MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

NATIONAL TECHNICAL UNIVERSITY
«KHARKIV POLYTECHNICAL INSTITUTE»

NUMERICAL AND APPLIED METHODS OF VEHICLE CALCULATIONS

Methodological instructive regulations for practical work

specialties
J8 Automobile transport
G11 Engineering

Kharkiv
NTU «KhPI»
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CONTAINS

INTRODUCTION

At present, the problems of constructing and studying mathematical models of systems for various purposes and signals of various kinds occupy one of the leading places in modern science. The penetration of mathematical methods into engineering, natural sciences and humanities follows the path of mathematical design and modelling of the corresponding objects, and the ever-increasing capabilities of computer technology ensure the great success of this scientific direction.

An important aspect of modelling is that the model must be an adequate substitute for the real situation, a real system. The model reduces the redundancy of information storage, automatically taking into account the values that correspond to the real object and characterise it, presented in the form of input and output values and state characteristics..

The MATLAB system provides an intelligent environment for organising computations for hundreds of thousands of professionals in engineering and scientific research. The MATLAB system is a leader in providing a convenient, integrated programming language for implementing a wide variety of mathematical algorithms and mathematical modelling tasks. The package also includes Simulink, a visual modelling tool that allows you to build and explore mathematical models without programming.

MATLAB programming offers the greatest flexibility, richness, and convenience for solving and exploring mathematical problems. In presenting the material, preference has been given to the simplest language constructs, by studying which you can create a wide variety of non-trivial mathematical algorithms.

PRACTICAL WORK №1

Approximating the characteristics of a car engine in MATLAB

Objective: To get acquainted with the methods of data approximation using the example of the external speed characteristic of a car engine.

Tasks:

- To become familiar with the method of approximating data by a polynomial.

- To become familiar with the method of approximating data using a spline.

- To determine the most appropriate of the two approximation methods considered.

Information material.

The process of approximating a function or tabular data by another function is called an interpolation. In this case, the function defined for the approximation is the approximator. The most famous and effective method of solving the problem of approximating a function or tabular data is the method of least squares, the essence of which is to find the coefficients of the approximating function under the condition of minimising the sum of the squares of the difference between the original function or tabular data and the values calculated using the approximating function.

The best known and most widely used in solving engineering problems are two variants of approximating functions - polynomial and spline approximation.

Suppose that as a result of an engineering or scientific experiment a system of points is obtained $\{(x_0, y_0),(x_1, y_1)...(x_n, y_n)\}$. Then, when approximating by a polynomial, it is necessary to find a power polynomial of the form: $Q(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + ... + a_m \cdot x^m$ such that the sum of the squares of the deviations of the polynomial from the given system of experimental points would be minimal. Such a problem is reduced to determining the coefficients of the polynomial $\{a_0, a_1, a_2...a_n\}$. The method that allows solving it is called the method

of least squares. The criterion for the mean square deviation in this case has the form: $\sum_{i=0}^{n}\left(Q(x_i)-y_i\right)^2=\sum_{i=0}^{n}\left(a_0+a_1\cdot x_i+a_2\cdot x_i^{\,2}+...+a_m\cdot x_i^{\,m}\right)^2\Rightarrow \min$.

A similar method is used when approximating a spline, which is essentially a fragment of a 3rd order polynomial that is carefully combined at the junction of the fragments, provided that the corresponding derivatives are obtained at the junction points. The criterion for the mean square deviation is the same as for a polynomial.

The Matlab package has a number of built-in functions for approximating. These are the functions:

`polyfit(X, Y, k)` – the function determines a vector p containing the coefficients of a polynomial of order `k`, which are calculated from the available tabular data of the vectors `X` and `Y`.

`polyval(p, x)` – function that determines the value of the approximating polynomial function `f1(x)` based on the vector of polynomial coefficients `p`, with argument values `x`.

`spline(X, Y, x)` – function that determines the value of the approximating spline function `f2(x)` based on available tabular vector data `X` and `Y` with argument values `x`.

Do it yourself: According to the option number in the magazine, use the data on the external speed characteristics of the motor from Table 1.1 and carry out the polynomial and spline approximation procedure..

Table 1.1 contains data on the engine crankshaft speed n ($хв^{-1}$) and their corresponding values of automobile engine torque M (Нм) according to the external speed characteristic (with the fuel supply control position corresponding to the maximum). The procedure is as follows.

1. In the first stage in the workspace *Workspace* initial data vectors are formed n and M (`X` and `Y`).

2. The order of the polynomial is assigned and a ranked vector of the

argument of the approximating function is formed n1 (x).

3. Using functions `polyfit(X, Y, k)`, `polyval(p, x)`, `spline(X, Y, x)` approximation is performed by polynomial and spline.

4. The initial data and approximating functions are constructed in the same coordinate system `f1(x)` and `f2(x)`.

Table 1.1 – Data on the external speed characteristics of automobile engines

Option	1,6,11	Option	2,7,12	Option	3,8,13	Option	4,9,14	Option	5,10,15
n, min^{-1}	M, Nm	n, min^{-1}	M, Nm	n, min^{-1}	M, Nm	n, min^{-1}	M, Nm	n, min^{-1}	M, Nm
1000	810	1200	755	1000	615	1100	625	1200	725
1100	825	1300	875	1100	647	1200	680	1300	755
1200	850	1400	910	1200	675	1300	710	1400	770
1300	880	1500	922	1300	692	1400	720	1500	780
1400	890	1600	905	1400	702	1500	710	1600	770
1500	885	1700	870	1500	698	1600	700	1700	740
1600	875	1800	828	1600	688	1700	680	1800	707
1700	865	1900	785	1700	675	1800	660	1900	670
1820	850	2000	750	1800	660	1900	635	2000	635
1900	840	2100	710	1900	642	2000	615	2100	600
2000	820			2000	620	2100	590		
2100	805			2100	600				
				2200	573				

Example program for polynomial and spline approximation:

```
k=4
p=polyfit(n, M, k);
n1=min(n):10:max(n);
f1=polyval(p, n1);
f2=spline(n, M, n1);
plot(n, M, 'o', n1, f1, '-', n1, f2, '-.')
grid on
```

PRACTICAL WORK №2

Frequency Analysis. Determining the amplitudes and frequencies of vibrations

Objective: To become familiar with the methods of frequency analysis of data based on Fourier transforms using the example of polyharmonic oscillations of the torque of a car engine.

Tasks:

- To become familiar with the direct Fourier transform.

- To find the amplitudes and frequencies of the polyharmonic oscillations of the torque of a car engine.

Information material.

The Fourier transform is a family of mathematical methods based on the decomposition of the original continuous function, sometimes into a set of basic harmonic functions (which are sinusoidal functions) of different frequency, amplitude and phase. From the definition it is clear that the main idea of the transform is that any function can be represented as an infinite sum of sinusoids, each characterised by its amplitude, frequency and initial phase.

The Fourier transform is the founder of spectral analysis. Spectral analysis is a signal processing technique that allows you to characterise the frequency composition of the signal being measured. Different Fourier transforms are used depending on how the signal is represented. There are several types of Fourier transform:

Continuous Fourier Transform (in the literature, Continue Time Fourier Transform - CTFT or abbreviated FT); Discrete Fourier Transform (in the literature, Discrete Fourier Transform - DFT); Fast Fourier Transform (in the literature, Fast Fourier Transform - FFT).

Before the advent of computers, calculating such transformations was very tedious, requiring manual execution of a large number of arithmetic operations that

depended on the number of points describing the wave function. It was not until 1965 that software was created that became popular as the "fast Fourier transform". It allows you to save time on calculations by reducing the number of multiplications during curve analysis. The Fast Fourier Transform method is based on the distribution of the curve over a large number of uniform sample values.

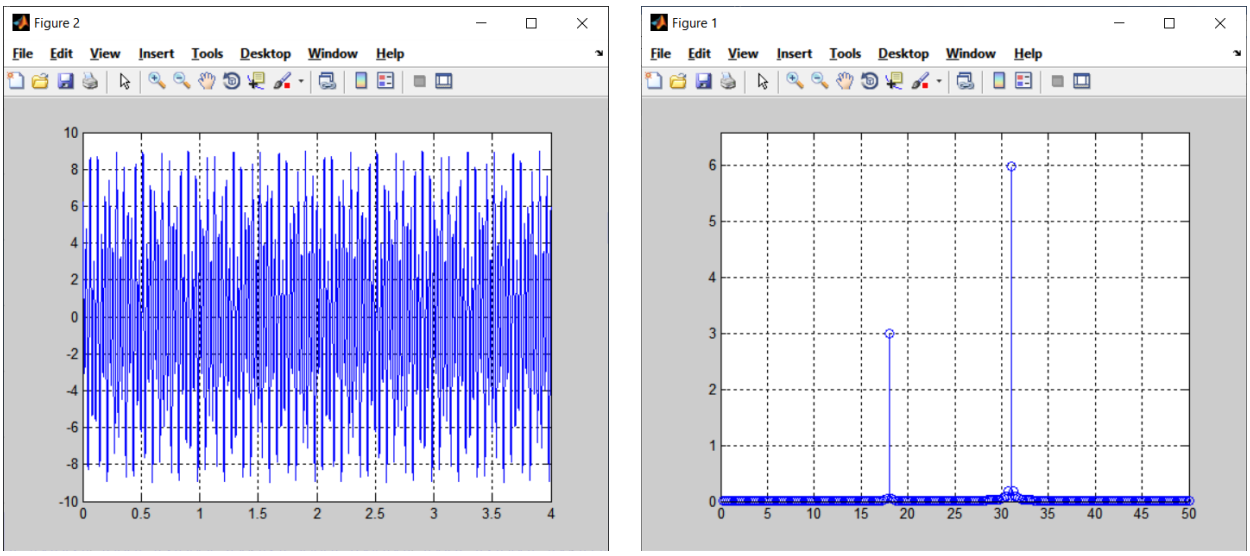
Matlab has a number of built-in functions for performing Fourier transforms. These are the functions:

`fft(x)` – function that forms the Fourier transform of the input signal `x;`

`fftshift(y)` – function that performs the Fourier transform of the vector y to determine the actual amplitudes of oscillations.

Do it yourself: According to the option by number in the magazine, using the data on the components of polyharmonic oscillations of the torque of a car engine from Table 2.1, analyse the amplitudes and frequencies of the oscillations and construct graphs of the output signal and frequency spectrum (Fig. 2.1) using the given fragment of the program.:

```
T=4; % Measurement time interval
h=0.001; % Measurement time step
t=0:h:T; % Time range
x=3*sin(2*pi*18*t)+6*cos(2*pi*31*t); % Input signal
y=fft(x); % Fourier transform of the input signal
Df=1/T; % Frequency step
F=1/h; % Maximum frequency
f=0:Df:F; % Frequency vector formation
f1=-F/2:Df:F/2; % Frequency vector reconstruction
v=fftshift(y); % Vector Fourier transform
a=abs(v); % Modulus of the reconstructed Fourier image
N=length(y); % Determining the number of elements of a Fourier transform
a=2*abs(v)/N; % Determining the real amplitudes of harmonics
figure
stem(f1, a); grid % Graph construction
axis([0 50 0 max(a)*1.1])
figure
plot(t, x); grid
```



a *b*

Figure 2.1 – Example of Fourier transforms:
a – output signal in time; b – frequency spectrum of the signal with real amplitudes

Table 2.1 – Data on the components of polyharmonic oscillations of the torque of an automobile engine

Option	Torque components	Measurement time, s	A step in time, s
1	5*sin(2*pi*15*t)+3*cos(2*pi*35*t)	4	0.001
2	2*sin(2*pi*10*t)+5*sin(2*pi*20*t)	5	0.002
3	3*sin(2*pi*5*t)+cos(2*pi*12*t)	6	0.001
4	4*sin(2*pi*4*t)+2*sin(2*pi*16*t)	4	0.005
5	8*sin(2*pi*5*t)+3*cos(2*pi*18*t)	5	0.001
6	6*sin(2*pi*3*t)+3*sin(2*pi*9*t)	6	0.002
7	5*sin(2*pi*1*t)+2*cos(2*pi*3*t)	4	0.001
8	2*sin(2*pi*8*t)+10*sin(2*pi*22*t)	5	0.005
9	7*sin(2*pi*4*t)+3*cos(2*pi*30*t)	6	0.001
10	5*sin(2*pi*3*t)+3*sin(2*pi*5*t)	4	0.002
11	9*sin(2*pi*16*t)+5*cos(2*pi*25*t)	5	0.001
12	6*sin(2*pi*8*t)+7*sin(2*pi*21*t)	6	0.005
13	2*sin(2*pi*13*t)+6*cos(2*pi*18*t)	4	0.001
14	4*sin(2*pi*2*t)+3*sin(2*pi*5*t)	5	0.002
15	8*sin(2*pi*9*t)+2*cos(2*pi*32*t)	6	0.001
16	12*sin(2*pi*6*t)+8*sin(2*pi*35*t)	4	0.005
17	7*sin(2*pi*3*t)+5*cos(2*pi*28*t)	5	0.001
18	5*sin(2*pi*11*t)+4*sin(2*pi*22*t)	6	0.002
19	3*sin(2*pi*4*t)+6*sin(2*pi*14*t)	4	0.001
20	9*sin(2*pi*17*t)+5*sin(2*pi*35*t)	5	0.005

PRACTICAL WORK №3

Calculating the motion of a pendulum mechanism

Objective: To become familiar with the methods for solving the differential equations of a simple rotating mechanical system.

Tasks:

- To become familiar with the physical model of a pendulum mechanism.
- To construct a mathematical model of the forces involved.
- Create a block diagram in Simulink to implement the mathematical model.
- Carry out simulation modelling of the motion of a pendulum mechanism under different initial integration conditions (initial positions of the mechanism).

Information material.

It is known that, for small deviations from the vertical, the motion of the pendulum is described with sufficient accuracy by a second-order linear differential equation with constant coefficients:

$$J \cdot \ddot{\varphi} + R \cdot \dot{\varphi} + m \cdot g \cdot l \cdot \varphi = 0,$$

where φ – angle of deviation of the pendulum from the vertical; J – moment of inertia of the pendulum relative to its axis of rotation; R – damping coefficient (dissipation of vibration energy); m – the mass of the pendulum weight concentrated at the end of the suspension; g – acceleration of free fall; l – the horizontal displacement of the centre of mass of the pendulum in relation to its axis of rotation.

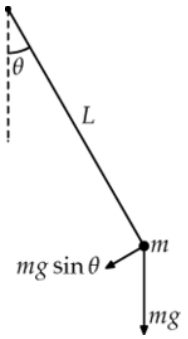
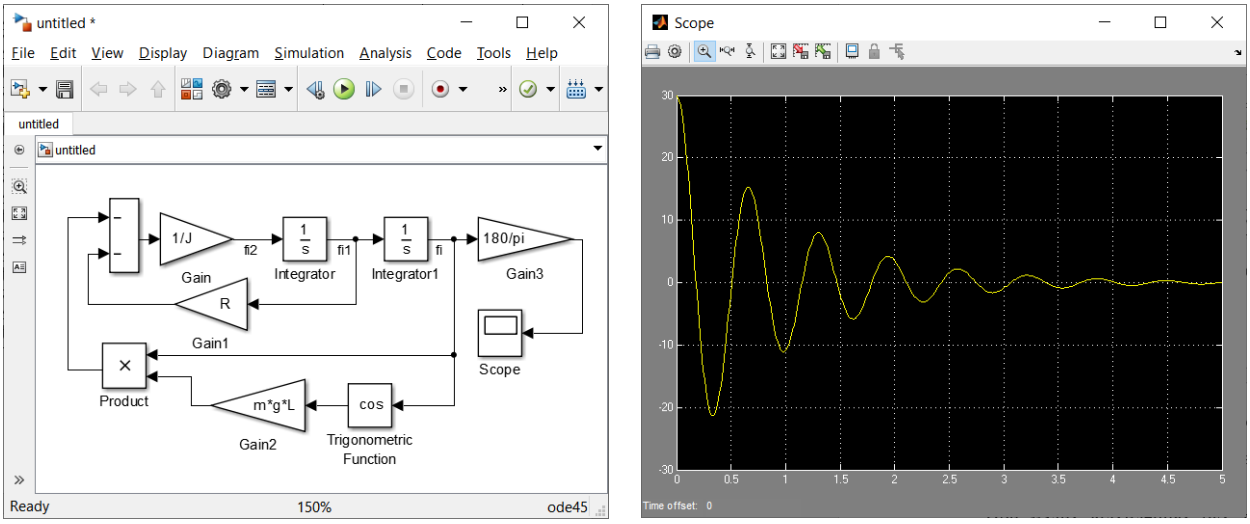


Figure 3.1 – Calculation diagram of the pendulum mechanism

Do it yourself: according to the option by number in the journal, using the

data on the indicators of the pendulum mechanism from Table 3.1, compile a mathematical description of the forces acting in the mechanism, create a block diagram of the model in Simulink and graphs of the movements of the pendulum weight (Fig. 3.2).



а *б*

Figure 3.2 – Implementation of the mathematical model of the pendulum mechanism: a – block diagram of the model; b – determined movement of the pendulum weight in time

Table 3.1 – Data for calculating pendulum weight oscillations

Option	Moment of inertia, kg*m^2	Damping coefficient, kg*m^2/s^2	Weight, kg	Suspension length, m
1	0,5	15	10	0,5
2	0,2	20	12	1,0
3	0,1	25	8	0,8
4	0,6	10	7	0,7
5	0,75	5	6	0,6
6	0,65	30	12	0,5
7	0,15	22	15	0,9
8	0,08	50	11	1,2
9	0,06	40	5	1,1
10	0,2	25	4	0,9
11	0,25	35	6	0,8
12	0,35	30	8	0,7
13	0,45	20	17	0,6
14	0,55	40	21	0,3
15	0,06	10	16	0,5
16	0,08	16	13	0,75
17	0,12	28	14	0,9
18	0,16	34	9	0,85
19	0,28	42	11	0,95
20	0,32	45	8	0,8

PRACTICAL WORK №4

Calculating the motion of a body on a spring

Objective: To become familiar with the methods for solving the differential equations of a simple mechanical system with translational motion.

Tasks:

- To become familiar with the physical model of the motion of a body on a spring.

- To construct a mathematical model of the forces involved.

- Create a block diagram in Simulink to implement the mathematical model of the motion of a body on a spring.

- Carry out simulation modelling of the motion of a body under different initial integration conditions (initial positions of the system).

Information material.

For small deviations, the motion of the body is described by a second-order linear differential equation with constant coefficients:

$$m \cdot \ddot{z} = -R \cdot \dot{z} - C \cdot z - m \cdot g \,,$$

where z – body movement; R – damping coefficient; m – body weight; g – acceleration of free fall; C – spring stiffness.

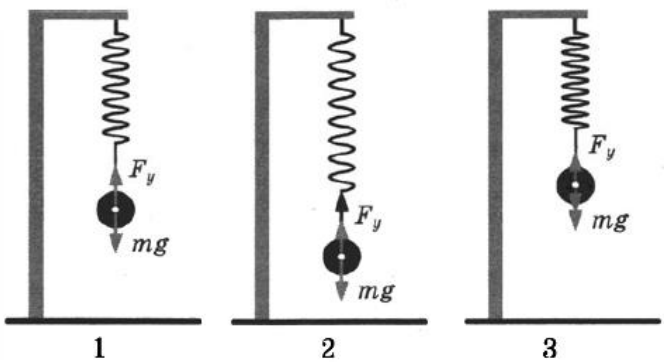
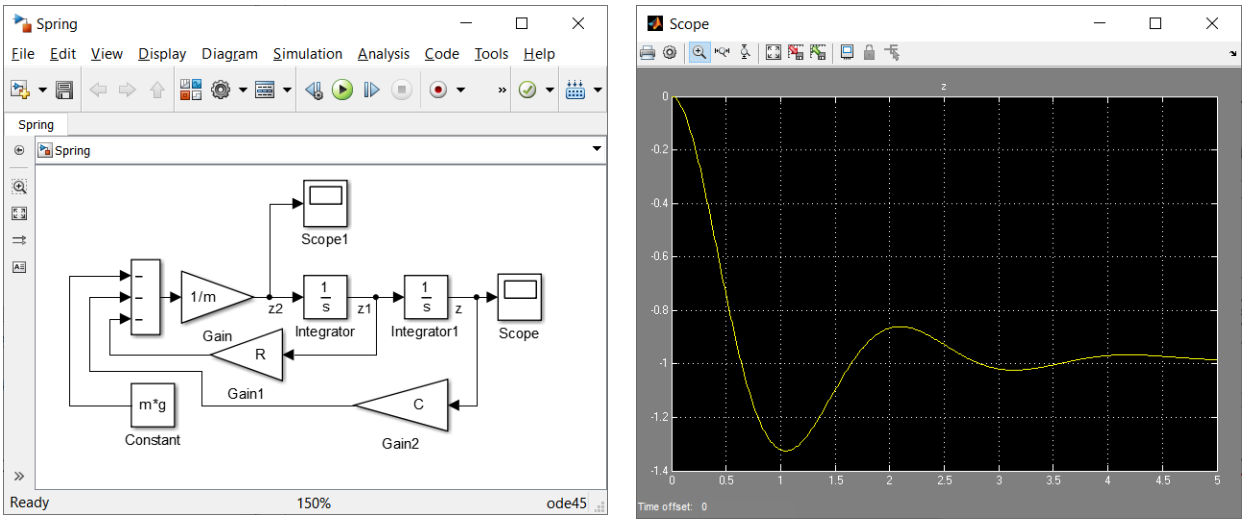


Figure 4.1 – Calculation model of body motion on a spring

Do it yourself: According to the option, using data on the model indicators from Table 4.1, compile a mathematical description of the forces acting in the system, build a block diagram of the model in Simulink, and plot graphs of the

kinematic and force parameters of the body's motion (Fig. 4.2).



a *b*

Figure 4.2 – Implementation of a mathematical model of mass motion on a spring: a – block diagram of the model; b – determined mass displacement in time

Table 4.1 – Data for calculating body vibrations on a spring

Option	Spring stiffness, N/m	Damping coefficient, N*s/m	Weight, kg	Initial position of the mass, m
1	500	15	10	0,5
2	200	20	12	1,0
3	100	25	8	0,8
4	600	10	7	0,7
5	750	5	6	0,6
6	650	30	12	0,5
7	150	22	15	0,9
8	80	50	11	1,2
9	60	40	5	1,1
10	200	25	4	0,9
11	250	35	6	0,8
12	350	30	8	0,7
13	450	20	17	0,6
14	550	40	21	0,3
15	60	10	16	0,5
16	80	16	13	0,75
17	120	28	14	0,9
18	160	34	9	0,85
19	280	42	11	0,95
20	320	45	8	0,8

PRACTICAL WORK №5

Calculating the motion of a body on an inclined plane

Objective: To become familiar with the methods for solving the differential equations of a simple mechanical system with translational motion.

Tasks:

- To become familiar with the physical model of the motion of a rod on an inclined plane.

- To construct a mathematical model of the forces involved.

- Draw up a block diagram in Simulink to implement the mathematical model of the motion of a beam on an inclined plane, improving the model as shown in Figure 5.1.

- Carry out simulation modelling of the motion of a beam on an inclined plane under different initial conditions (initial positions of the system).

Information material.

The motion of a beam on an inclined plane is described by a second-order differential equation:

$$m \cdot \ddot{z} = P_{\text{пр}} - m \cdot g \cdot \sin(\alpha) - \mu \cdot m \cdot g \cdot \cos(\alpha) \cdot sign(\dot{z}),$$

where $z, \dot{z}, \ddot{z}$ – coordinate, velocity and acceleration of the bar in the chosen reference system; $P_{\text{пр}}$ – spring elastic deformation force; α – surface inclination angle; m – mass of the bar; g – acceleration of free fall; μ– coefficient of friction between the block and the inclined surface.

The model of a beam moving along an inclined plane can contain a number of inconsistencies. First, the beam is not fixed to the spring, so they may not always interact (Fig. 5.1.a). Secondly, in the event of significant deformation, the spring may lock (compress to a state where the leg rests on the leg (Fig. 5.1.b)), which is similar to the breakdown of the vehicle suspension. Thirdly, if the shock absorber is installed parallel to the spring, the rod is not fixed to the spring or the shock absorber, so they may not always interact (Fig. 5.1.c). Fourthly, the shock

absorber may have an asymmetrical characteristic on the compression and rebound stroke (fig. 5.1.d).

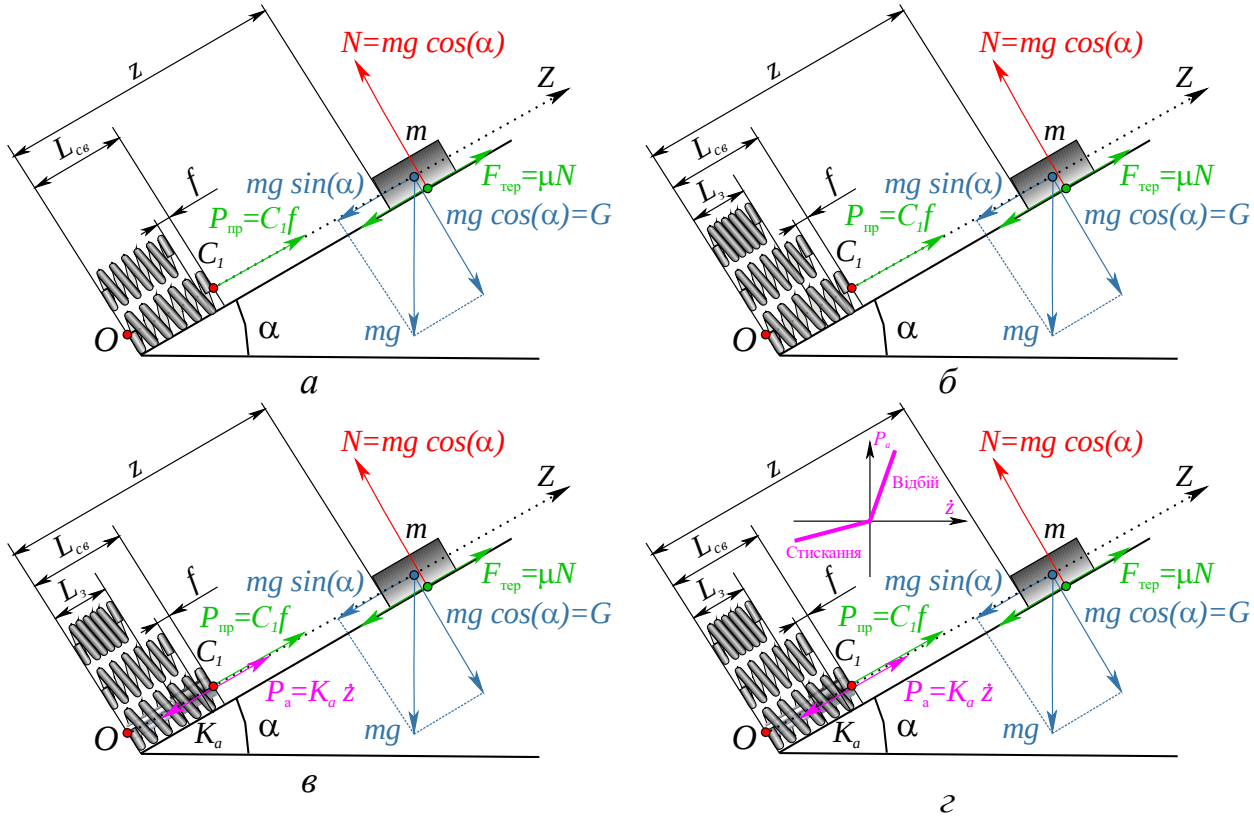


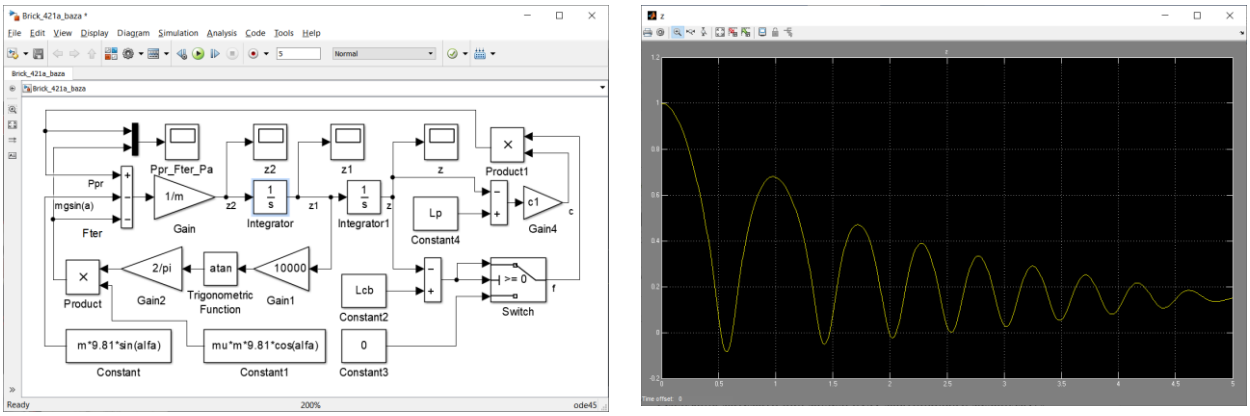
Figure 5.1 - Scheme for calculating the motion of a beam along an inclined surface:

a - model with non-linearity of the contact between the beam and the spring; b - model with additional non-linearity when the spring is closed; c - model with additional non-linearity when the damper is mounted parallel to the spring; d - same as in Figure 5.1.c, but with asymmetric damper characteristics.

Do it yourself: Using the data on the indicators of the pendulum mechanism from Table 5.1, set up a mathematical description of the forces acting in the system, create a block diagram of the model in Simulink and graph the kinematic and force parameters of the rod motion (Fig. 5.2).

As the work progresses, it is necessary to improve the model progressively according to the factors shown in Figure 5.1.a-d. At each stage of improvement it is essential to check the simulation results under different initial integration

conditions (initial positions of the system). The simulation results should be saved as separate graphics and data in mat files.



a *b*

Figure 5.2 – Implementation of a mathematical model of the motion of a bar along an inclined plane: a – block diagram of the model; b – determined movement of the bar in time

Table 5.1 – Data for calculating the movement of a bar along an inclined plane

Option	Friction coefficient, μ	Coef. shock absorber, K (N*s/m)	Mass, m (kg)	Free spring length, (m)	Spring closing length, (m)	Spring stiffness, C_1 (N/m)
1	0,5	15	10	0,5	0,2	1000
2	0,2	20	12	0,5	0,2	2000
3	0,1	25	8	0,5	0,2	3000
4	0,6	10	7	0,5	0,2	1200
5	0,75	5	6	0,5	0,2	500
6	0,65	30	12	0,5	0,2	2500
7	0,15	22	15	0,5	0,2	5000
8	0,08	50	11	0,5	0,2	4000
9	0,06	40	5	0,5	0,2	3000
10	0,2	25	4	0,5	0,2	2000
11	0,25	35	6	0,5	0,2	1000
12	0,35	30	8	0,5	0,2	1500
13	0,45	20	17	0,5	0,2	2400
14	0,55	40	21	0,5	0,2	4800
15	0,06	10	16	0,5	0,2	4200
16	0,08	16	13	0,5	0,2	3500
17	0,12	28	14	0,5	0,2	4000
18	0,16	34	9	0,5	0,2	2800
19	0,28	42	11	0,5	0,2	3200
20	0,32	45	8	0,5	0,2	2400

PRACTICAL WORK №6

Calculation of the motion of the transmission shaft line

Objective: To get acquainted with the methods of writing differential equations for the rotational motion of a simple transmission.

Tasks:

- To construct a mathematical model of the acting forces.

- Create a block diagram in Simulink to implement the mathematical model of the motion of the transmission shaft line.

- Carry out simulation modelling of the motion of the transmission shaft line under different initial integration conditions.

Information material.

The motion of the transmission shaft is described by differential equations according to the number of masses of the system (Fig. 6.1):

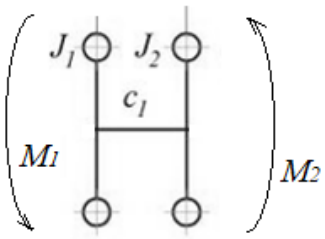


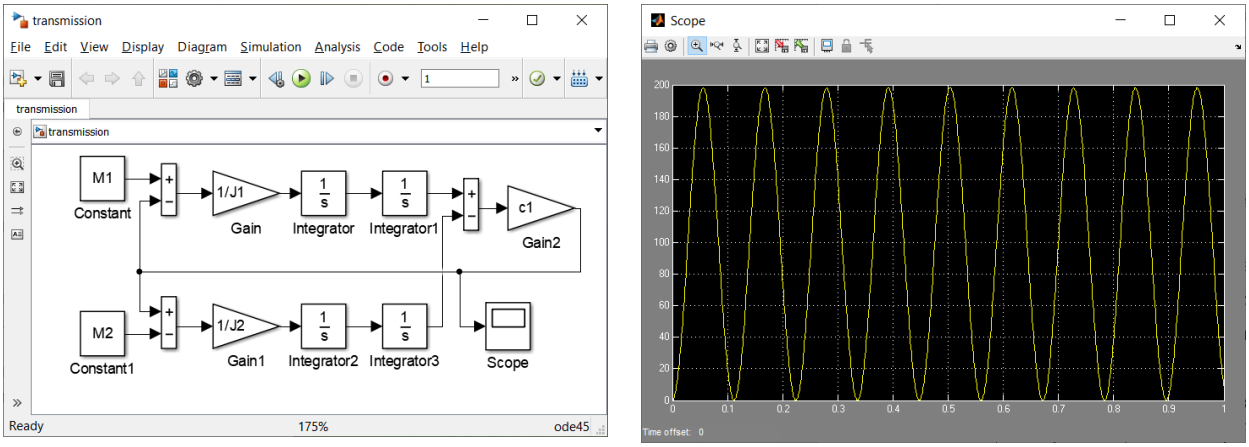
Figure 6.1 – Calculation diagram of the transmission shaft line

Then the motion of the transmission shaft will be described by a system of equations:

$$\begin{aligned} J_1 \cdot \ddot{\varphi}_1 &= M_1 - c_1 \cdot (\varphi_1 - \varphi_2); \\ J_2 \cdot \ddot{\varphi}_2 &= c_1 \cdot (\varphi_1 - \varphi_2) - M_2 \cdot, \end{aligned} \qquad (6.1)$$

where J_1, J_2 – the moments of inertia of the masses of the engine flywheel and the drive wheels are given, respectively; φ_1, φ_2 – engine flywheel and drive wheel rotation angles; c_1 – shaft stiffness; M_1, M_2 – Torques applied to the transmission from the engine flywheel and the drive wheels (active engine torque and driving resistance torque on the drive wheels).

Do it yourself: Using the data from Table 6.1, create a mathematical model of the motion of the transmission shaft, create a block diagram of the model in Simulink and graph the kinematic and force parameters of the motion of the system masses. Set the moments M_1, M_2 independently. During the work, it is necessary to carry out simulations with different initial integration.



a *b*

Figure 6.2 – Implementation of the mathematical model of shaft drive motion: a – block diagram of the model; b – elastic moment in the transmission

Таблиця 6.1 – Дані для розрахунку руху валопроводу трансмісії

Option	Moment of inertia J_1, kg*m^2	Moment of inertia J_2, kg*m^2	Shaft line stiffness, (x100) Nm/rad
1	0,5	10	15
2	0,2	12	20
3	0,1	8	25
4	0,6	7	10
5	0,75	6	5
6	0,65	12	30
7	0,15	15	22
8	0,08	11	50
9	0,06	5	40
10	0,2	4	25
11	0,25	6	35
12	0,35	8	30
13	0,45	17	20
14	0,55	21	40
15	0,06	16	10
16	0,08	13	16
17	0,12	14	28
18	0,16	9	34
19	0,28	11	42
20	0,32	8	45

PRACTICAL WORK №7

Calculation of the motion of the engine crankshaft

Objective: To familiarize yourself with the methods of compiling differential equations of motion of the engine crankshaft.

Tasks:

- To familiarize yourself with the physical model of the transmission.

- To build a mathematical model of the acting force factors.

- To create a block diagram in Simulink to implement the mathematical model of the motion of the transmission shaft.

- To conduct simulation modeling of the motion of the transmission shaft under different initial integration conditions.

Information material.

The motion of the engine crankshaft (Fig. 7.1) is described by the equation:

$$J \cdot \dot{\varepsilon}_{\omega} \cdot \omega_{\text{nom}} = \left(M_{\text{norm}} + \frac{\varepsilon_r - \varepsilon_{\omega}}{k_x - 1} - \varepsilon_M \right) \cdot M_{\text{nom}}, \qquad (7.1)$$

where J – crankshaft moment of inertia; M_{nom}, ω_{nom} – nominal rated torque and nominal angular speed of the crankshaft; M_{norm} – normalized torque according to the external speed characteristics; k_x=1,1; $\varepsilon_r \in [\omega_{\min}/\omega_{\text{ang}}, 1]$ – coefficient characterizing the position of the fuel supply control (accelerator pedal); ε_{ω}, ε_M – relative angular velocity of the crankshaft and engine load factor by torque, respectively:

$$\varepsilon_{\omega} = \frac{\omega}{\omega_{\text{nom}}}; \qquad \varepsilon_M = \frac{M_c}{M_{\text{nom}}},$$

where M_c, ω – the moment of resistance to motion applied to the crankshaft and its current angular velocity, respectively.

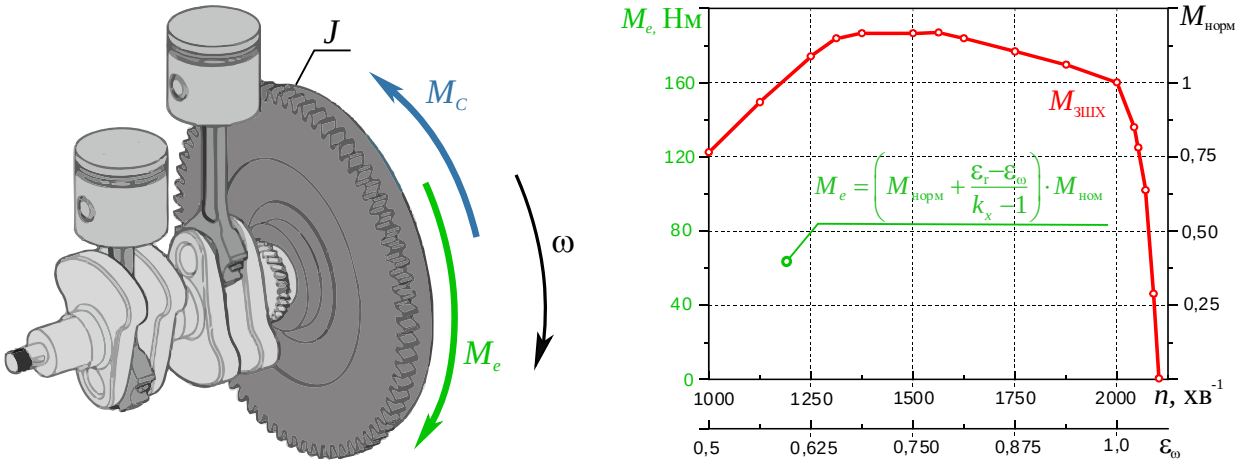
Equation (7.1) can be rewritten as:

$$J \cdot \dot{\varepsilon}_{\omega} \cdot \omega_{\text{nom}} = \left(M_{\text{norm}} + \frac{\varepsilon_r - \varepsilon_{\omega}}{k_x - 1} \right) \cdot M_{\text{nom}} - \varepsilon_M \cdot M_{\text{nom}} = \cdot M_e - \cdot M_{\text{c}},$$

where M_e – effective engine torque.

Equation (7.1) is a differential equation for the motion of the crankshaft of a diesel engine in relative variables, taking into account the load and the position of the fuel supply control device.

When implementing equation (7.1), the engine is loaded with a resistance moment ε_M can be absolutely any, up to an overload not exceeding the engine's torque adaptability coefficient $k_м$. If the fuel supply changes quite intensively, it may turn out that $\varepsilon_r > \varepsilon_\omega$, which is unacceptable. In this case, the engine switches to the external speed characteristic (correction branch) and the condition is accepted $\varepsilon_r = \varepsilon_\omega$, at the same time, the torque developed by the engine does not depend on the position of the fuel supply control element. For a diesel engine equipped with an all-mode regulator, which is actually described above, the partial characteristics are regulatory branches and fragments of the external speed characteristic.



а *б*

Figure 7.1 – Calculation scheme for calculating the crankshaft motion: a – power factor scheme; b – engine characteristic

During the simulation, the value ε_ω may vary within $\varepsilon_\omega \in [\omega_{min}/\omega_{nom}, k_x]$ and the coefficient characterizing the position of the fuel supply control element $\varepsilon_r \in [\omega_{min}/\omega_{nom}, 1]$. The proposed method differs from existing ones in that it allows

to realize any patterns of changes in external resistance over time ε_M and fuel supply control ε_r.

Do it yourself: Using the data from Table 7.1, create and implement a mathematical model of the engine crankshaft motion in the Simulink environment. Torque curve M_{ESCh} (Fig.7.1) external speed characteristics of the engine (M_{norm} = M_{ESCh} / M_{nom}) can be set independently. To implement any pattern of change in external resistance over time ε_M, and fuel control ε_r to set in a block Slider Gain or Sine Wave.

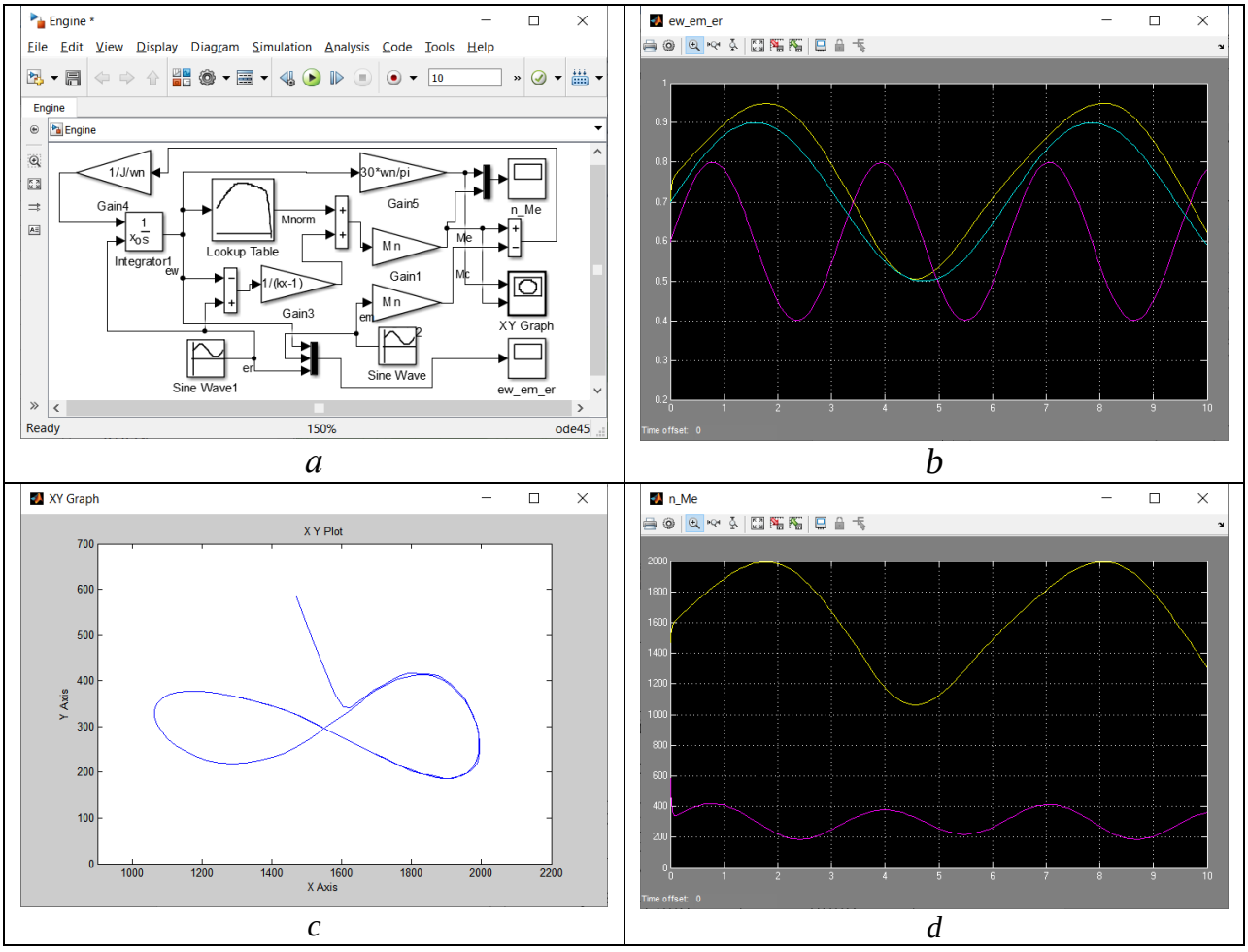


Figure 7.1 – Implementation of a mathematical model of crankshaft motion: a – block diagram of the model; b – engine operating parameters; c – change in engine operating modes; d – change in engine speed and torque over time

Table 7.1 – Data for calculating engine crankshaft motion

Option	Moment of inertia, kg*m^2	Nominal rated torque, Nm	Nominal angular velocity, rad/s
1	0,5	500	220
2	0,2	200	250
3	0,1	100	200
4	0,6	600	240
5	0,75	750	210
6	0,65	650	220
7	0,15	150	250
8	0,08	80	200
9	0,06	60	240
10	0,2	200	210
11	0,25	250	220
12	0,35	350	250
13	0,45	450	200
14	0,55	550	240
15	0,06	60	210
16	0,08	80	220
17	0,12	120	250
18	0,16	160	200
19	0,28	280	240
20	0,32	320	210

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