This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

eurizon
European network for developing new horizons for RIs

2024 IEEE 5th KhPI Week on Advanced Technology

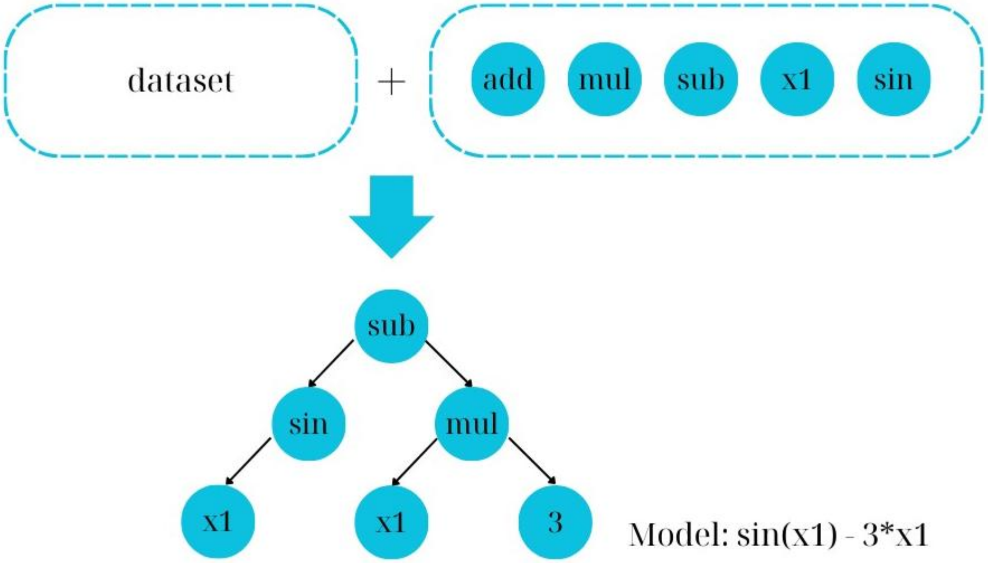
Symbolic Regression-Based Models for Hyperelastic Material

National Technical University «Kharkiv Polytechnic Institute»

Nataliia Fomenko, Oleksiy Larin

10 October 2024 Kharkiv, Ukraine

Symbolic regression



This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Problem statement

Tasks:

- generate synthetic data based on the hyperelastic material in a way that can be typically obtained from experiment. The dataset has 3 variables: time, strain values, stress values;
- conduct a series of comparative calculations to study different accuracy metrics;
- test the same metrics for two variants of input data: one hysteresis loop and two.

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Problem statement

$$W = C_1(\bar{I}_1 - 3) + C_2(\bar{I}_2 - 3) \qquad (1)$$

where C_1 and C_2 are empirically determined material constants, and $\bar{I}_1$ and $\bar{I}_2$ are the first and second invariant of Finger tensor (the inverse of the left Cauchy-Green tensor).

$$\sigma_{uniax} = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left[C_{10} + \frac{C_{01}}{\lambda}\right] \qquad (2)$$

where C_{10} and C_{01} are material parameters.

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Technical stack

- gplearn
- Deep Symbolic Optimization (DSO)
- PySR
- symreg
- QLattice
- Operon

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Hyperparameters

1. Functional of quality that is formulated from the sense of metrics that is used.
2. A policy optimizer, which is an algorithm for optimizing the parameters.
3. Token optimization options.

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Error metrics

$$E_1 = -\frac{1}{N}\sum_{i=1}^{N} log\left(1 + \left|\frac{\Delta x_i}{\Delta y_i} - \frac{\delta x_i}{\delta y_i}\right|\right)$$

where N is the number of data points, $\frac{\Delta x_i}{\Delta y_i}$ is an implicit derivative estimated from the data, and $\frac{\delta x_i}{\delta y_i}$ is the implicit derivative derived from the candidate implicit equation.

$$E_2 = \frac{1}{1 + \frac{1}{\sigma_y}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - f(X_i)\right)^2}}$$

where n – dataset size, f – possible approximation function, (X,y) – given dataset, σ_y – standard deviation of the target values.

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Input data

- Function set: ["add", "sub", "mul", "div", "n2"]
- Number of samples 500000 and 1000000
- Constants required for data generation:

$E = 5,\ \varepsilon_0 = 3.8,\ \eta = 0.2,\ C10 = 1.4,\ C01 = -0.3.$

- 1000 iterations

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Application of symbolic regression to the approximation of hyperelastic behavior (one loop)

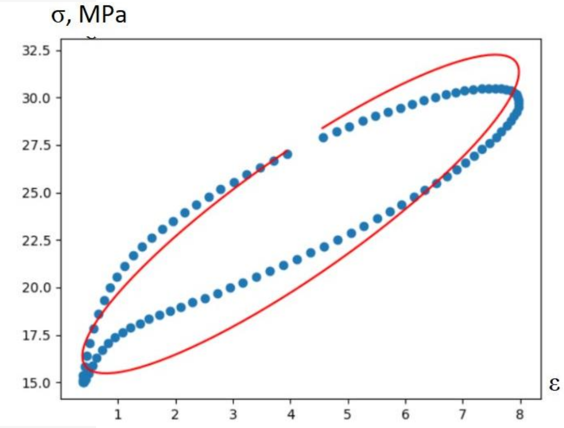


Fig. 1. Approximation using E2 metric without constants search for one hysteresis loop

$$R = 0.7901$$

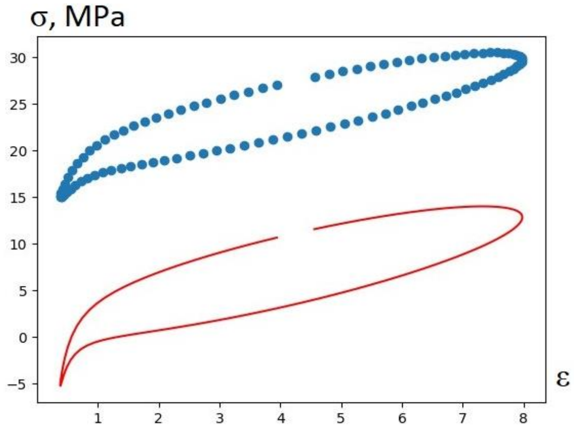


Fig. 2. Approximation using E1 metric without constants search for one hysteresis loop

$$R = 0.7712$$

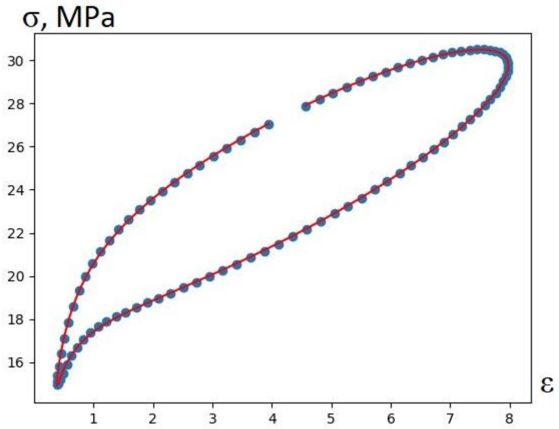


Fig. 3. Approximation using E1 metric with constants search for one hysteresis loop

$$R = 0.9126$$

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Application of symbolic regression to the approximation of hyperelastic behavior (two loops)

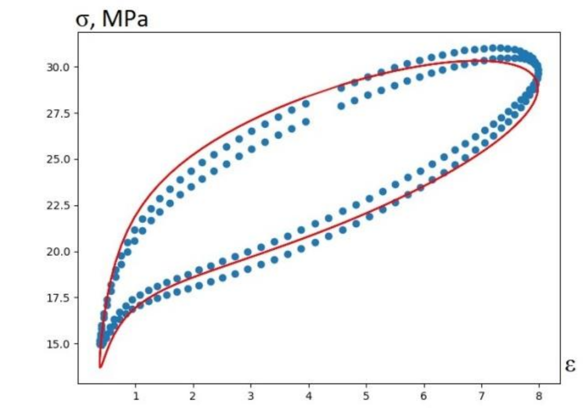


Fig. 4. Approximation using E2 metric without constants search

$R = 0.7817$

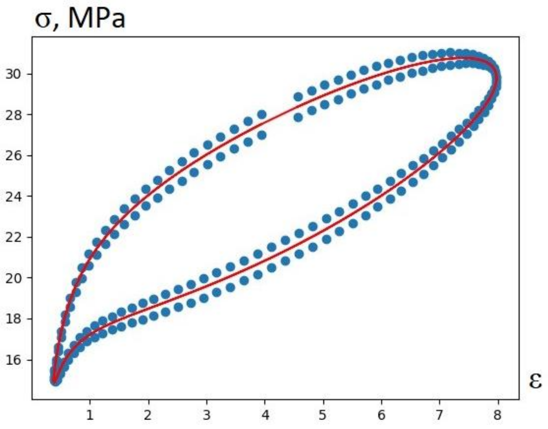


Fig. 5. Approximation using E2 metric with constants search

$R = 0.7817$

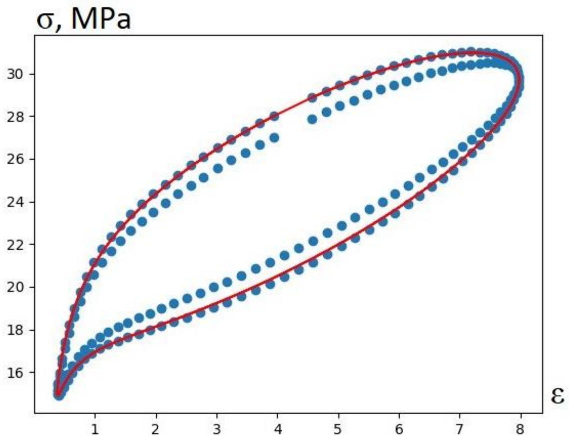


Fig. 6. Approximation using E1 metric with constants search for 2 hysteresis loops

$R = 0.8617$

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Conclusions

- We investigated an error metric based on implicit derivatives (E_2) alongside the standard inverse normalized root mean squared error (*NRMSE*, E_1).
- SR successfully captured the qualitative trends of the material behavior (R-squared up to 0.86), the novel E_1 metric did not consistently outperform the established E_2 metric. However, E_1 offered a visually better fit in specific scenarios (single hysteresis loop without constant search).

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Thank you for your attention!

Nataliia Fomenko
nataliia.fomenko@infiz.khpi.edu.ua

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.

Accuracy of models in different configurations

	Without const search	With const search
E_1 and 1 loop	$R = 0.7712$	$R = 0.9126$
E_2 and 1 loop	$R = 0.7901$	$R = 0.9933$
E_1 and 2 loops	$R = 0.7817$	$R = 0.8617$
E_2 and 2 loops	$R = 0.8674$	$R = 0.9356$

This project has received funding through the EURIZON project, which is funded by the European Union under grant agreement No.871072.