13^\circ} = 6,6 - j8,8 \text{ A};$$

$$\underline{I}_B = \underline{I}_b = \frac{\underline{U}_b}{\underline{Z}_\beta} = \frac{220 e^{-j120^\circ}}{40 e^{-j36,87^\circ}} = 5,5 e^{-j83,13^\circ} = 0,66 - j5,46 \text{ A};$$

$$\underline{I}_C = \underline{I}_c = \frac{\underline{U}_c}{\underline{Z}_\gamma} = \frac{220 e^{j120^\circ}}{20 e^{j90^\circ}} = 11 e^{j30^\circ} = 9,53 + j5,5 \text{ A},$$

where the RMS values of phase currents equal to line are:

$$I_a = I_A = 11 \text{ A}; \quad I_b = I_B = 5,5 \text{ A}; \quad I_c = I_C = 11 \text{ A};$$

initial phases of currents are:

$$\psi_{i_a} = -53,13^\circ; \quad \psi_{i_b} = -83,13^\circ; \quad \psi_{i_c} = 30^\circ.$$

The current in the neutral wire:

$$\underline{I}_N = \underline{I}_a + \underline{I}_b + \underline{I}_c = 6,6 - j8,8 + 0,66 - j5,46 + 9,53 + j5,5 = 16,79 - j8,76 =$$

$$= 18,94 e^{-j27,55^\circ} \text{ A},$$

where the RMS value of the current in the neutral wire is  $I_N = 18,94$  A, and its initial phase equals  $\psi_{i_N} = -27,55^\circ$ .

As can be seen, the calculated values of the current and its initial phase almost coincide with the values obtained from the phasor diagram.

Active and reactive power of the whole circuit:

$$P = P_\alpha + P_\beta + P_\gamma = U_a \cdot I_a \cdot \cos \varphi_\alpha + U_b \cdot I_b \cdot \cos \varphi_\beta + U_c \cdot I_c \cdot \cos \varphi_\gamma =$$

$$= 220 \cdot 11 \cdot \cos 53,13^\circ + 220 \cdot 5,5 \cdot \cos(-36,87^\circ) + 220 \cdot 11 \cdot \cos 90^\circ = 2420 \text{ W};$$

$$Q = Q_\alpha + Q_\beta + Q_\gamma = U_a \cdot I_a \cdot \sin \varphi_\alpha + U_b \cdot I_b \cdot \sin \varphi_\beta + U_c \cdot I_c \cdot \sin \varphi_\gamma =$$

$$= 220 \cdot 11 \cdot \sin 53,13^\circ + 220 \cdot 5,5 \cdot \sin(-36,87^\circ) + 220 \cdot 11 \cdot \sin 90^\circ = 3630 \text{ var}.$$

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## APPENDIX

### Sample of the title page of an individual work report

Ministry of Education and Science of Ukraine  
National Technical University  
“Kharkiv Polytechnic Institute”

Department of Applied Electrical Engineering

Calculation and graphic work

on the topic

CALCULATION OF ELECTRIC CIRCUITS

Variant № \_\_\_\_\_

The work was performed by a student \_\_\_\_\_; group \_\_\_\_\_

(surname, initials)

(index) (date of execution)

The report was accepted by \_\_\_\_\_ with a grade of \_\_\_\_\_

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Kharkiv

\_\_\_\_\_  
(year of execution)



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ГОНЧАРОВ Євген Вікторович  
КОРИТЧЕНКО Костянтин Володимирович  
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МАРКОВ Владислав Сергійович  
ПОЛЯКОВ Ігор Володимирович

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(англ. мовою)

**З КУРСУ “ЕЛЕКТРОТЕХНІКА, ЕЛЕКТРОНІКА  
ТА МІКРОПРОЦЕСОРНА ТЕХНІКА”**  
для студентів неелектротехнічних спеціальностей  
денної та заочної форми навчання

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