Syllabus
Course Program

Special chapters of higher mathematics

Specialty
113 - Applied mathematics

Institute
IBE of Computer Modeling, Applied Physics and Mathematics

Educational program
Computer and mathematical modeling

Department
Applied Mathematics

Level of education
Bachelor's degree

Course type
Mandatory (*SP 14)*

Semester
4

Language of instruction
English

Lecturers and course developers

Yuri Mikhlin

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Doctor of Physical and Mathematical Sciences, Professor, Professor of the Department of Applied Mathematics

Researcher and educator with over 50 years of teaching, theoretical, and practical experience in the field of nonlinear dynamics, theory of nonlinear normal modes of oscillations, stability theory of motion, dynamics of thin-walled elastic systems, vibration damping, etc. Author of over 300 scientific publications and co-author of 6 monographs. Has frequently given talks and lectures at universities in various countries and at leading international conferences. Main courses: Special Chapters of Higher Mathematics, Differential Equations, Linear Algebra, Stability of Dynamic Systems.
More about the lecturer on the department's website

General information

Summary

Course Description
The course "Special Chapters of Higher Mathematics" provides knowledge essential for the theoretical and practical training of bachelor's and master's students in solving various theoretical and engineering problems arising in the theory of oscillations, modeling of physical processes, statics, strength, and dynamics of machines and mechanisms, optimization tasks, and more. The acquired knowledge will be applied when studying mathematical physics equations, variational calculus, specialized courses, and during the preparation of term papers, qualification, and diploma projects.

Course objectives and goals

The purpose of the course is to familiarize students with the fundamental concepts of the theory of functions of a complex variable, to form an understanding of its methods and interconnections with real variable analysis, as well as with other mathematical disciplines. The course also aims to provide skills in solving basic problems in the theory of complex variable functions, to introduce the basics and methods of functional analysis, and to develop students' skills in analyzing and solving the most important problems of functional analysis.

Format of classes

Lectures, practical classes, independent work, consultations. Final assessment – exam.

Competencies

GC06: Ability for abstract thinking, analysis, and synthesis.
PC01: Ability to use and adapt mathematical theories, methods, and techniques to prove mathematical statements and theorems.

Learning outcomes

LO02: Master the basic principles and methods of mathematical, complex, and functional analysis, linear algebra and number theory, analytical geometry, theory of differential equations, including partial differential equations, probability theory, mathematical statistics and random processes, numerical methods.

Student workload

Total: 180 hours (6 ECTS credits): lectures – 48 hours, practical classes – 32 hours, independent work – 100 hours

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Course prerequisites

Students should possess basic knowledge and skills in mathematical analysis, linear algebra, and the theory of ordinary differential equations.

Features of the course, teaching and learning methods, and technologies

Features of the discipline include individual homework assignments comprising both standard and more complex tasks (optional) for the most advanced students. Students are encouraged to use computer software to solve proposed problems.

Program of the course

Topics of the lectures

Topic 1. Theory of Functions of a Complex Variable
1.1. Forms of writing complex numbers. Operations with complex numbers in algebraic form.
1.2. Operations with complex numbers in trigonometric and exponential forms.
1.3. Geometric representations on the complex plane.
1.4. Basic elementary functions of a complex variable.
1.5. Cauchy-Riemann conditions (D'Alembert-Euler). Geometric meaning of the modulus and argument of the derivative.
1.6. Conformal mappings.
1.7. Linear and fractional-linear mappings.
1.8. Integration of functions of a complex variable.
1.9. Calculation of integrals of complex functions using Cauchy's integral formulas.
1.10. Taylor series for complex functions.
1.11. Laurent series.
1.12. Singular points, classification of singular points. Calculation of residues of complex functions at singular points.

Topic 2. Introduction to Functional Analysis
2.1. Metric and normed spaces. Banach's theorem - contraction mapping principle.
2.2. Application of the contraction mapping principle for solving differential and integral equations.

2.3. Unitary and Hilbert spaces. Schwarz and triangle inequalities. Orthogonal systems of functions. Schmidt orthogonalization.
2.4. Generalized Fourier series. Minimal property of Fourier series. Orthogonal polynomials.
2.5. Fourier integral transform.
2.6. Laplace integral transform and its properties.
2.7. Application of the Laplace transform to solving differential and integral equations.
2.8. Operators in functional spaces. Linear operators and linear functionals. Spaces of linear operators.
2.9. Eigenvalue and eigenvector problems for linear operators. Applications in differential equations. Sturm-Liouville problem.
2.10. Adjoint, self-adjoint, and orthogonal linear operators and their properties.
2.11. Nonlinear operators. Bifurcations of solutions, examples in differential and integral equations.
2.12. Elliptic functions, Jacobi functions, and their applications. Euler's gamma function. Bessel functions.

Topics of the workshops

None.

Topics of the laboratory classes

Topic 1. Theory of Functions of a Complex Variable
1.1. Operations with complex numbers.
1.2. Geometric representations on the complex plane.
1.3. Basic elementary functions of a complex variable.
1.4. Cauchy-Riemann conditions. Finding the real part of a complex function from its imaginary part or vice versa.
1.5. Linear and fractional-linear mappings.
1.6. Integration of functions of a complex variable.
1.7. Taylor and Laurent series.
1.8. Singular points, classification of singular points. Calculation of residues at singular points.

Topic 2. Introduction to Functional Analysis
2.1. Application of the contraction mapping principle for solving differential and integral equations.
2.2. Orthogonal systems of functions. Schmidt orthogonalization.
2.3. Generalized Fourier series. Orthogonal polynomials.
2.4. Laplace integral transform and its properties.
2.5. Application of the Laplace transform to solving differential and integral equations.
2.6. Eigenvalue and eigenvector problems for linear operators. Applications in differential equations. Boundary value problems. Sturm-Liouville problem.
2.7. Adjoint, self-adjoint, and orthogonal linear operators.
2.8. Nonlinear operators. Bifurcations, examples in differential and integral equations.

Self-study

Independent work involves mastering the presented theoretical material, solving additional problems that consolidate practical material, and completing individual assignments (IA) for each section of the course. Assignments are checked remotely with scores assigned for each IA, contributing to the final grade. Students may be invited to revise or correct their assignments if necessary.

Course materials and recommended reading

Main Literature
1. Veretelnyk V.V., Tymchenko H.M. Theory of Functions of a Complex Variable (Textbook), Kharkiv: NTU "KhPI", 2012, 207 p.
2. S.M. Yezhov, M.A. Razumova. Theory of Functions of a Complex Variable: "Kyiv University", 2012, 191 p.
3. Shvets V.T. Higher Mathematics: Theory of Functions of a Complex Variable. Odesa: BMW Publishing, 2014, 284 p.
4. Mastynovskyi Yu.V., Shyshkanova H.A. Theory of Functions of a Complex Variable. Zaporizhzhia: ZNTU, 2012, 160 p.

5. Boyaryshcheva T.V., Hudyvok T.V., Pohorilyak O.O. Functional Analysis. Textbook for students. Uzhhorod: Invazor, 2013, 125 p.
6. Horodnii M.F. et al. Practice Tasks in Functional Analysis. Kyiv: Kyiv University Press, 2006, 103 p.
7. Fedak I.V. Lecture Course on Functional Analysis and Measure Theory. Textbook. Basic Structures of Functional Analysis. Ivano-Frankivsk: Vasyl Stefanyk PNU, 2020, 48 p.
8. Vasilyshyn T.V., Hoy T.P., Fedak I.V. Integral Equations. Ivano-Frankivsk: Holiney, 2016, 224 p.
9. Bilokolos Ye.D., Yurachkivskyi A.P., Sheka D.D. Special Functions in Problems of Mathematical Physics. Textbook. Taras Shevchenko Kyiv National University, 2000, 91 p.
Additional Literature
10. Complex Analysis: Textbook. Goldberg A.A., Sheremeta M.M., Zabolotskyi M.V., Skaskiv O.B.; Ivan Franko National University of Lviv, 2002, 204 p.
11. Ablowitz, M.J., Fokas, A.S. Complex Variables: Introduction and Applications. Cambridge, 2003.
12. Bachman, G., Narici, L. Functional Analysis. Academic Press, 1966 (reprint Dover Publications).
13. Hutson, V., Pym, J.S., Cloud, M.J. Applications of Functional Analysis and Operator Theory, 2nd edition, Elsevier Science, 2005.
14. Kolmogorov, A.N., Fomin, S.V. Elements of the Theory of Functions and Functional Analysis. Dover Publications, 1999.
15. Temme, N.M. Special Functions: An Introduction to the Classical Functions of Mathematical Physics. John Wiley & Sons, 1996.

Assessment and grading

Criteria for assessment of student performance, and the final score structure

Description of the Final Assessment Structure, Mandatory Assignments, and Scoring Procedure
Particular attention is paid to independent work and individual assignments.

Control tests – 20 points
Colloquia – 20 points
Individual assignments (IA) – 30 points
Final exam – 30 points.

Grading scale

Total points	National	ECTS
90–100	Excellent	A
82–89	Good	B
75–81	Good	C
64–74	Satisfactory	D
60–63	Satisfactory	E
35–59	Unsatisfactory (requires additional learning)	FX
1–34	Unsatisfactory (requires repetition of the course)	F

Norms of academic integrity and course policy

The student must adhere to the Code of Ethics of Academic Relations and Integrity of NTU "KhPI": show discipline, good manners, goodwill, honesty, responsibility. Conflict situations should be openly discussed in study groups with the teacher, and if it is impossible to resolve the conflict, they should be brought to the attention of the staff of the Institute's directorate.
Regulatory and legal support for the implementation of the principles of academic integrity of NTU "KhPI" is available on the website: http://blogs.kpi.kharkov.ua/v2/nv/akademichna-dobrochesnist/

Approval

Approved by	Date, signature	Head of the department Vyacheslav BURLAENKO
	Date, signature	Guarantor of the educational program Gennadiy LVOV