7. STEADY-STATE PROCESSES IN NONLINEAR ELECTRIC CIRCUITS

7.1. CHARACTERISTIC FEATURES OF NONLINEAR ELEMENTS

So far we have analyzed fundamental properties and calculation methods of the linear electric circuits. The nature of the linear electric circuits enabled the application of superposition principle.

It should be noted that the current through any linear element corresponds to voltage across it and is calculated using algebraic or differential first-order equations. Linear resistance, inductance or capacitance is defined according to linear dependence of currents on voltages, flux linkage on current and electric charge on voltage.

In this section will analyze fundamental properties and methods of calculation of nonlinear electric circuits. Any electric circuit is nonlinear if it includes at least one nonlinear element. A nonlinear element is the element in which current and voltage have nonlinear dependence on each other. Processes in nonlinear electric circuits are described by nonlinear algebraic or differential high-order equations. We should remember that in general case it is impossible to apply the superposition principle to the nonlinear circuits.

In accordance with possible dissipation of electric energy into thermal energy or accumulation of magnetic or electric energy, there are distinguished nonlinear resistances, inductances and capacitances.

Static characteristics of nonlinear elements can be obtained experimentally, or can be defined in the form of a chart or table by approximate analytic expressions. They express the nonlinear voltage-current dependence (current-voltage *U*-*I* curve of nonlinear resistance), flux linkage or magnetic flux with respect to current (weber-ampere *B*-*H* curve of nonlinear inductance), the charge in versus of voltage (coulomb-volt *Q*-*U* curve of nonlinear capacitance).

By response to the control action, they are divided into unregulated and regulated nonlinear elements. Unregulated nonlinear elements can contain single nonlinear curve only. Regulated nonlinear elements have a set of nonlinear curves.

7.1.1. GRAPHICAL REPRESENTATION OF NONLINEAR ELEMENTS

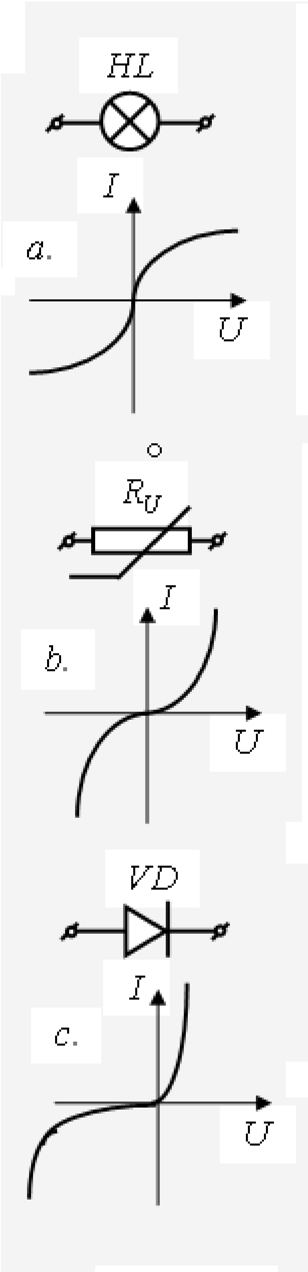

Fig.7.1

A graphical representation of the nonlinear element characteristics is the most visible.

Depending on the type of the current-voltage characteristics there are distinguished nonlinear elements symmetric with respect to the origin, and the asymmetric characteristics.

The operating modes of nonlinear elements with symmetrical characteristics are not changed if the signs of current and voltage are changed simultaneously. The operating modes of the nonlinear elements with asymmetric characteristic are essentially dependent on the signs of the current and voltage at the terminals.

a. Nonlinear resistances. Current-voltage curves of symmetric nonlinear elements are given in Fig.7.1.*a*, *b*. For example an incandescent lamp with a metal thread has a shape of characteristics Fig.7.1.*a*. Change of the angle of *U-I* curve is explained by the fact that as the current increases, the filament is being heated more, and internal resistance of metallic filament also increases.

A varistor is semiconductor device having the current-voltage characteristics, Fig.7.1.*b*. With increasing applied voltage the change rate of the varistor conductivity is decreasing regardless of the applied voltage sign. Symmetrical shape of the varistor characteristics is used in both DC and AC circuits.

The typical form of current-voltage characteristics (CVC) of semiconductor diode is shown in Fig.7.1.*c*. Forward-bias branch of a current-voltage characteristic (the first quadrant) corresponds to the forward displacement of the *p-n* junction. With the voltage increase the direct current rapidly increases. This current is conditioned by the motion of the majority carriers through the lowered potential barrier. However, once the main charge carriers have moved to the opposite area, they become minority carriers.

Such process of the introduction of majority carriers from the opposed areas without changing the sign of the voltage and current by lowering the height of the potential barrier is called injection.

The reverse branch of CVC (third quadrant) corresponds to the reverse bias *p-n* junction. Diffusion current with growth (in absolute magnitude of reverse voltage) decreases exponentially and when voltage is a tenth part of a volt it is practically equal to zero. From this consideration follows an important property of unilateral conductivity of the *p-n* junction, which is used in electric instrument engineering.

In a more general form the *p-n* junction can be considered as a nonlinear element whose resistance depends on the value and polarity of the applied voltage. Namely this nonlinearity of CVC underlies the operation of semiconductor devices that are used for rectification of the AC, frequency conversion, limiting of amplitudes.

Bipolar transistor, Fig.7.2.*a*, has two *p-n* junctions and a set of nonlinear curves input characteristics, Fig.7.2.*b*, and output ones, Fig.7.2.*c*. The changing effect from one or more of the voltages or currents of transistor circuits are shown graphically and these graphs are called the input Fig.7.2.*b* and output Fig.7.2.*c* characteristics.

There are five main variables (currents through collector, base and emitter, and voltages across the collector and base, emitter and base) and also three connection circuits, so there may be a large number of characteristics. One of possible connections is circuit with common-emitter (Fig.7.2.a).

In the common-emitter circuit the input characteristic is the dependence of the base current on the voltage between the emitter and base. Base current varies significantly less than the corresponding emitter current and for the *n-p-n* transistor the input characteristic has a shape which is shown in Fig.7.2.*b*. Such a characteristic can be obtained for *p-n-p* transistor, which has a reverse polarity.

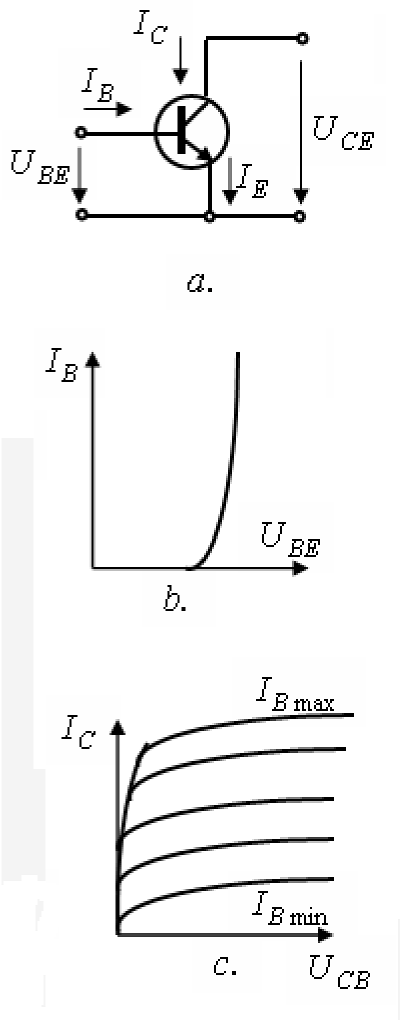

Fig.7.2

The set of output characteristics in common-emitter circuit can be obtained by changing the base current and when keeping the voltage between the emitter and base at a constant value, and some of these for the *n-p-n* transistor are shown in Fig.7.2.*c*.

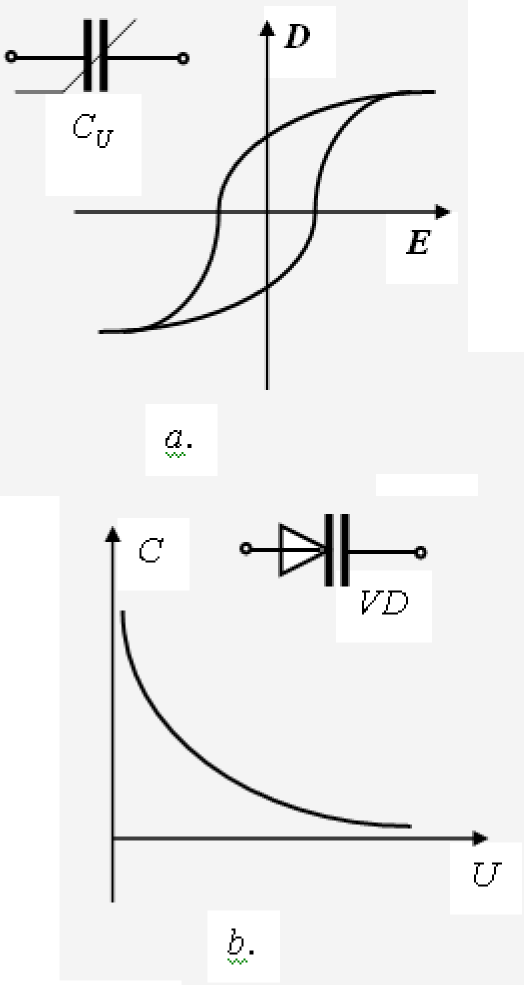

Fig.7.3

b. Nonlinear capacitances. There are two types of nonlinear capacitances: varicond and varicap (varactor). In the varicond ferroelectric with variable dielectric permeability is used as dielectric (Fig.7.3.a). The relation between the electric displacement density, ***D***, and the corresponding electric field intensity, ***E*,**

$$\boldsymbol{D} = \varepsilon_a \boldsymbol{E}$$

is characterized by the fact that the relative dielectric permeability ε of ferroelectric materials is not a constant value, but is a function of the electric field intensity ***E***.

The varicap is a planar diode. Its characteristic is shown in Fig.7.3.b.

c. Nonlinear inductance. The relation between the magnetic induction (flux density) **B** and the corresponding field intensity, ***H*,**

$$\boldsymbol{B} = \mu_a \boldsymbol{H}$$

is characterized by the fact that the relative permeability μ of magnetic materials is not a constant value, but is a function of the magnetic field intensity. In essence, all magnetic materials exhibit a phenomenon called saturation, whereby the flux density increases proportionally to the magnetic field intensity, but only to a certain limiting value of the latter. It may be noted that since the *B-H* is nonlinear curve, the value of μ (which determines the slope of the magnetization curve) depends on the intensity of the magnetic field.

To understand the reasons for the saturation of the magnetic material, we will briefly examine the mechanism of magnetization. The basic idea of the magnetic material theory is that the spin of electrons constitutes the motion of charge, and therefore leads to magnetic effects.

In most materials the electron spins cancel out, and on the whole this effect does not appear. In ferromagnetic materials, on the other hand, atoms can be oriented so that the electron spins cause magnetic effect. In such materials there exist small areas with strong magnetic properties (called magnetic domains), the actions of which are neutralized by unmagnetized material by other similar areas that are oriented differently in a random pattern. When the material is magnetized, the magnetic domains are generally oriented in the same direction that is determined by the degree of the intensity of the applied magnetic field.

As a result, a large number of miniature magnets within the material are polarized by the external magnetic field. As the field increases, more and more domains are oriented in the same direction. When all of the domains have become oriented in the same direction, any further increase in the magnetic field strength does not increase the flux density in a magnetized material. Thus, the relative permeability μ will tend to 1 in the saturation area.

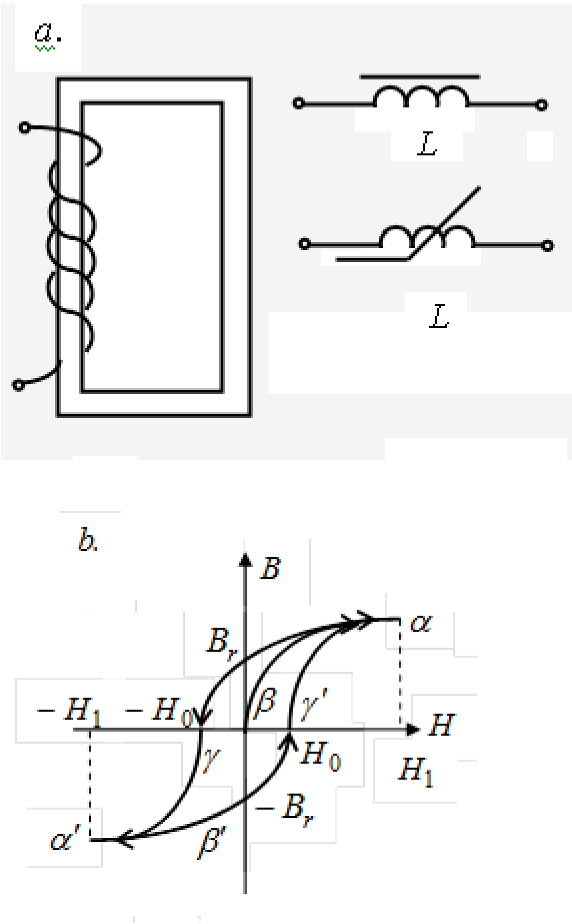

Fig. 7.4.

Saturation phenomenon has some interesting properties with respect to the operation of magnetic circuits: in agreement with results of the previous sections, it would seem that an increase in the MMF (i.e., an increase in the excitation current of the coil) will lead to a proportional increase in the magnetic flux. This is true in the linear region, however, ferromagnetic materials may reach saturation, and further increase in the drive current (or, equivalently, the MMF) does not yield further increase in the magnetic flux.

There are two features that distinguish the magnetic materials from the ideal model having a linear dependence *B-H*: eddy currents and hysteresis. The first phenomenon consists of the currents caused by any variation in time of the magnetic flux through the core material. As you know, a time varying flux will induce voltage and hence the current. When this occurs inside the magnetic core the induced voltage will cause the "eddy" currents (the terminology should be clear) through the core, these currents are dependent on the resistivity of the core. The effect of these currents is expressed in energy dissipating as heat. Eddy currents are reduced if you choose high-resistance core material, or by laminating the core, which provides non-ferromagnetic tiny gaps between the ferromagnetic layers of the core. The core lamination reduces eddy currents without significantly affecting the magnetic properties of the core.

Hysteresis can be explained by another loss mechanism in magnetic materials, which reflects a rather complex behavior associated with the magnetization of the material properties. The curve in Fig.7.4.*b* shows that the *B*-*H* curve for magnetic material when magnetized (when *H* is increased) is offset relative to the curve that is observed when the material has demagnetized. To understand the hysteresis process, we will consider a core that has been energized for some time, with field intensity of H_1 A/m. Since the current required maintaining the appropriate MMF, H_1 is reduced, we follow along the hysteresis curve from point α to point β.

When the MMF is exactly zero, the material has the remanent (or residual) magnetization B_r. To bring the flux density to zero, we must further decrease the MMF (i.e. by applying negative current) until the field strength reaches the values $-H_0$ (point γ on the curve). When the MMF is made more negative, the curve eventually reaches the point α'. If the excitation current of the coil is now increased, then the magnetization curve will take the path $\alpha' = \beta' = \gamma' = \alpha$, eventually returning to the original point in the *B*-*H* plane, but in another way.

As a result of this process requiring an excess of magnetomotive force to magnetize or demagnetize the material there arise pure energy losses. These losses are difficult to estimate precisely, but we can show that this is due to the area between the curves in Fig.7.4.*b*. There are experimental methods that allow estimating these losses.

7.1.2. STATIC AND DIFFERENTIAL RESISTANCES

For a comprehensive analysis and calculation of the nonlinear elements properties, two concepts of resistance are used: static and dynamic (or differential) resistances. The static resistance is defined as the ratio of the DC voltage U_0 across the nonlinear element to the DC I_0 at the same point

$$R_{st} = \frac{U_0}{I_0}.$$

Differential resistance is the ratio of small (theoretically infinitely small) voltage increment dU on the CVC of the nonlinear element to the corresponding current increment dI

$$R_d = \frac{dU}{dI} \approx \frac{\Delta U}{\Delta I}.$$

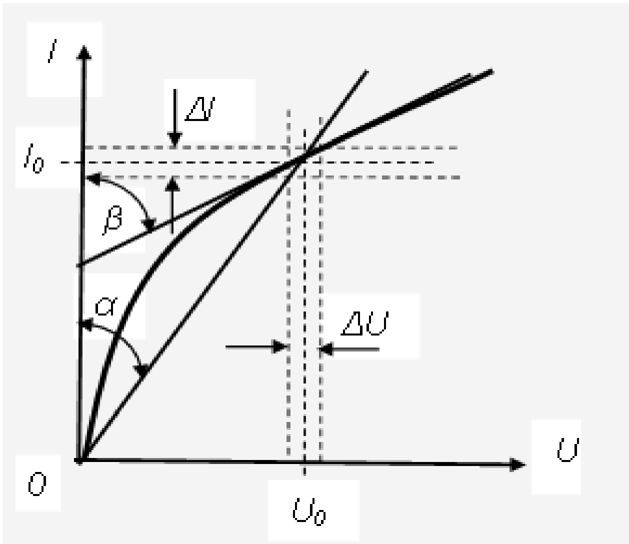

Fig. 7.5.

In general case, the differential resistance determines the steepness of CVC of nonlinear element at each point. The static and differential resistances are not equal to each other and coincide for linear elements only. The static resistance is proportional to (see Fig. 7.5) tangent of the angle α, formed by the secant drawn from the origin to the observation point on the characteristic, relative to the current axis

$$R_{st}=m_r tg\alpha,$$

where $m_r=m_u/m_i$ is the scale of resistance calculated through the voltage m_u and current m_i scales.

Differential resistance is proportional to the tangent of the angle β formed by the tangent at this CVC point with respect to the current axis

$$Rd= m_r tg\beta.$$

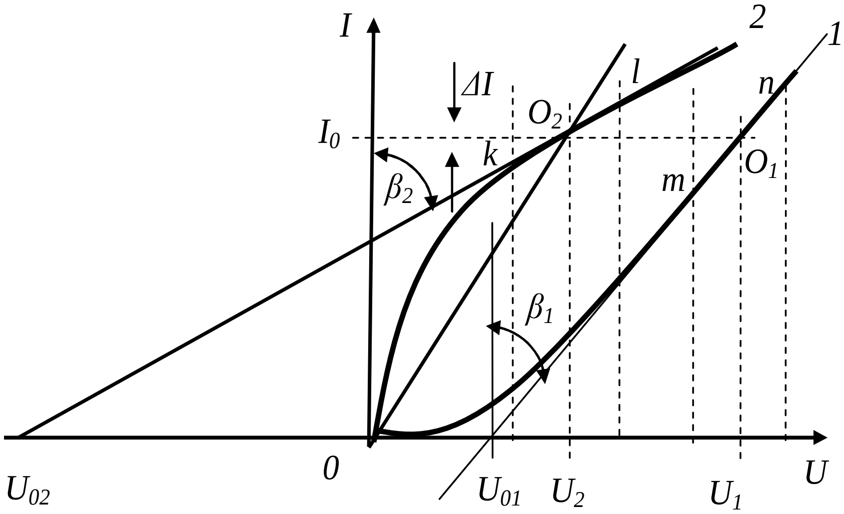

Fig. 7.6.

Static resistance is always positive. Differential resistance can be both positive and negative, for instance on the falling part of the non-linear characteristic with a negative slope, where the increase of the voltage at value ΔU leads to a decrease of the current on ΔI.

7.2. NONLINEAR DC CIRCUITS

In DC circuits show up only static resistances of nonlinear elements.

7.2.1. LINEARIZATION OF NONLINEAR ELEMENT BY LINEAR RESISTANCE AND EMF

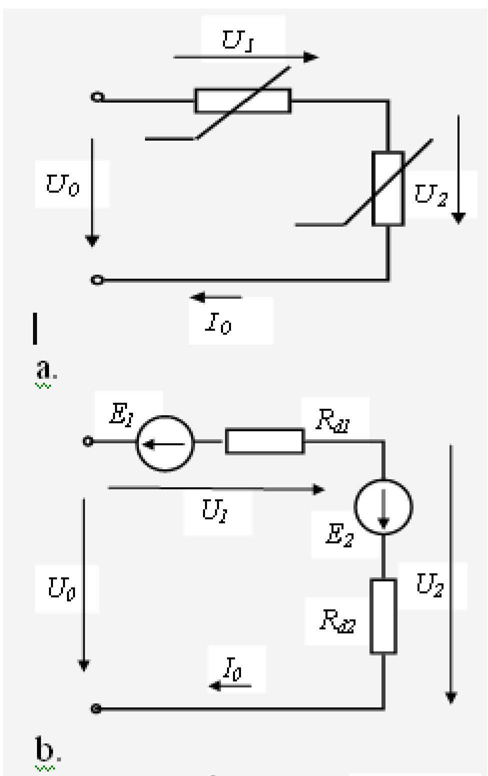

Fig. 7.7.

Such replacement can be performed if the circuit mode is chosen so that the operational segment of the non-linear element does not extend beyond the segment of the CVC, where with a certain degree of approximation it may be replaced by a straight line. These segments are characterized by a constant value of the differential resistance.

In Fig. 7.6 the current-voltage characteristics of the two non-linear elements are shown. Segments *mn* and *kl* of these CVC are characterized by constant differential resistance:

$$R_{d1} = m_r tg\beta_1; R_{d2} = m_r tg\beta_2.$$

Operational points O_1 and O_2 are respectively located within the segments *mn* and *kl* of CVC. If these segments are continued they will cross the vertical axis (the voltage axis) at points U_{01} and U_{02} respectively.

Then the equations of straight lines are written as follows:

$$U_1=U_{01}+IR_{d1};\ U_2=-U_{02}+IR_{d2}.$$

Having accepted that $U_{01}=E_1$ and $U_{02}=E_2$, we will obtain

$$U_1=E_1+IR_{d1};\ U_2=-E_2+IR_{d2}.$$

Consequently, each nonlinear element can be replaced by an EMF source of DC and linear resistance which is equaled to differential resistance of nonlinear element at the straight-line segment. In Fig. 7.7 *a, b* are shown an electrical circuit containing two nonlinear elements and a corresponding equivalent circuit.

Attention should be paid to the fact that the EMF E_1 is directed oppositely to the positive direction of the current (CVC1), and EMF E_2 is directed according to the positive direction of the current (CVC 2). After replacing the nonlinear resistance by linear resistance and EMF, the electric circuit is calculated as a linear circuit.

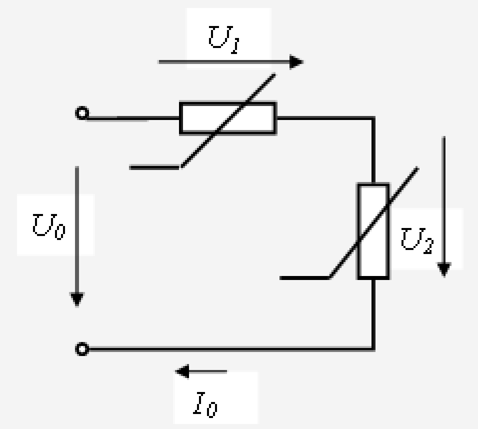

Fig.7.8.

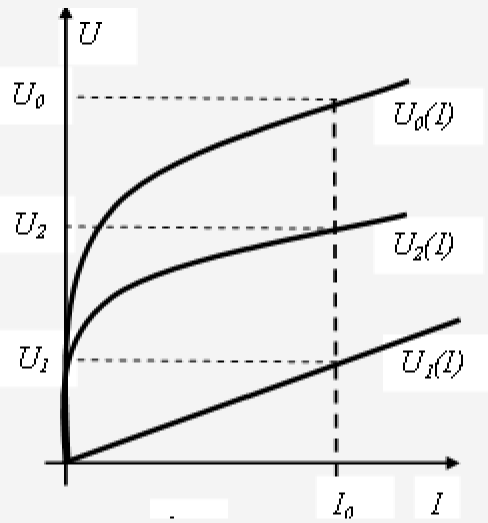

Fig.7.9.

7.2.2. SERIES, PARALLEL AND MIXED CONNECTIONS OF NONLINEAR ELEMENTS

Kirchhoff's laws can be used for the calculation of nonlinear circuits. If the analytical dependence *U(I)* of nonlinear elements are given, the analytical calculation can be performed in this case only.

Nonlinear circuits having simple configuration are obviously calculated using the graphical method. In this case, the nonlinear current-voltage characteristics of circuit elements must be specified in the form of plots.

Below is given the graphical method of nonlinear circuits calculation with series-parallel connection, based on the Kirchhoff's laws.

a. Series connection. When several linear and non-linear elements are connected in series, the total current-voltage characteristic is drawn up by summing the ordinates of the individual element characteristics in accordance with the Kirchhoff's voltage law

$$U = U_1 + U_2 + \ldots + U_n .$$

Circuit with a series connection of two nonlinear elements and the sequence of plotting the resulting current-voltage characteristic is shown in Fig. 7.8, 7.9.

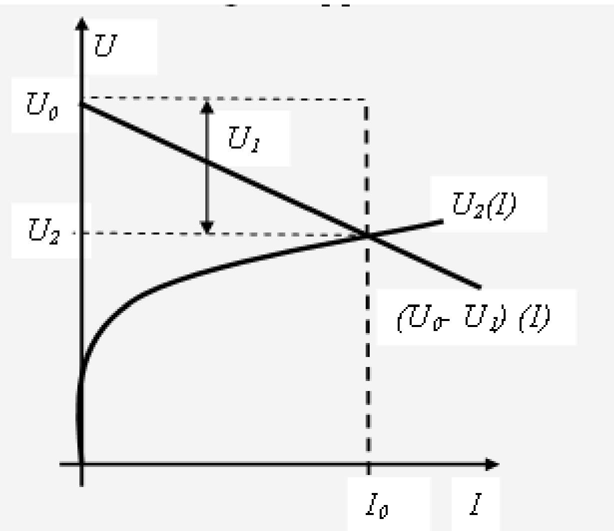

Fig.7.10.

To find the resulting current-voltage characteristics, arbitrary values of the current were assigned and the corresponding ordinates of the characteristics of non-linear elements were summarized. Thus, to receive the current value, $U = U_1 + U_2$ is determined. Two series-connected nonlinear elements are replaced by one with a CVC *U (I)*, as a result of the calculation. The resulting current-voltage characteristics allow you to find the current through circuit and the voltage across the nonlinear elements, for any value of the voltage *U* applied to the circuit.

For a given value of the applied voltage *U*, current and voltage drops can be determined without drawing up the resulting current-voltage characteristics of the circuit. For this purpose, one of the predetermined CVC is shifted upwards along the axis *Y* (voltage axis) from the origin to the value of the applied voltage *U* and constructed so as to obtain a mirror image of the curve with respect to the horizontal line (see Fig. 7.10). The intersection point of the mirror reflection characteristics of the nonlinear element *U₁(I)*, with another characteristic of the nonlinear element *U₂(I)* circuit determines the desired current through the circuit *I* and voltages across nonlinear elements U_1 and U_2.

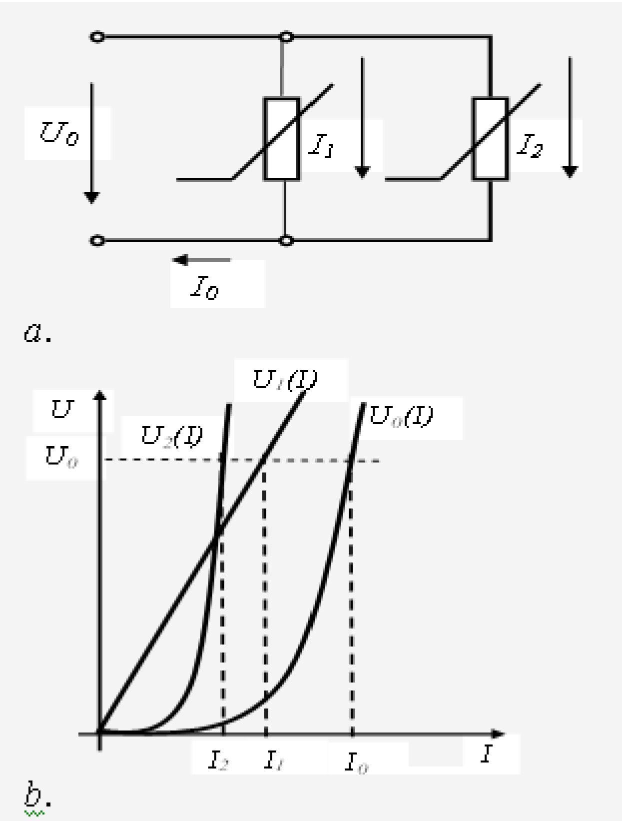

Fig.7.11.

b. Parallel connection. When several linear and nonlinear elements are connected in parallel, the total current-voltage characteristic is drawn up by summing the abscissas characteristics of individual elements in accordance with Kirchhoff's currents law

$$I = I_1 + I_2 + ... I_n.$$

In Fig. 7.11.*a* is shown the parallel connection of two nonlinear elements, and their characteristics are given. To construct the resultant CVC of the circuit, arbitrary values of voltage should be chosen and the summing of relevant characteristic abscissa of the given nonlinear elements is performed. Thus for the given value of voltage U_0, the current $I = I_1 + I_2$ is defined, Fig. 7.11.*b*.

The resultant CVC plot corresponds to replacing two nonlinear elements connected in parallel by one element with a *U(I)* characteristic. According to this characteristic, for a given value of current *I* through the unbranched part of the circuit the voltage applied to the nonlinear elements is determined and then the currents through them.

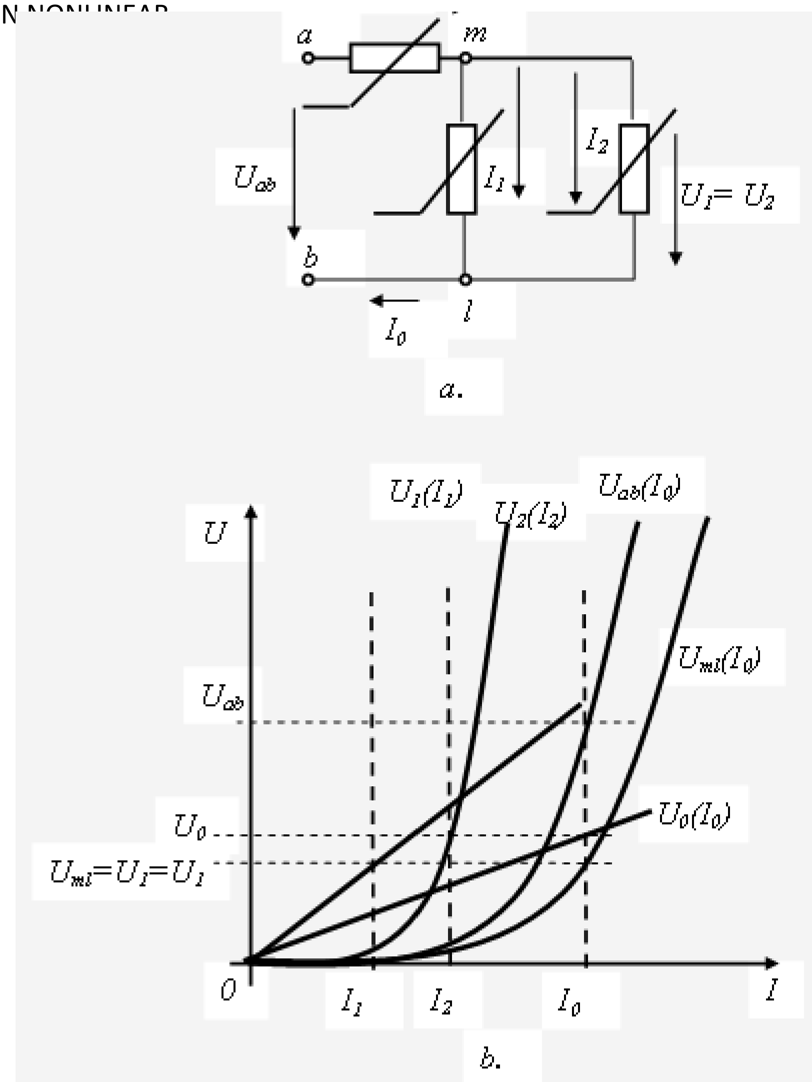

Fig . 7.12.

c. Mixed connection. The circuit with three nonlinear elements having mixed connections is shown in Fig. 7.12 *a*. The circuit segment, consisting of two elements connected in parallel, is replaced by a non-linear element with equivalent CVC $U_{ab}(I)$. A result of the circuit conversion is presented as a non-linear element $U_{ab}(I)$ and a non-linear element $U_1(I)$ connected in series according to which the resulting characteristics of the circuit *U(I)* is constructed, Fig. 7.12 *b*.

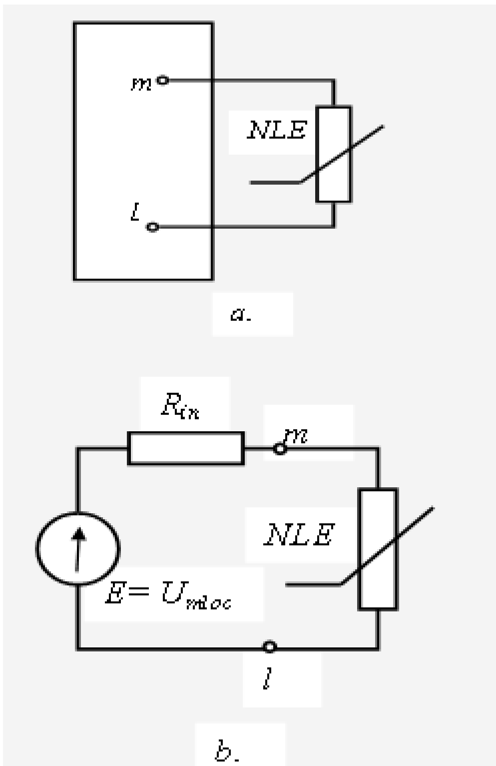

Fig.7.13.

7.2.3. CALCULATION OF CIRCUIT WITH NONLINEAR ELEMENT BY USING METHOD OF OPEN AND SHORT CIRCUIT

This method is based on using of Thevenin's and Norton's theorems.

If the complicated electrical circuit has only one nonlinear element then it is rational to determine the current through the branch with the nonlinear element by open (*oc*) and short circuit (*sc*) method.

For the calculation it is necessary to allocate the branch with nonlinear element and to present the rest linear circuit in the form of an active one-port (Fig. 7.13,*a*).

As is known, with respect to terminals *m* and *l* of selected branch the linear active one-port can be replaced by equivalent generator (Fig. 7.13,*b*) of EMF with voltage equal to one in open circuit across terminals *m* and *l* ($E=U_{mloc}$) and an internal resistance equaling to the input one-port resistance (R_{in}) or by a source current (I_{sc}) with the same internal resistance (R_{in}). At that

$$R_{in} = \frac{U_{mloc}}{I_{sc}}$$

where I_{sc} is the short circuit current across terminals *m* and *l*.

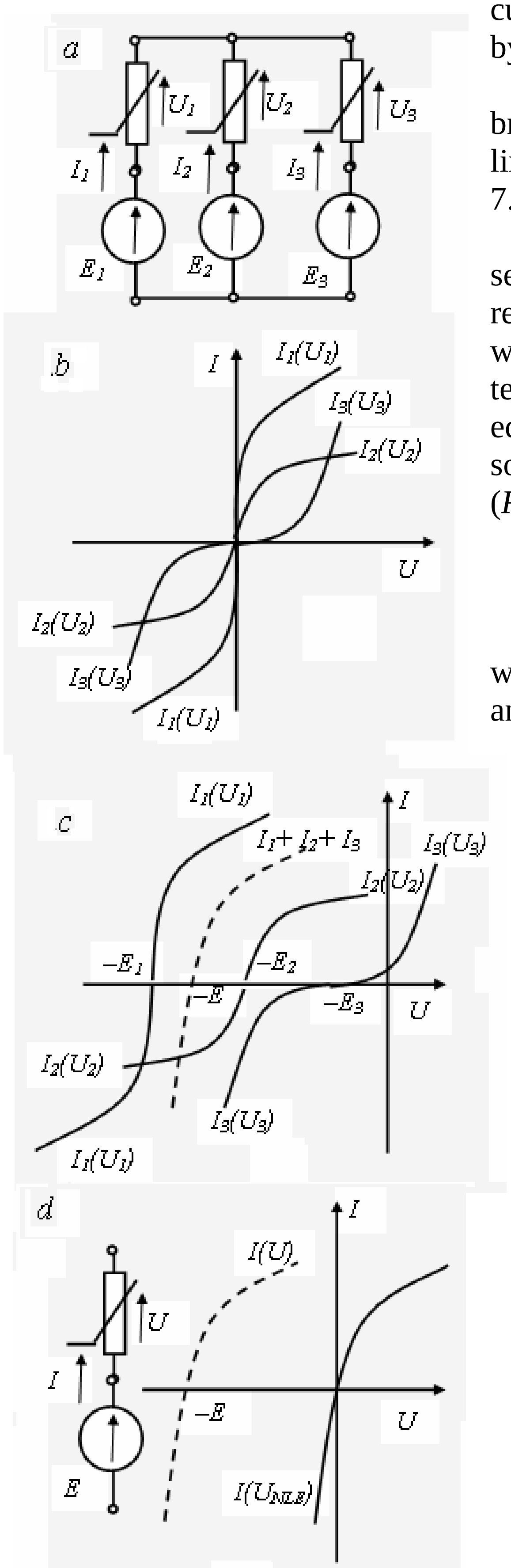

Fig. 7.14

7.2.4. CALCULATION FEATURES OF NONLINEAR CIRCUIT WITH TWO NODES

a. Graphical method. In the previous section we have shown the possibility of obtaining the characteristic of the nonlinear element, which is equivalent to two parallel-connected linear and nonlinear elements.

Similar plotting is possible for the several parallel branches which along with nonlinear elements can comprise direct current sources connected in series with the nonlinear element (see Fig. 7.14, *a*).

To this, preliminarily current volt characteristics of each branch are built, in our case this is achieved by shifting the nonlinear characteristics (see Fig. 7.14,*b*) by value of the electromotive forces to the left from the origin (see Fig. 7.14,*c*). Thereafter, the resultant characteristic $I_1+I_2+I_3$ of parallel-connected branches is built, which is shown in Fig. 7.14,*c* and *d* by dotted line. The CVC has shifted to the left from the origin by the value *E* which can be regarded as the electromotive force *E* of the equivalent circuit.

In Fig. 7.14,*d* are shown the characteristics of the equivalent circuit consisting of the DC EMF E with a series-connected nonlinear resistance. The characteristic of this nonlinear resistance is shown in Fig. 7.14,*d* by solid line.

Since current sum $I=I_1+I_2+I_3$ in node equals zero, therefore the current through equivalent circuit is absent. Consequently, E equals to the voltage across the upper node relative to the lower node of the original circuit.

From this we can find the voltage across each nonlinear element:

$$U_1=E_1-E;\ U_2=E_2-E;\ U_3=E_3-E.$$

The current through each nonlinear element is determined by the corresponding current-volt characteristics.

Equivalent characteristic of the circuit presented in Fig.7.14.*d* can be used for calculation if we add a new branch (with linear or nonlinear element) to the given two-nodal circuit.

The described calculation method can also be applied in case when EMF sources are not contained in all parallel branches.

Graphic method of calculation requires great accuracy at plotting of the drawing. With inadequate accuracy or small scale the received result can be unsatisfactory.

b. Numerical method. Numerical method gives possibility of accurate solution in the minimum of graphic plotting. Calculation is conveniently carried out in tabular form:

E	$U_1=E_1-E$	$U_2=E_2-E$	$U_3=E_3-E$	$\sum I=0$

Any of the expected values of the voltages E across the upper and lower nodes is set after the calculation is performed in the specified sequence, and the corresponding values ΣI are defined. By assigning other voltage values across nodes, we continue this process until ΣI changes the sign. Then, in accordance

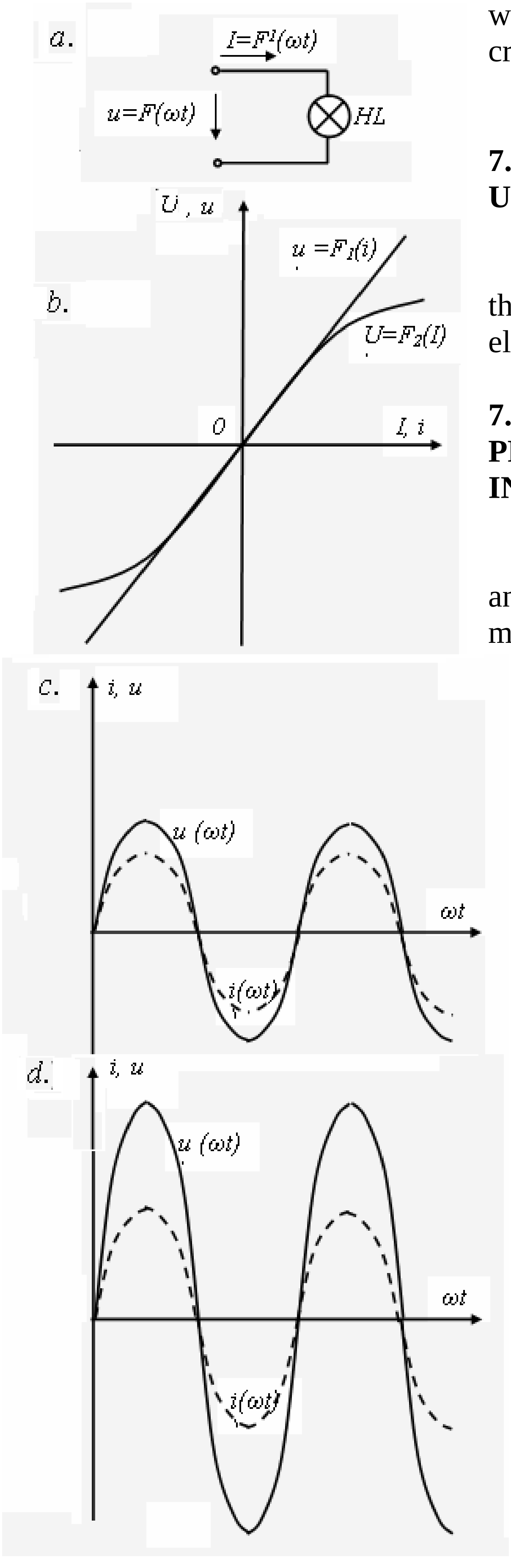

Fig. 7.15.

with the table data, a curve segment $\Sigma I=f(E)$ is created near of transition through zero in large scale.

The desired value *E* corresponds to $\sum I = 0$.

7.3. NONLINEAR ELECTRIC CIRCUIT UNDER AC

In AC circuits must take into account not only the static and dynamic characteristics of non-linear elements, but also the inertial properties of their.

7.3.1. SPECIFIC FEATURES OF PERIODICAL PROCESSES IN ELECTRIC CIRCUITS WITH INERTIAL NONLINEAR ELEMENTS

In nonlinear elements under AC, along with static and dynamic resistances, own inertance can be manifested. The nonlinear properties of the inertial elements are caused by changes in temperature due to the heat losses from flowing of current. Since thermal processes (heating and cooling) are inertial processes, even at relatively low frequency (e.g. 50 Hz) the temperature of these nonlinear elements and therefore resistance over a period practically does not change. Therefore, the dependence *i(u)* between the instantaneous values of current and voltage remains linear, while the dependence of *I(U)* between the effective value of the current and voltage will be nonlinear. Such nonlinear elements are inertial ones. These include incandescent, iron-hydrogen resistors, semiconductor resistances, etc.

Inertial elements with resistances are, for example, elements with great thermal inertia (e.g. incandescent, Fig.7.15,*a*.). An electromechanical element can serve as an example of inertial element.

Let all the nonlinear elements included in the circuit be inertial ones. This means that at steady state mode the parameters of all circuit elements remain constant over the period of currents and voltages variation. Hence, to describe the given and unchanged steady-state process, one can use the same techniques that have been developed to describe processes in linear circuits. When a sinusoidal voltage is applied to the circuit then voltages and currents at all the branches will also be sinusoidal, and to describe the process, one can use the complex form of record and vector diagram.

In periodic non-sinusoidal process after the applied voltage has been expanded into the Fourier series, we will have the same values of the parameters *R*, *L*, *C* of circuits for all harmonics as in the linear circuits, if we assume that these parameters do not change with frequency as we have been doing it before.

However, when the steady-state mode changes, for example, due to changes in the actual voltage at the terminals of the circuit, or even when you keep the actual voltage but change the amplitude spectrum of its harmonics, the effective voltages and currents at the circuit branches with non-linear elements are changed. Because at these branches the dependence $U = F(I)$ is not linear, their parameters $R_E = U_R / I$, $\omega L_E = U_L / I$ and $1 / (\omega C_E) = U_C / I$ are changed and, therefore the current distribution throughout the circuit is changed.

Thus the possibility of using for calculating such a circuit by superposing method and all calculation methods of complex circuits based on the principle of superposition is excluded. Kirchhoff's laws are applicable and can be written in complex form at the sinusoidal voltage. But in these equations complex nonlinear inertial resistance elements, i.e. modules and arguments of these impedances, are functions of actual currents through these elements. Therefore, algebraic equations written in complex form under Kirchhoff's laws are now non-linear ones. The difficulty in solution lies in the fact that in the general case the modulus and argument of the nonlinear element may depend on the actual current through the nonlinear element. But even if only modulus of this impedance changes then calculation will be complicated sufficiently, since this change leads to a redistribution of the amplitudes and phases of current and their change at all the branches of the circuit.

When sinusoidal voltage is being applied the following method of successive approximations can be recommended. Having set some probable values of complex impedances $\underline{Z}_S = Z_S e^{j\varphi_S}$ of nonlinear element and believing that they are constants, the calculation of the circuits is performed. Having determined the actual currents through the non-linear elements, we check the correspondence of the given parameters of the elements to the values of these parameters obtained from the actual characteristics of nonlinear elements at the determined values of currents. When the parameters mismatch then correction is made and calculation is repeated. This calculation should be carried out until the parameter values adopted for the calculation are close enough to their values obtained from the characteristics.

Sinusoidal steady-state modes in complicated power systems in some cases can also be described by systems of nonlinear algebraic equations. This is possible when the parameters of the values of generated and consumed power are given, but not lines and loads parameters and EMFs. When the power consumption is given, voltage and current are linked through the non-linear resistance or conductance.

In complicated power systems in terms of performance due to the nature of connection of generators and consumers it is the most appropriate to choose the voltage at the nodes of the system as unknown values. With this allowance, the most wide-spread calculation of such systems is nodal voltage method.

7.3.2. NONLINEAR INDUCTANCE DRIVEN BY SINUSOIDAL VOLTAGE

When connecting the coil with a steel core to a source of harmonic voltage one can obtain nonharmonic current. This phenomenon depends on the magnitude of the applied voltage to the induction coil with a ferromagnetic core and directly depends up the characteristics of the core ferromagnetic material. We define the current curve of inductor when these factors are taking into account.

7.3.2.1. THE INFLUENCE OF SATURATION AND HYSTERESIS EFFECTS ONTO CURRENT FORM OF INDUCTANCE WITH FERROMAGNETIC CORE

Inductance coils with ferromagnetic cores are rather widely applied in AC circuits. In such circuits under the presence of sinusoidal power supply there appear both sinusoidal and non-sinusoidal currents and voltages, depending on the properties of ferromagnetic material. We will explore this phenomenon.

Relative permeability μ of steel core inductance coil does not have constant value. Magnetic flux density B is related with the magnetic field intensity H through the nonlinear dependence which is described by nonlinear characteristic curve called hysteresis curve $B= \mu \mu_0 H$.

Magnetic flux density B is linked with magnetic flux Φ through the core cross-section area of steel S of the magnetic circuit (ferromagnetic) $\Phi =B S$.

The current flowing through the inductor is linked with the magnetic field strength H in accordance with Ampere's circuital law $Iw=Hl_a$ (w is the number of turns in winding; l_a is the length of the median magnetic line of the magnetic circuit).

At this, the connection between the flux Φ and the current through the coil I is similar to the dependence of the magnetic flux density B on magnetic field strength H and is represented by a curve identical to the hysteresis loop.

In the magnetically soft steels which, as a rule, are used in magnetic cores of electrical machines and apparatus, the area of the hysteresis loop tends to zero. In the magnetically hard steel which are usually used to created permanent magnets the hysteresis loop area is the highest possible.

At the beginning of the study, we make assumption that the magnetic circuit is made of magnetically soft steel with zero area of the hysteresis loop, leakage fluxes, and the resistive impedance of the coil with a ferromagnetic core equal to zero, i.e. $\Phi_S = 0$, $R_S = 0$, in which case the voltage applied to the coil is counterbalanced by coil self-induction EMF

$$u=-e_L=-d\psi/dt=Wd\Phi/dt,$$

where u is voltage applied to coil; e_L is EMF of coil self-induction; ψ is coil flux linkage; W is the number of coil turns and Φ is coil flux linked with flux linkage

$$\psi=W\Phi.$$

If the applied voltage u changes under the harmonic law $u=U_M\ sin\omega t$ then magnetic flux will also change under the harmonic law and the flux is shifted by the phase relative to the circuit voltage (Fig.7.16.). Indeed,

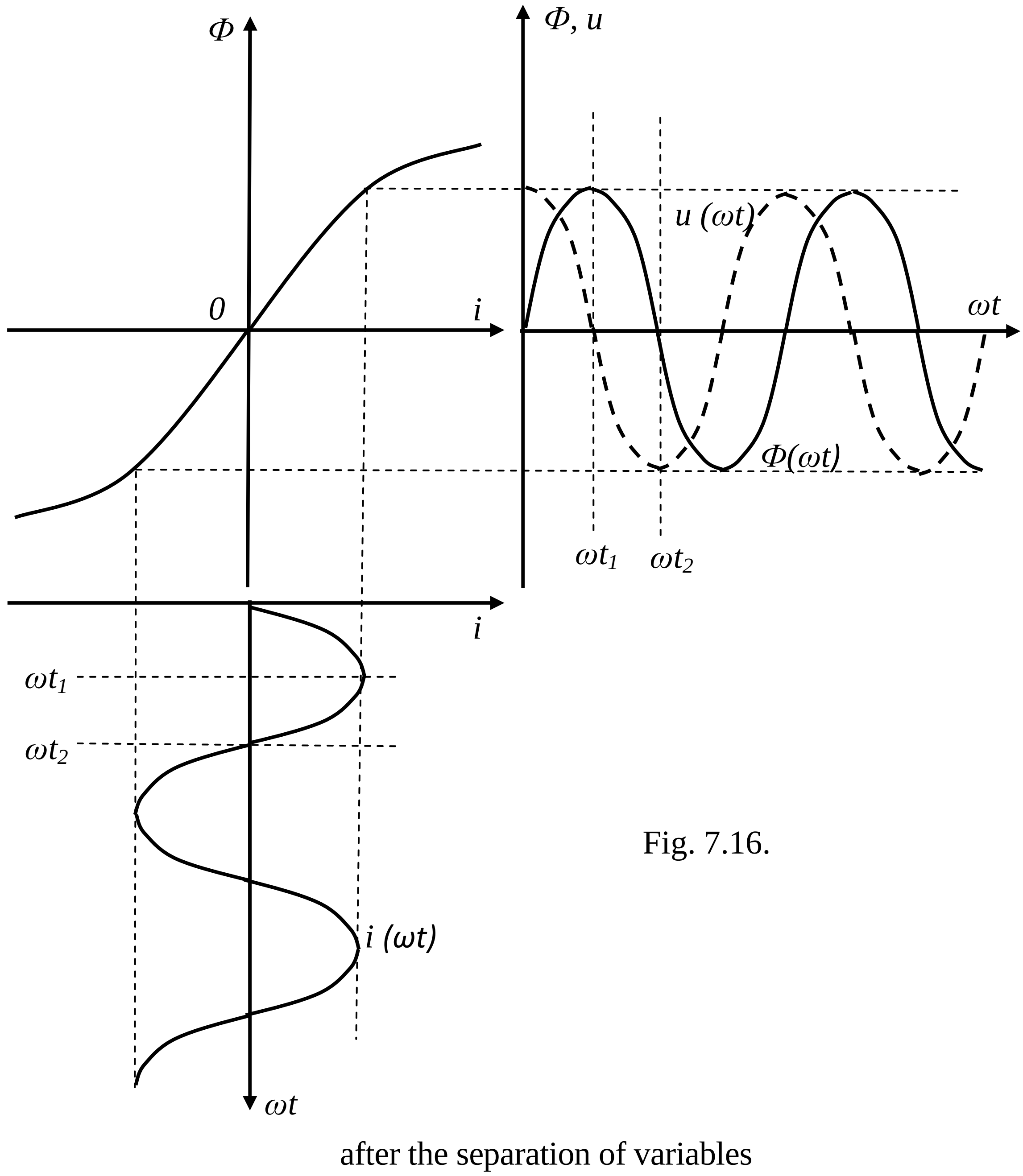

Fig. 7.16.

after the separation of variables

$$d\Phi=1/\ W\ udt=1/\ W\ U_M\ sin\omega tdt$$

and after integration we obtain

$$\Phi=\Phi_M \sin(\omega t-\pi/2)= U_M/ W/\omega \sin(\omega t-\pi/2).$$

Thus, when the lossless coil having soft-magnetic steel core is connected under sinusoidal voltage, the magnetic flux through the core has also sine wave and lags in phase from voltage by the angle $\pi/2$, and its amplitude is

$$\Phi_M= U_M/ W/\omega= U\sqrt{2}/ W/2/\pi/f.$$

Under the above assumptions and the absence of magnetic circuit saturation, the dependence between the instantaneous values of current coil and magnetic flux of core is unambiguous and linear, Fig.7.16.

Thus, if we neglect the small losses and saturation of the magnetic circuit, which is made of magnetically soft steel, as well as in the absence of leakage fluxes and a sinusoidal shape of voltage power source, the coil current and the magnetic flux through the core are also changing sinusoidally, and magnetic flux lags in phase from current by the angle of $\pi/2$, like in case of an ideal linear inductive element.

With these assumptions we take into account the impact of magnetic saturation of magnetization curve on the current waveform. For this purpose, we increase the amplitude of the applied sinusoidal voltage, which leads to the increase in the amplitude of magnetic flux and to the saturation of magnetic circuit of the coil, Fig. 7.17. The current curve becomes non-sinusoidal curve due to the decrease of coil reactive resistance in the zone of saturation of the magnetic circuit. The current waveform differs considerably from the sine

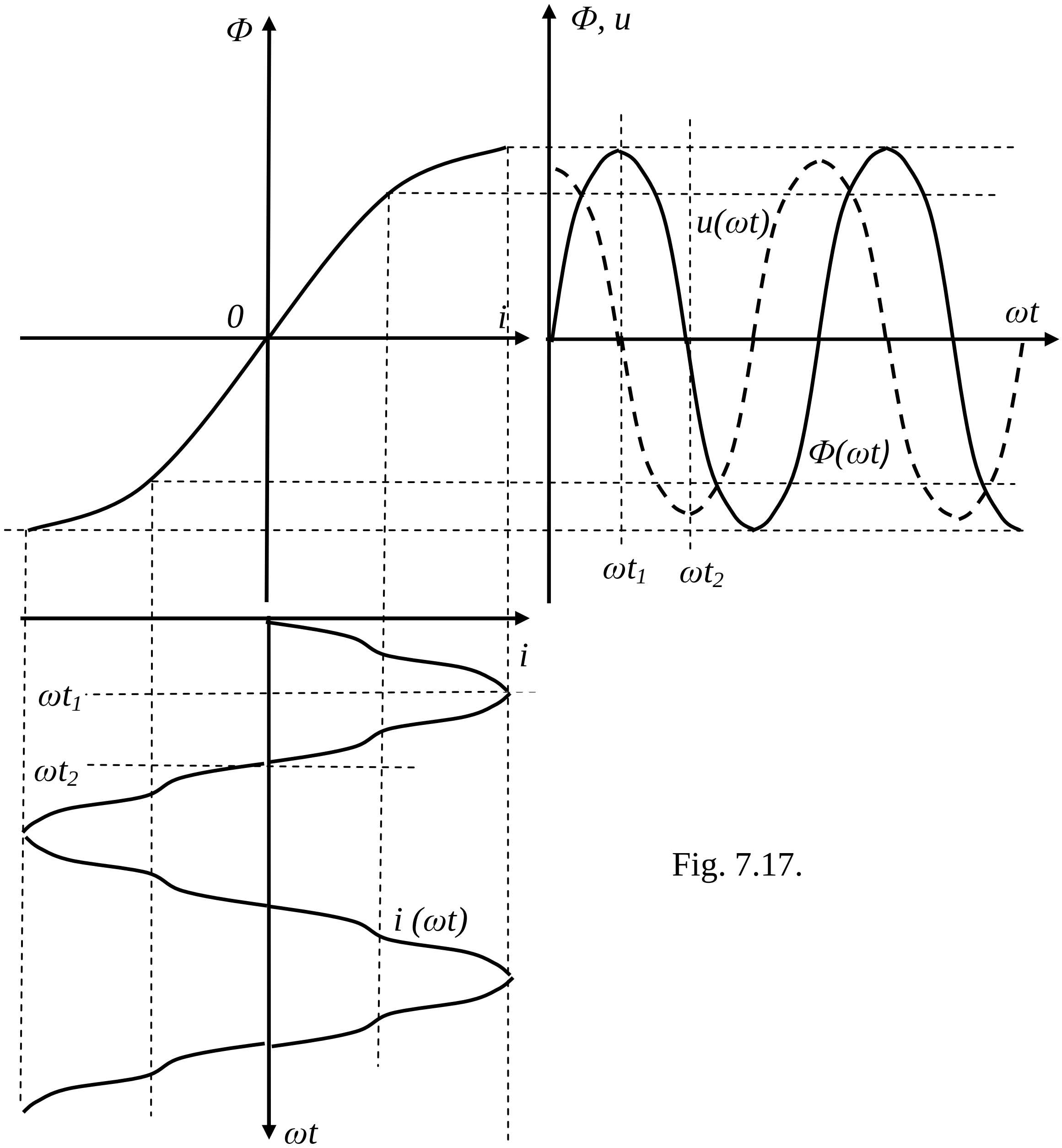

Fig. 7.17.

wave, and approaching to the triangular shape, i.e. has a pointed shape.

Nonsinusoidal current curve is symmetrical about the origin and the abscissa axis, therefore at expansion in a Fourier series only odd harmonic components are presented, Fig.7.17

$$i = I_{M1}\sin(\omega t+\psi_1) + I_{M3}\sin(3\omega t+\psi_3) + I_{M5}\sin(5\omega t+\psi_5) + I_{M7}\sin(7\omega t+\psi_7) + \ldots$$

The main (first) harmonic component of the current waveform is in phase with the magnetic flux and lags the voltage by an angle of π/2. At the instant when they cross abscissa and when they reach values

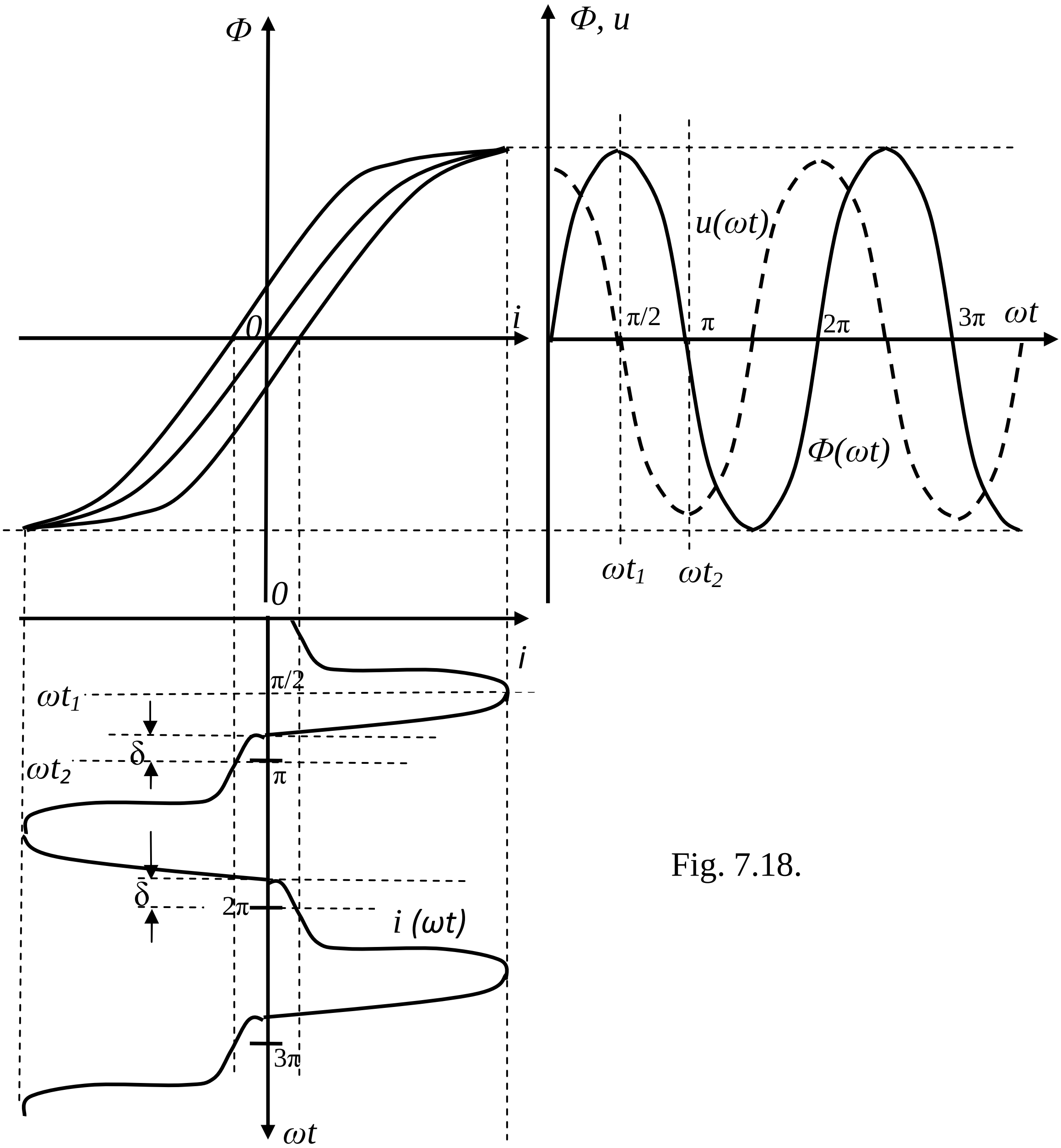

Fig. 7.18.

of the amplitude, the curves of the current and the magnetic flux coincide in time. When there is a deeper saturation of the magnetic circuit, the current curve differs from the sinusoidal waveform to an even greater extent.

Since the current curve is symmetrical to the origin and the abscissa, then a total of the electromagnetic energy accumulated in the coil with a steel core during the magnetization of the magnetic circuit will be returned back into the source during the demagnetization of the magnetic circuit, and the average power consumed from the power supply is equal to zero.

We take into account the hysteresis phenomenon and magnetic circuit saturation, neglect leakage fluxes and the resistive impedance of the winding turns, Fig.7.18. Steel magnetization depends not only on instantaneous value of magnetization current but also on the state of magnetic core in which the steel of magnetic circuit was before. This phenomenon is uniquely described by hysteresis loop and at periodic magnetization reversal is accompanied by absorption of energy manifested in the form of heat loss within the magnetic dipoles of the magnetic circuit.

If magnetic saturation under sinusoidal voltage sharpens the current curve without breaking its symmetry, then this symmetry is broken when we take into calculation the hysteresis phenomenon. The amplitudes of magnetic flux and current coincide in time, but the current curve leads in time of the magnetic flux curve by the angle δ. The average power consumption of the coil with ferromagnetic core under the admitted assumptions is not equal to zero, because there are heat losses appearing at the rotation of magnetic dipoles in the ferromagnetic core, which are called steel reverse magnetization losses.

7.3.2.2. EQUIVALENT CIRCUIT AND PHASOR DIAGRAM OF INDUCTANCE COIL WITH FERROMAGNETIC CORE

Consider a coil with a steel core that is connected under the sinusoidal voltage, assuming that the resistive impedance of the coil winding R_C and leakage inductance L_S are equal to zero. Under these assumptions the relations $u=U_M \sin(\omega t+\pi/2)$; $\Phi=\Phi_M$

$\sin\omega t$; $i=I_{M1}\sin(\omega t+\psi_{1+}$ $I_{M3}\sin(\omega t+\psi 3)+\ldots$ are valid (see Fig.7.19).

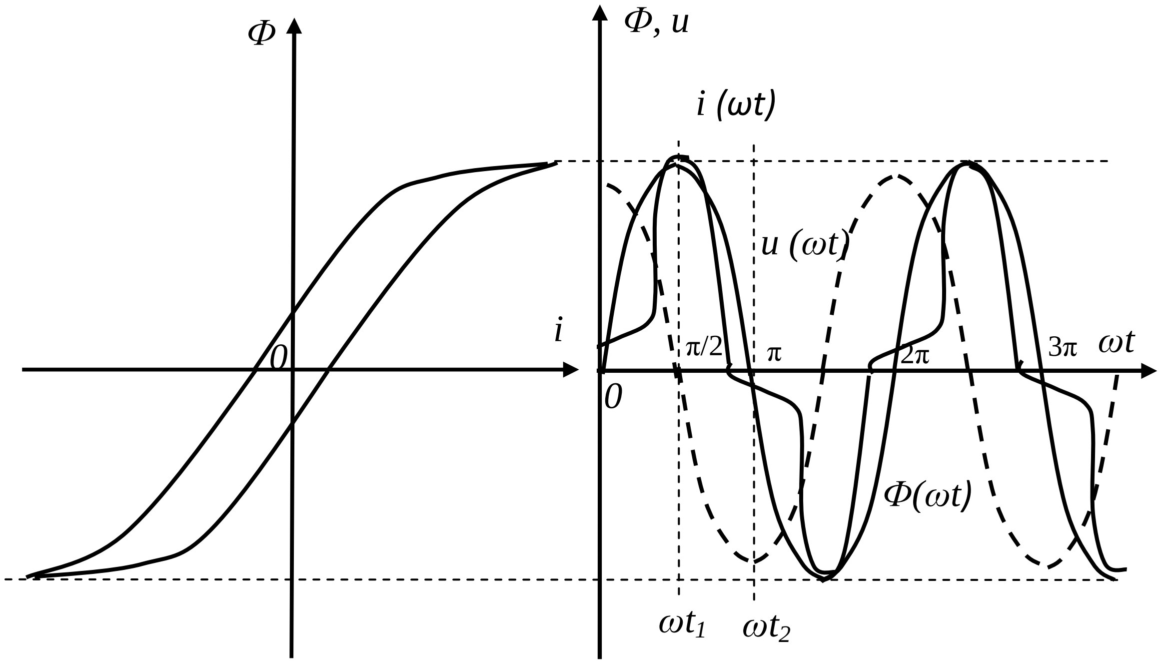

Fig. 7.19.

Graphical method of the current curve calculates in accordance with given hysteresis loop and, followed by harmonic analysis of the resulting curve, is inconvenient in practice. In the analysis of the circuit a simpler way will be used, based on the application of the notion of equivalence sinusoids. In this case, we take into account hysteresis phenomenon, but ignore non-sinusoidality of current curve. At that the RMS value of non-sinusoidal current and power of heat loss should be similar to the equivalent sinusoidal current.

Using such approach, a magnetic circuit is replaced by some conditionally linear element wherein the sinusoidal magnetic flux is induced under the action of the equivalent sinusoidal current i_E.

The conditions of equivalence are:

1. The equality of effective values of equivalent current and initial non-sinusoidal current

$$I_E^2=(I_1^2+I_3^2+\ldots).$$

2. The equality of losses conditioned by equivalent and initial non-sinusoidal currents

$$P=UI_1\cos(\psi_{u1}-\psi_{i1})=R_{ST}\,I_E^2,$$

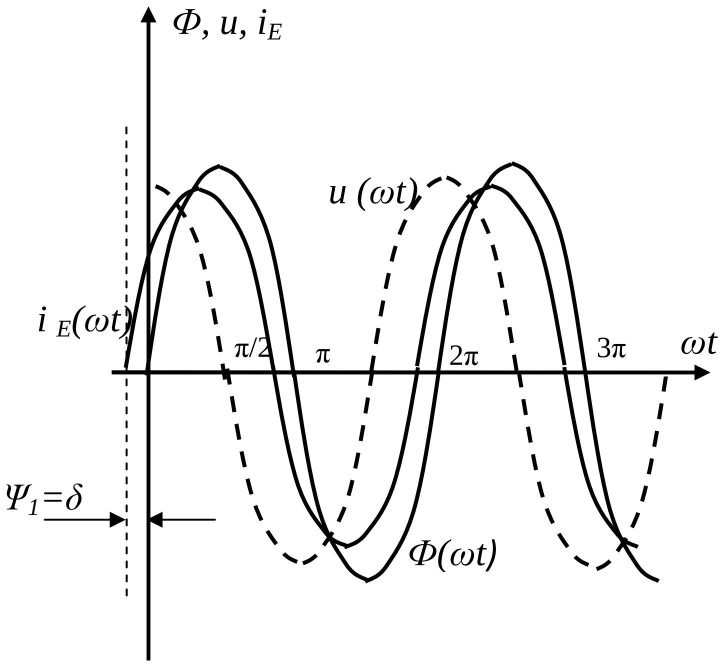

Fig. 7.20.

where R_{ST} – is some equivalent resistive impedance which is conditioned by losses of average power within the steel magnetic conductor of the coil.

Replacing the real non-sinusoidal current waveform

$$i = I_{M1} sin(\omega t+\psi_1)+ I_{M3} sin(3\omega t+\psi_3)+ I_{M5} sin(5\omega t+\psi_5)+ I_{M7} sin(7\omega t+\psi_7)+\ldots$$

by equivalent sinusoidal curve (see Fig.7.20)

$$i_E = I_{ME} sin\,(\omega t + \psi_{1E})$$

allows us to use a phasor method and phasor diagrams in calculation of the circuit containing a coil with a ferromagnetic core.

A vector diagram is plotted, Fig.7.21, in accordance with obtained functions of the sinusoidal supply voltage, the magnetic flux of the coil and the equivalent current.

In the vector diagram are denoted: $\underline{U}_{\Phi}$ is part of applied voltage $\underline{U}$ which is counterbalancing EMF of coil self-inductance $\underline{E}_L$; $\underline{I}_E$ is equivalent current vector consisting of the vector sum of loss current $\underline{I}_L$ and the current of the magnetizing loop $\underline{I}_{\mu}$; $\underline{\Phi}$ is vector of the magnetic flux; $\Psi_1=\delta$ is the lead angle of the magnetic flux by the equivalent current due to a non-zero area of the hysteresis loop.

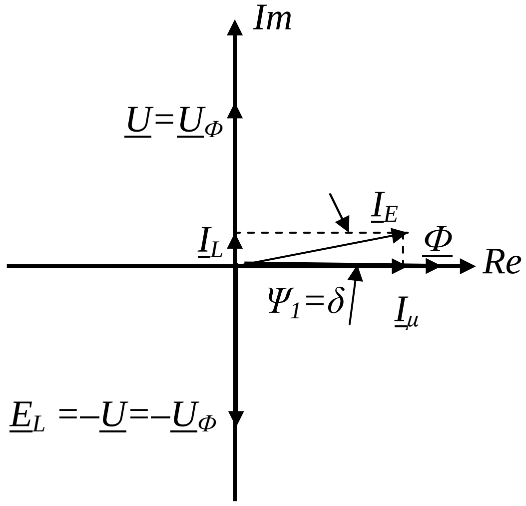

Fig. 7.21.

The phasor diagram (Fig. 7.21) corresponds to the equivalent circuit (Fig. 7.22) where, parallel to the ideal nonlinear inductance L_{μ} , loss ohmic impedance R_L is connected in which the average losses of remagnetization of the hysteresis loop, the so-called losses in steel (ferromagnetic core), are imitated.

Equivalent circuit and phasor diagram of the non-linear inductance, under the assumptions made, with series-connected resistances imitating losses in iron and perfect non-linear inductance are shown in Fig.8.9 and 8.10.

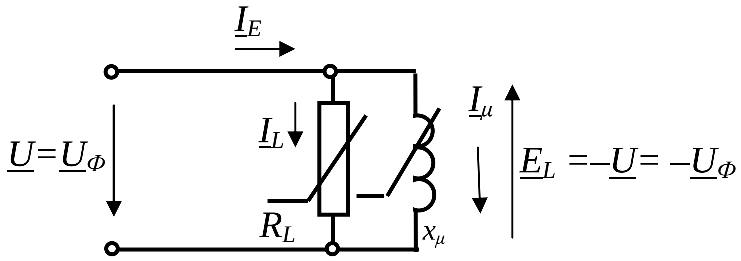

Fig. 7.22

The restrictions imposed at the initial study of the coil with a steel core are removed. Consider the coil with a ferromagnetic core with taking into account the leakage flux and the internal resistive impedance of the coil winding. In this case, the instantaneous value of the applied voltage of power supply is balanced by the sum of instantaneous values of the voltage drop across the internal resistive impedance of the coil winding u_{Rc}, and voltage counterbalancing the EMF of self-inductance from the magnetic leakage flux u_S, and voltage counterbalancing EMF of self-inductance from the magnetic flux in the iron core of coil u_Φ

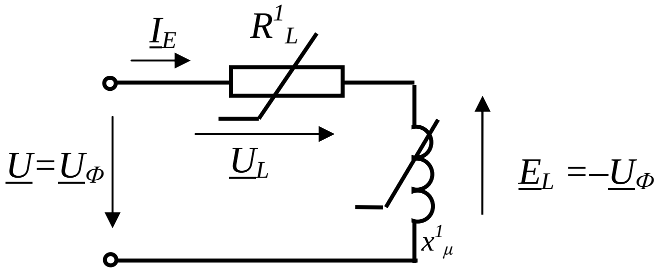

Fig. 7.23.

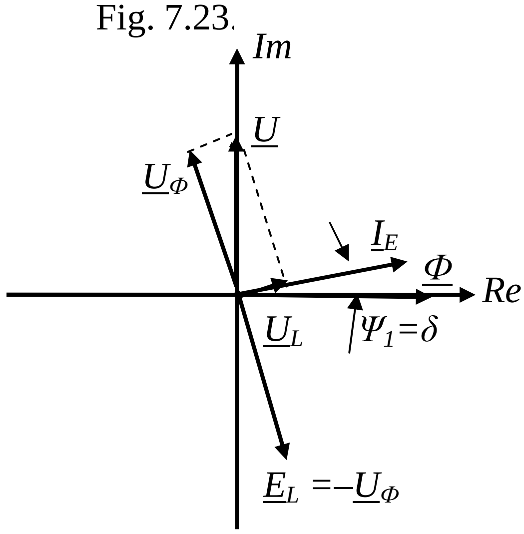

Fig. 7.24

$$u=u_{RC}+u_S+u_\Phi.$$

The voltage drop across the ohmic impedance of the coil winding is in phase with the equivalent current of coil

$$\underline{U}_{RC}=R_C\,\underline{I}_E.$$

Voltage counterbalancing EMF of self-induction from the leakage fluxes e_S leads the coil equivalent current i_E by the angle $\pi/2$

$$u_S=-e_S=Wd\Phi_S/dt=d\psi_S/dt=d\psi_S/di_E\,di_E/dt=$$
$$=L_S\,di_E/dt,$$

where $Φ_S$ is leakage fluxes; $ψ_S= WΦ_S$ is flux linkage of the leakage fluxes; W is number of turns in the coil winding; $L_S= dψ_S/di_E$ is leakage loop inductance.

Since the leakage fluxes are mostly closed up across air outside of the ferromagnetic core, the inductance L_S is a linear quantity. In complex notation, we obtain

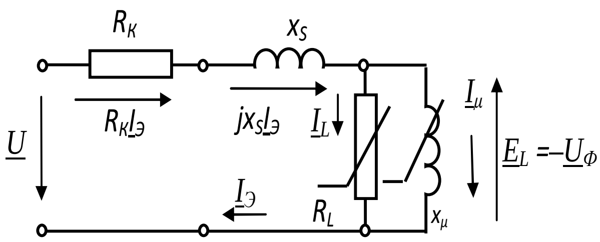

Fig.7.25.

$$\underline{U}_S=jωL_S\underline{I}_E=jx_S\underline{I}_E,$$

where x_S is inductive reactance of the coil leakage fluxes.

Self-induction EMF from the magnetic flux through the iron core which voltage counterbalancing is

$$u_Φ=-e_L=WdΦ/dt= dψ/dt= dψ/di_μ\, di_μ/dt=L_μ\, di_μ/dt,$$

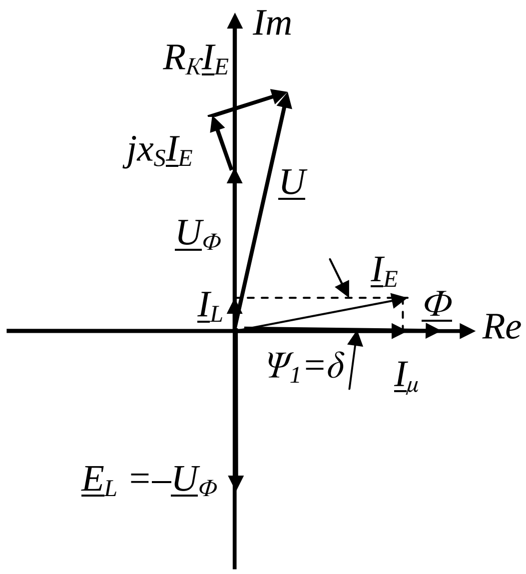

Fig. 7.26

where $Φ$ is flux through the steel magnetic conductor; $ψ= WΦ$ is flux linkage of main flux; $L_μ= dψ_S/di_μ$ is inductance of magnetization loop.

In complex notation, we obtain

$$\underline{U}_Φ=jωL_μ\underline{I}_μ=jx_μ\underline{I}_μ,$$

where $x_μ$ is the inductive reactance of the magnetization loop.

In phasor notation, the complex value of applied voltage $\underline{U}$ balanced by the sum of complex values across internal resistive impedance of the coil winding $\underline{U}_{Rc}$, by voltage counterbalancing the EMF of the self-induction from the leakage fluxes $\underline{U}_S$ and by voltage counterbalancing EMF of the self-induction from magnetic flux through steel core of the coil $\underline{U}_Φ$, is

$$\underline{U}=\underline{U}_{Rk}+\underline{U}_S+\underline{U}_Φ= R_k\underline{I}_E+ jx_S\underline{I}_E+ jx_μ\underline{I}_μ= \underline{I}_E(R_k+ jx_S) + jx_μ\underline{I}_μ= \underline{I}_E(R_k+ jx_S) + \underline{U}_Φ.$$

Thus, the equivalent circuit of the coil with a steel core differs from the perfect coil by the presence of two resistances ($Rc + jx_S$) which are taking into account the presence of the ohmic impedance of the coil winding and the leakage fluxes. Both these resistors are flowed by equivalent sinusoidal current of coil.

In Fig.7.25 and Fig.7.26 are shown equivalent circuit and phasor diagram of non-linear inductance in which the resistive impedance of coil winding and the leakage fluxes are taken into account, as well as loss resistances in steel and the resistance of perfect nonlinear inductance connected in parallel.

The ohmic impedance R_L is called coil steel resistance, and the resistance R_C is called coil copper resistance.

7.3.2.3. EQUATIONS, PHASOR DIAGRAM AND THE EQUIVALENT CIRCUIT OF TRANSFORMER WITH FERROMAGNETIC CORE

As a rule, transformer windings are arranged on a ferromagnetic core which provides increasing magnetic coupling between the windings. To this end, windings are situated as close as possible. Consider a transformer with two windings that are electrically non-connected, having the number of turns w_1 and w_2, Fig. 7.27.

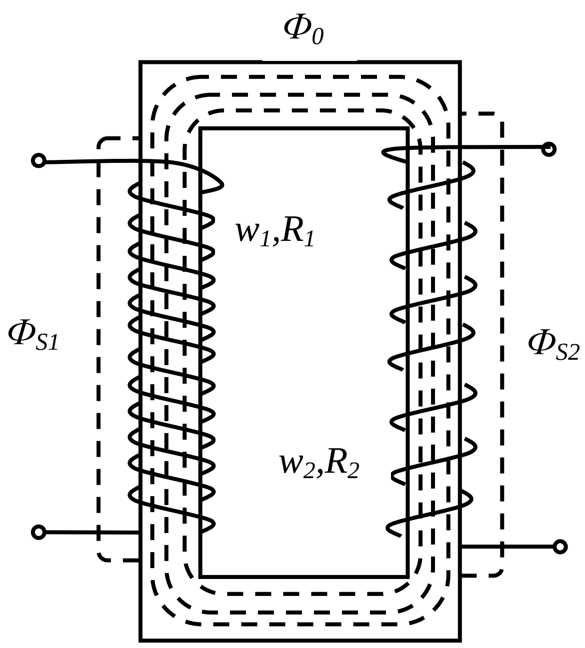

Fig. 7.27.

The actual picture of the magnetic field in the transformer is rather complicated. Some magnetic lines are entirely closed through the core and they embrace all the turns of both windings. The rest are closed entirely or partly through the air, and embrace some of the windings turns. Taking into account only the voltage across the terminals of the winding, and not considering the voltages distribution between the separate turns, one can replace the complex picture of the field by equivalent simplified one as shown in Fig. 7.27. The lines of flux Φ_0 embrace all the turns of both windings. The lines of flux Φ_{S1} embrace all the turns of the first winding only. The lines of flux Φ_{S2} embrace all the turns of the second winding only. The flux Φ_{S2} is called the main one and the fluxes Φ_{S1} and Φ_{S2} are called the leakage fluxes.

The flux Φ_0 is non-linearly dependent on magneto-motive force $i_1w_1+ i_2w_2$ and is determined by both currents. The flux Φ_{S1} is proportional to the current i_1, and the flux Φ_{S2} is proportional to the current i_2. For the flux linkages of the first and second coils one can write down

$$\Psi_1= \Psi_{S1}+ \Psi_{01}=L_{S1}i_1+ \Phi_0w_1;$$
$$\Psi_2= \Psi_{S2}+ \Psi_{02}=L_{S2}\, i_2+ \Phi_0w_2.$$

Here L_{S1} and L_{S2} are inductances of primary and secondary windings which are determined by the leakage fluxes.

Let's assume that voltage u_1 is applied at the terminals of the transformer primary winding, and the terminals of the secondary winding are connected at a load. Voltage u_1 has R_1i_1 component which is equal to the voltage drop across the ohmic impedance of the primary winding and $d\Psi_1/dt$ component which is balanced by the EMF induced by flux linkage Ψ_1

$$u_1= R_1i_1+ d\Psi_1/dt.$$

Electromotive force $d\Psi_2/dt$ induced by flux linkage Ψ_2 in the secondary winding is counterbalanced by the voltage drop R_2i_2 across the secondary winding of the ohmic impedance and by the voltage u_2 across the terminals of the load:

$$-d\Psi_2/dt= R_2i_2+ u_2$$

or

$$-d\Psi_0/dt= R_2i_2+ +L_{S2}\, di_2/dt+ u_2.$$

Using the relationship of leakage fluxes and the main flux, the transformer equation can be rewritten in the following form

$$u_1= R_1i_1+ L_{S1}\, di_1/dt+ w_1d\Phi_0/dt= R_1i_1+ +L_{S1}\, di_1/dt+u_\Phi;$$
$$-w_2d\Phi_0/dt= R_2i_2+ L_{S2}\, di_2/dt+u_2.$$

Denote electromotive forces induced by the flux Φ_0 within the primary and secondary windings by

$e_1 = -u_\Phi = -w_1 d\Phi_0/dt$ and $e_2 = -w_2 d\Phi_0/dt$.

The transformer equations are nonlinear, because of the existing nonlinear relationship between the flux Φ_0 and magneto-motive force $i_1w_1+i_2w_2$. Therefore, periodic currents, fluxes and voltages are non-sinusoidal. After introduction of the equivalent sine wave, one can write the equation for the transformer in the phasor form:

$$\underline{U}_1 = R_1\underline{I}_1 + j\omega L_{S1}\underline{I}_1 + \underline{U}_\Phi;$$
$$\underline{E}_2 = R_2\underline{I}_2 + j\omega L_{S2}\underline{I}_2 + \underline{U}_2,$$

where $\underline{U}_2=\underline{Z}_L\underline{I}_2$, $\underline{Z}_L$ is the load impedance.

If the number of turns in the windings w_1 and w_2 are substantially different from one another, then different electromotive forces e_1 and e_2 are induced across the windings w_1 and w_2. For the convenience of the analysis, the windings of the transformer are reduced to the equal number of turns. In this case all the quantities of the transformer's secondary circuit are reduced to its primary circuit. These reduced quantities will be denoted by the prime sign ('). Reduction is performed by replacing the transformer with the actual w_2 number of turns in the secondary winding by the transformer having an equivalent number of turns in the secondary winding $w'_2 = w_1$.

Thus, instead of actual transformer with turn ratio $n=w_1/w_2$ we will consider the equivalent transformer with turn ratio equal to unity.

Reduction should be done so that it did not affect the operation mode of the primary circuit. For this, it is necessary and sufficient that magneto-motive force of the secondary winding has not changed as a result of reduction, that is, the condition $w_1i'_2 = w_2i_2$ or $i'_2 = i_2/n$ must be observed, because in such case the main flux Φ_0 remains unchanged.

Thus, if the flux Φ_0 is not changed then the electromotive force of the secondary winding varies in proportion to the number of turn. We have $e'_2=e_2 w'_2/w_2=ne_2$. Obviously, all voltage drops across the secondary circuit should be recalculated similarly proportional to the turn ratio n.

The resistive and inductive impedances in the secondary circuit are recalculated in proportion to n^2, because the voltage varies in proportion to n, and current varies in reciprocal proportion to n.

Susceptances and capacitances are recalculated in reciprocal proportion to n^2.

After reduction, the transformer equations can be written in the following form

$$\underline{U}_1 = R_1\underline{I}_1 + j\omega L_{S1}\underline{I}_1 + \underline{U}_\Phi;$$
$$\underline{E}'_2 = R'_2\underline{I}'_2 + j\omega L'_{S2}\underline{I}'_2 + \underline{U}'_2,$$

where $\underline{U}'_2 = \underline{Z}'_L\underline{I}'_2$

Relationship between the vector amplitude of the induced flux Φ_{om} and magneto-motive force $i_1w_1+i_2w_2$ can be written in vector form

$$\underline{\Phi}_{om} = (\underline{I}_{1m}w_1 + \underline{I}_{2m}w_2)/\underline{Z}_M = (\underline{I}_{1m}w_1 + \underline{I}'_{2m}w'_2)/\underline{Z}_M = (\underline{I}_{1m} + \underline{I}'_{2m})\, w_1/\underline{Z}_M = \underline{I}_{0m}w_1/\underline{Z}_M$$

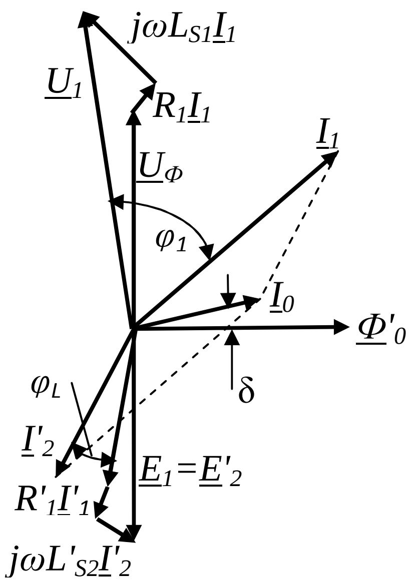

Fig. 7.28.

where $Z_M = R_M + jX_M$ is complex magnetic impedance of core, where X_M takes into account core losses due to hysteresis and eddy currents.

Current value $\underline{I}_{0m} = \underline{I}_{1m} + \underline{I}'_{2m}$ is called magnetizing current.

We should notice that the currents i_1 and i_2 are flowing through the windings, and current i_0 exists if $i_1 \neq 0$ and $i_2 \neq 0$ only as some rated current. The current i_0 is equal to the current i_1 only when $i_2=0$.

As a result of the losses present in the core, the main flux Φ_0 lags in phase from current i_0 by angle δ. According to the last transformer equations, one can plot the transformer phasor diagram (Fig.7.28).

The phasor diagram in Fig.7.28 is plotted for case $\varphi_L > 0$, i.e. when the load impedance has resistive-inductive characteristic.

In accordance with the last transformer equations one can draw up an equivalent circuit of the transformer, in the form presented in Fig. 7.29.

The constant values R_1, L_{S1}, R_2, L_{S2} do not depend on currents and form a linear part of the circuit. The susceptance B_0 depends on voltage $U_Ф$ and is the nonlinear inductive susceptance. The conductance G_0

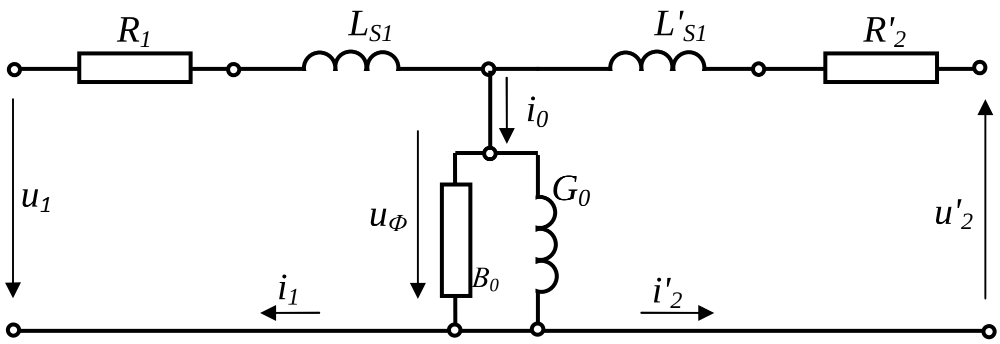

Fig. 7.29.

is constant quantity under condition that losses in core are proportional to $U^2_Ф$, i.e. are proportional to B^2_m.

If the hysteresis losses are changed disproportionately to the squared amplitude of the magnetic induction, then the G_0 value to some extent will depend on $U_Ф$, and is also nonlinear in powerful transformers under rated load, current I_0 is only a few percent from the current value I_1 which arises from relatively small magnetic impedance of the core due to the high permeability of steel. Under rated voltage and rated load the voltage drop across the windings $I_1(R_1^2 + \omega^2 L_{S1}^2)^{0.5}$ and $I_2(R'^2_2 + \omega^2 L'^2_{S2})^{0.5}$ in transformers are usually a few percent from the voltage U_1. As to U_1 to U_2 ratio, it is close to the transformation ratio n.

Transformer with ferromagnetic core is nonlinear two-port scheme. Therefore, when transformer parameters are determined from open circuit and short circuit tests, in the open circuit test it is necessary to accept voltage U_1 as equal to voltage $U_Ф$ under the rated load, because there is dependence of B_0 on $U_Ф$. The voltage drop across circuit section R_1, L_{S1} in open circuit test is very little. The parameters B_0 and G_0 are determined from the open circuit test. The short circuit test is done under rated current value. Because voltage U_1 , and consequently $U_Ф$, is much less than their value in rated operation, then current $I_0 << I_1$, and it can be neglected. Therefore parameters $R_1 + R'_2$ and $L_{S1}+L'_{S2}$ are determined from the short circuit test. The R_1 and R'_2 values, as well as L_{S1} and L'_{S2} values usually have the same order of magnitudes. Taking into account the above ratio between the currents I_1 and I_0, as well as between the voltage drops across the windings U_1 and the $U_ф$, we will not be making a big mistake by believing that in the equivalent circuit $R'_2=R_1$ and $L'_{S2}= L_{S1}$.

7.3.3. FERRORESONANCE PHENOMENON IN ELECTRIC CIRCUITS

In electric circuits containing coils with ferromagnetic core and capacitors there occur special phenomena related with nonlinear properties of these circuits.

Herein and in the subsequent sections are considered phenomena in such circuits by simplest examples of coil with ferromagnetic core and capacitor connected in series and parallel.

7.3.3.1. FERRORESONANCE PHENOMENON IN SERIES-CONNECTED COIL WITH FERROMAGNETIC CORE AND A CAPACITOR

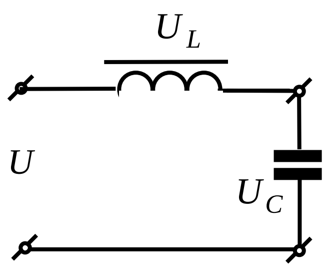

Fig. 7.30.

Let there be given a circuit consisting of series-connected coils with reactive ferromagnetic core and a capacitor C (see Fig. 7.30). Assume that losses in the circuit are absent, and non-sinusoidal voltage waveforms and current ones are replaced by equivalent sinusoids, having chosen them equal to the first harmonic of the actual curves, in other words, we are neglecting the presence of higher harmonics. Under the above conditions, the voltage across the terminals of the coil U_L and the voltage across the terminals of the capacitor U_C are directly opposed in phase to one another; voltage U across the circuit terminals is equal to the absolute value of their difference $U=|UL–UC|$, where there can be as a case of predominance of U_L over U_C and as well as a case of predominance of U_C over U_L.

We are presenting the voltages U_L and U_C in function on current I, at that $U_L = F(I)$ is depicted by a coil characteristic, and $U_C = I/(\omega C)$ is depicted by a straight line passing through the origin, we obtain

$$U=\left|U_L-U_C\right|=\left|F(I)-I/(\omega C)\right|=\varphi(I).$$

The dependency $U=|U_L–U_C|=\varphi(I)$ is nonlinear characteristic of the whole circuit.

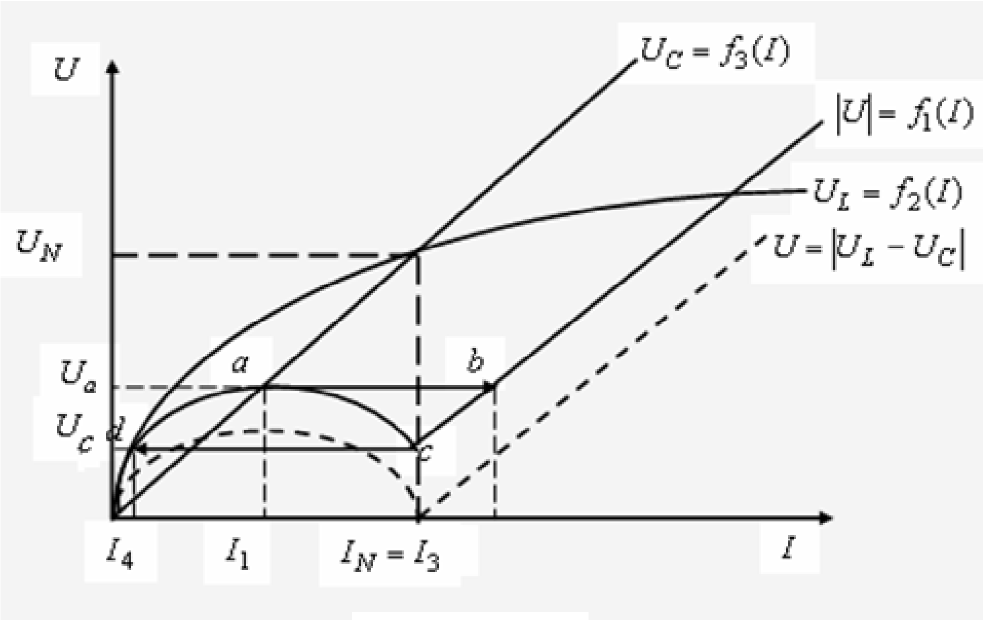

Fig. 7.31.

A graph of the difference U_L-U_C is found by subtracting ordinates of the straight line $U_C = I/\omega C$ from the respective ordinates of the curve $U_L = F(I)$ (Fig. 7.31).

Current at a given value of voltage U is defined by finding the point of intersection of the curve $|U_L$-$U_C|$ with the straight line running parallel to the x-axis at a distance of U from it. As can be seen from Fig. 7.31, there may be three such points, whence it follows that under the same voltage at the terminals of the circuit, generally speaking, three different modes of current can occur. Such uncertainty absolutely not typical to circuits with constant parameters, cannot take place in the given circuit, if the characteristic of the coil does not intersect with the characteristic of the capacitor. But even when the characteristics intersect, the uncertainty occurs only in a limited range of voltages. Namely, if the applied voltage U is less than the value at which the line is touching to the curve $|U_L$-$U_C|$, at this we have three points of curve $|U_L\text{-}U_C|$ intersection with straight line U, where the first two points, starting from the origin, correspond to the predominance of self-induction reaction in a circuit, and the third – the predominance of the capacitor reaction. If the applied voltage exceeds the specified limit, the curve $|U_L\text{-}U_C|$ intersects the line U at only one point and, therefore, in the circuit there is only one well defined current mode. A singular point A of the characteristics $U=\varphi(I)$ lying on the abscissa axis is the resonance point, since at that point the voltages U_L and U_C are mutually compensated. It follows that, unlike circuits with constant parameters, the resonance in the circuit under study can be achieved by changing the value of the applied voltage. This is explained by the fact that the inductance of the coil with a ferromagnetic core depends on the current value, and hence varies with the voltage at the terminals of the circuit. This phenomenon is called ferroresonance. In this case a ferroresonance phenomenon occurs in a series circuit.

Because of the presence of losses and higher harmonics in the circuit, which we have neglected previously, the actual characteristic of circuit takes the form shown in Fig. 7.31 (solid line). The shape of this curve shows that by gradually increasing the voltage, we come to the *a* point of characteristics, and further the jump from *a* point into *b* point will occur accompanied by a sharp increase in current. When the voltage is being further increased, gradual increase of the current occurs. As the voltage decreases, the current gradually decreased until it reaches the *c* point of the characteristic in which there is a jump to the *d* point, accompanied by a sharp decrease in the current. These jumps are accompanied by a change in the sign of the angle offset in the circuit.

At constant voltage U across the terminals the dropping branch of the circuit characteristics is an area of unstable operation modes. Indeed, suppose that $U = const$ and some point on the characteristics of the dropping part of branch corresponds to the operation mode of the circuit. Then any accidental increase in current leads to the reduction of the voltage drop in the circuit and therefore to a further increase in the current. Conversely any random decrease of current leads to the increase of the voltage drop in the circuits and, consequently, in subsequent current decrease. In both cases current will change before it achieves value determined by the corresponding point of intersection of the straight line *U=const* with one of the increasing branches of characteristic. In either of these points the operation mode will have steady state, because random current increase will lead to increase in the voltage drop and the current will decrease, and random current decrease will lead to the decrease in the voltage drop and current will increase. Having connected a large additional linear resistance in series with circuit, one can receive the steady state mode of circuit on dropping part of its characteristic as well.

We show how one can to take into account the ohmic impedance *R* of this circuit while constructing the characteristic of a circuit consisting of coil with ferromagnetic core and capacitor connected in series. Denote the applied voltage by the symbol *U*, while its active component and magnitude of reactive component by $U_a=IR$ and $|U_r|=|U_L-U_C|$.

The nature of dependence of $|U_r|$ on *I* has been previously determined, which is a circuit characteristic with neglected ohmic impedance and higher harmonics. We will assume that this relationship is known. For further operations it will be more convenient to use dependence $|U_r|$ not directly on the current, but on the value $U_a=IR$ proportional to the current, while the scale on both axes of coordinates must be the same, i.e. the dependence $|U_r|=F(U_a)$.

Using the equivalent sine waves, we must take into account that voltage drops $|U_a|$ and $|U_r|$ are dephased relatively to one another by angle $\pm\ \pi/2$. Accordingly, we have

$$U_a^2 + U_r^2 = U^2.$$

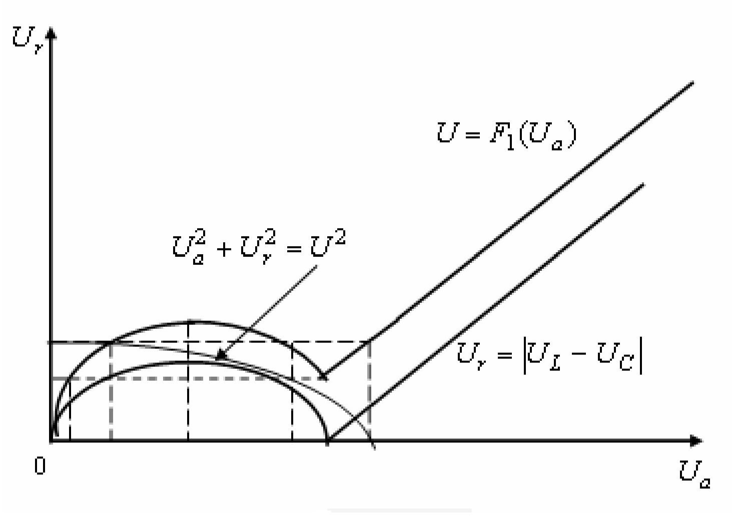

Fig. 7.32.

The equation $|U_r| = F(U_a)$ shows that $|U_r|$ and U_a are related by dependence described by the characteristic of the circuit in the case when $R = 0$. The equation $U_{2a}+ U_{2r}= U_2$ shows that, in addition, between $|U_a|$ and $|U_r|$ there is a relationship defined by a circle centered at the origin, with radius equal to the voltage U at the terminals of the circuit. Both of these conditions for $|U_r|$ and U_a are performed at the points of intersection of the circle having the radius U with the curve $|U_r| =F(U_a)$, while the number of such points is equal to the number of current operation modes which are possible in the circuit under the given U (Fig. 7.32).

From the point of intersection of the circle of radius U with ordinate axis we draw a line parallel to the x-axis, and fulfill the projection of the values of U_a corresponding to a given value of U on it. Having done this operation for a number of circles corresponding to different values of U, and having connected these points by a smooth curve, we build the dependence $U = F_1(U_a)$, which due to the direct proportionality between U_a and I at the same time will give the relationship between U and I, i.e. the desired circuit characteristics with regard to its resistive impedance.

In Fig. 7.32 is shown the circuit characteristic found by such method, where a circle of radius U is plotted by the dashed lines touching the curve and limiting the range of voltages in which a single voltage value can correspond to three different modes of current.

A circle of radius U corresponding to the resonance is especially highlighted, and the resonance point is marked on the characteristics by the cross.

7.3.3.2. FERRORESONANCE PHENOMENON IN PARALLEL-CONNECTED COIL WITH FERROMAGNETIC CORE AND A CAPACITOR

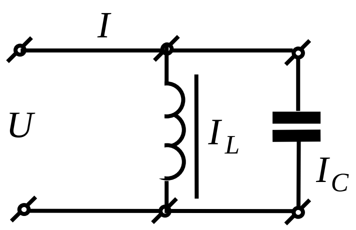

Fig. 7.33.

Considering the coil with a ferromagnetic core and a capacitor connected in parallel (Fig. 7.33), we neglect losses in the circuit and the presence of higher harmonics, just like in previous case. Then reactive current I_L through the coil and current I_C through the capacitor will be opposite one another in phase, and current I through the unbranched circuit part will be equal to the absolute value of their difference $I = |I_L–$

I_C|, at that are possible cases when I_L predominate over I_C, as well when I_C predominate over I_L.

Representing the currents I_L and I_C as functions of voltage at the circuit terminals, where $I_L = F(U)$ is depicted by a characteristic of reactive coil, and $I_C = \omega CU$ is depicted by a straight line passing through the origin of coordinates, we obtain

$$I = |I_L – I_C| = | F(U) – \omega CU | = \varphi (U),$$

which is a nonlinear characteristic of the whole circuit.

The graph of difference $I_L – I_C = F(U) – \omega CU$ is defined by subtracting abscissas of straight line I_C

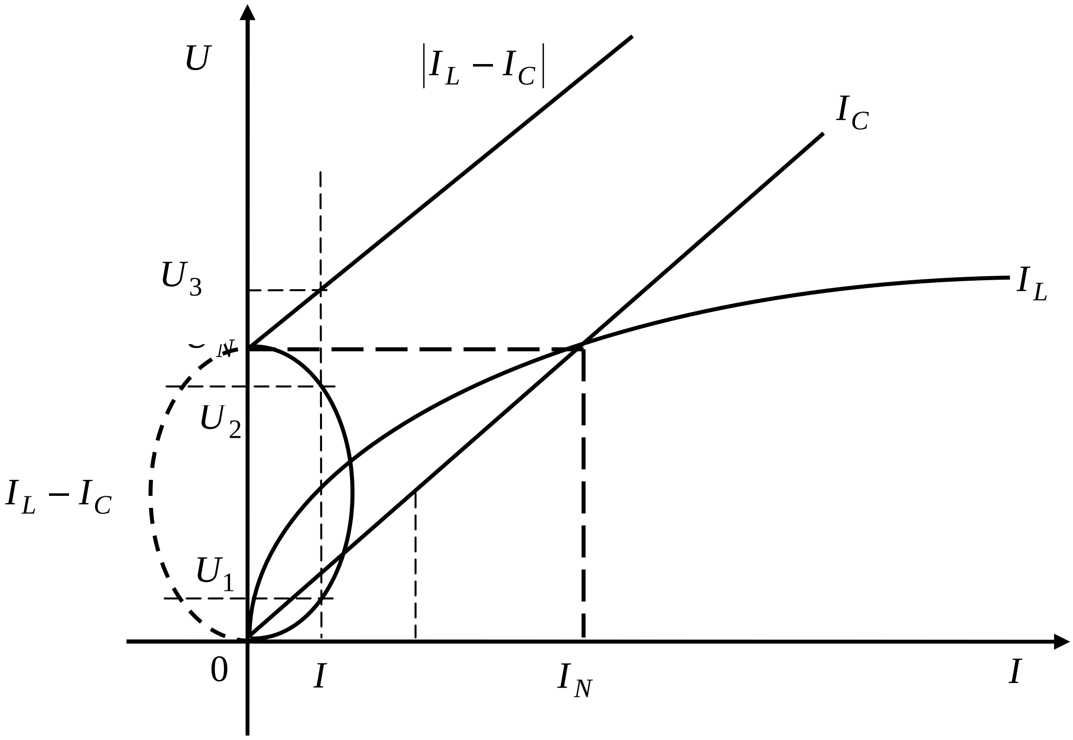

Fig. 7.34.

$=\omega CU$ (Fig. 7.34) from corresponding abscissa of curve $I_L = F(U)$. The voltage, at which the current in the external circuit is equal to a given value of I, is defined by finding the point of intersection of the curve $|I_L – I_C|$ with straight line running parallel to the y-axis at a distance of I from it. As can be seen from Fig. 7.34, there can be three such points, from this follows that the same current through the circuit, generally speaking, can be obtained under three different voltages across its terminals. This specific feature of the circuit will disappear if coil characteristic does not intersect with the capacitor characteristic. But even if the characteristics intersect then the multiple-valued dependence of voltage on current occurs only in the limited area of current values.

Namely: if current I in the unbranched part of the circuit is less than value in which straight line I touches the curve $|I_L–I_C|$, then we have three points of the curve $|I_L–I_C|$ intersection with straight line I, while two first points counting from the origin of coordinates correspond to the predominance of the capacitor reaction in the circuit, and the third one – to the predominance of the self-induction reaction. If current I exceeds the indicated limit, then the curve $|I_L–I_C|$ intersects with straight line I only in one point and, consequently, this current can exist through the circuit only in single defined value of voltage at the circuit terminals.

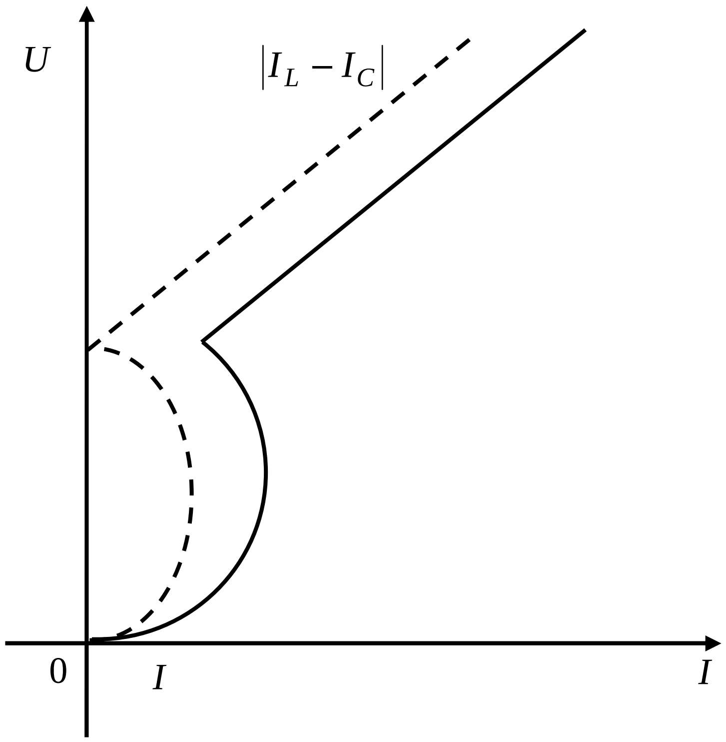

Fig. 7.35.

A singular point A of characteristic $I=\varphi(U)$ lying on the y-axis is the resonance point, since at this point the currents I_L and I_C mutually compensate one another. It follows that when the capacitor and coil with a ferromagnetic core are connected in parallel, unlike circuits with constant parameters, resonance can be achieved by changing the value of the applied voltage.

This phenomenon also relates to ferroresonance, at that case we deal with ferroresonance in parallel circuit.

Because the losses in the circuit and the higher harmonics are present, which we have neglected, the actual characteristic of the circuit takes the form shown in Fig. 7.35 by the solid line. From the form of this curve is follows with the gradual increase of the current through the circuit, as well as with the decrease of its values, there will be similar jumps in elements connected in series, also accompanied by the change of the sign of phase shift angle in the circuit. However to obtain these jumps in practice it is necessary to have a device which regulates the current rather than voltage. Practically, this can be achieved if the circuit shown in Fig. 7.33 is not connected directly to a variable voltage source, but through a large linear resistance *R* considerably exceeding the loop resistance of the circuit where a coil with ferromagnetic core and a capacitor is connected in parallel. In this case, the current *I* will be determined by the resistance *R* and when it changes, surges at the terminals of the circuit will occur. If the loop of the parallel-connected coil with a ferromagnetic core and a capacitor is connected directly to the voltage source with low internal impedance, then with changing *U*, the whole curve (Fig. 7.35) can be obtained in practice without jumps.

Applying the considered techniques to construct the characteristics of a coil with a ferromagnetic core and a capacitor connected in series and in parallel, one can construct the characteristics of more complicated unlinear circuits presenting the mixed connections of reactive elements.

7.3.4. INDUCTIVE UNCONTROLLABLE ELEMENTS. FERROMAGNETIC VOLTAGE STABILIZERS

Features of circuits containing coils with ferromagnetic cores and capacitors are used to create ferro-magnetic voltage stabilizers that serve the purpose of maintaining constant voltage across the terminals of the receiver when changing the supply voltage. The main part of all stabilizers consists of two series-connected resistors – linear and nonlinear ones.

To obtain the characteristics of ferromagnetic voltage stabilizers we are applying the graphic method described in 7.3.3.1, like we did when considering the ferro-resonance phenomenon.

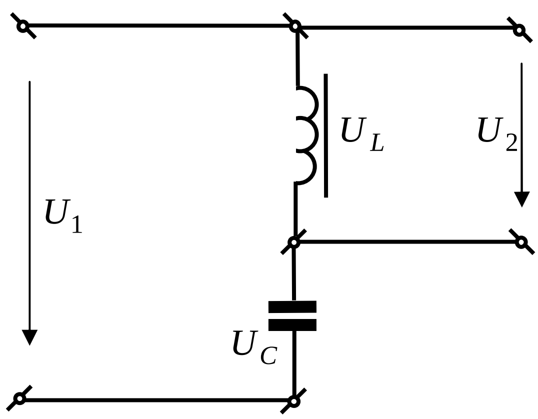

Fig. 7.36.

Consider the simplest stabilizer (Fig.7.36) consisting of series-connected capacitor *C* and coil *L* with ferromagnetic core at open circuit mode.

The voltage U_1 is applied to the terminals of this circuit, and coil terminals are the output terminals of the stabilizer and, consequently, the output voltage U_2 is equal to the voltage at the terminals of the coil U_L.

Knowing the capacitance C of the capacitor and the coil characteristic $U_L = F_2 (I)$, one can construct (see Fig. 7.37) the dependence of $U_1 = | U_L - U_C | = F_1(I)$, where U_C is voltage at the terminals of the capacitor. Assume that the voltage changes from U'_1 to U''_1. Then, using the curves $U_1 = F_1 (I)$ and $U_2 = U_L = F_2 (I)$, one can find the corresponding values of the output voltage U'_2 and U''_2. These values U'_1 and U'_2 correspond to current I', and the values U''_1 and U''_2 correspond to current I'' through the stabilizer circuit. From Fig. 7.37 it is obvious that the considerable voltage change $\Delta U_1 = U'_1 - U''_1$ in network entails a relatively small change in the output voltage $\Delta U_2 = U'_2 - U''_2$.

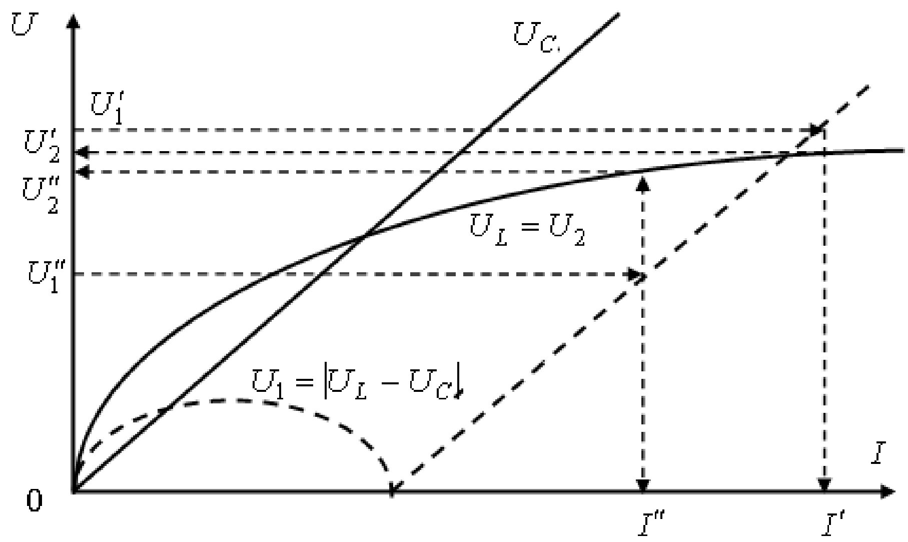

Fig. 7.37.

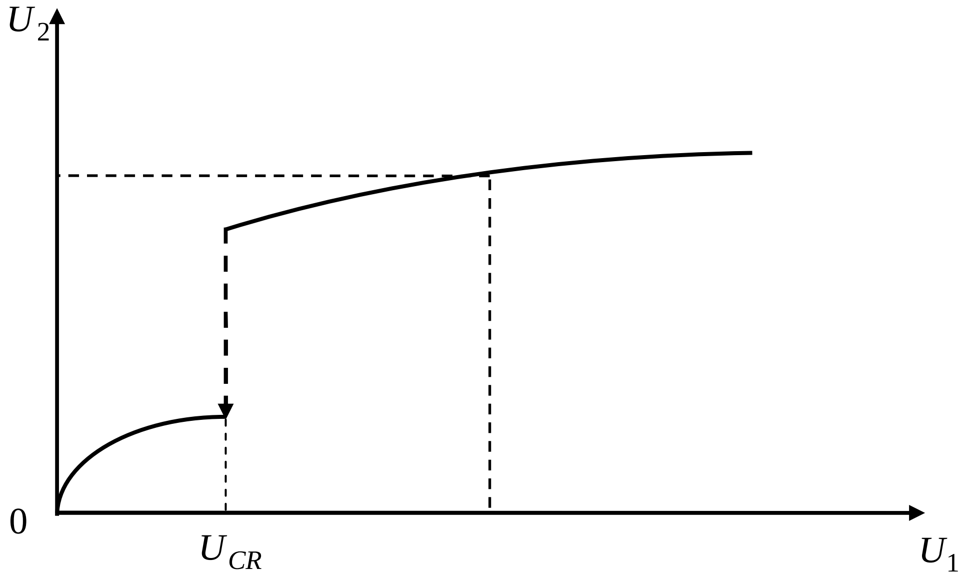

Fig. 7.38.

Having determined the corresponding values of U_2 for several values of U_1, one can plot dependence $U_2 = F (U_1)$, which shows (see Fig. 7.38) that the circuit under consideration can stabilize voltage only when the supply voltage is exceeding the critical voltage U_{CR}. From Fig. 7.38 it is clear that the flatter the final part of characteristic coil, the better the stabilizer.

The described stabilizer does not have practical application due to unsatisfactory working characteristics.

In practice, a stabilizer is often used, the main part of the circuit of which is shown by solid lines in Fig. 7.39. In this circuit a coil L_1 with an unsaturated ferromagnetic core is used as the linear impedance, and a circuit consisting of parallel-connected capacitor C_2 and coil L_2 with saturated ferromagnetic core is used as nonlinear impedance. The relevant characteristics $U_{L1} = F(I)$ of the coil L_1, $U_2 = F_2(I)$ of the circuit branched part and $U_1 = F_1(I)$ of the total circuit is shown in Fig. 7.40.

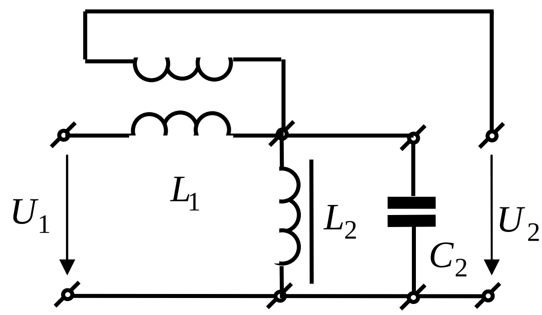

Fig. 7.39.

Further improvement of this circuit can be achieved by subtracting the voltage U'_L, which is part of U_{L1} voltage at the terminals of the coil L_1, from the voltage across the loop L_2, C_2. This can be done by imposing the additional coil with appropriate number of turns on the core with coil L_1, and connect it as is shown in Fig. 7.39 by dotted lines. Thus one can receive nearly complete voltage stabilization. Note that connection of the load to the stabilizer impairs its characteristic making it less gently sloping. It should be borne in mind that the rated power of the stabilizer elements greatly exceeds the allowable power of load.

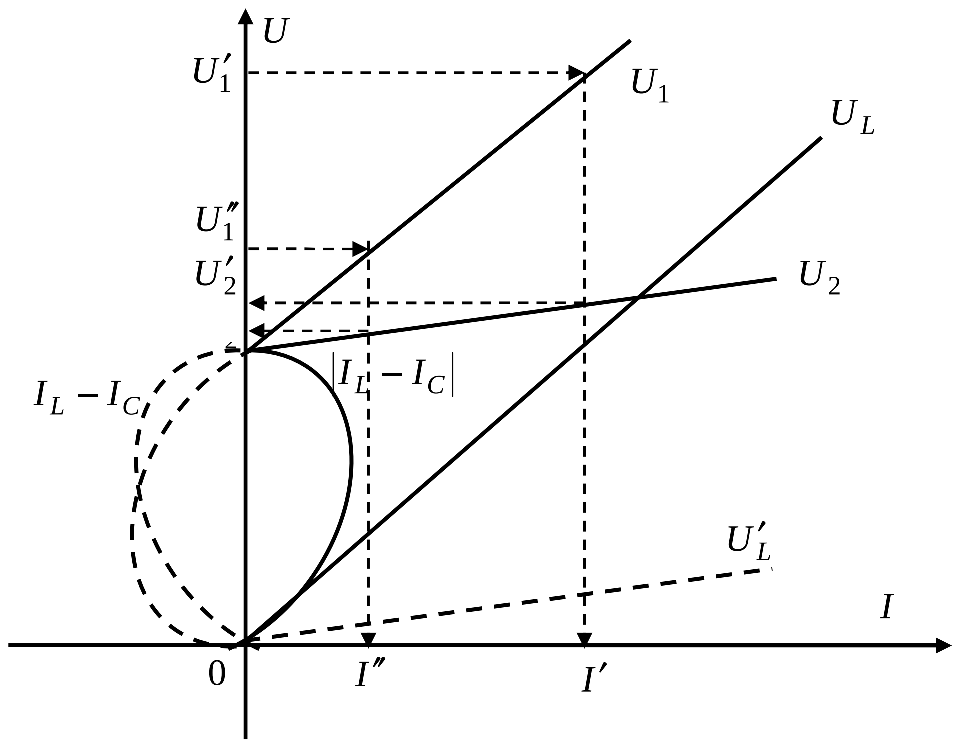

Fig. 7.40.

Among the disadvantage of conventional ferromagnetic stabilizers there is also dependence of the output voltage on the frequency. It is possible to reduce this dependence via additional complication of the stabilizer circuit.

7.3.5. INDUCTIVE CONTROLLABLE ELEMENTS OF NONLINEAR CIRCUIT

7.3.5.1. FERROMAGNETIC POWER AMPLIFIER

If an additional winding is superimposed upon ferromagnetic core of the coil, then by varying the current through this winding it is possible to influence the form of the characteristic of the coil from the side of the main winding. Thus, we can control the process in the circuit of main winding by varying the control current through additional winding.

Such controlled inductive elements may be used in AC circuits, e.g. for voltage regulation in power lines or as a variable inductive load in devices for electrical machine testing, and so on.

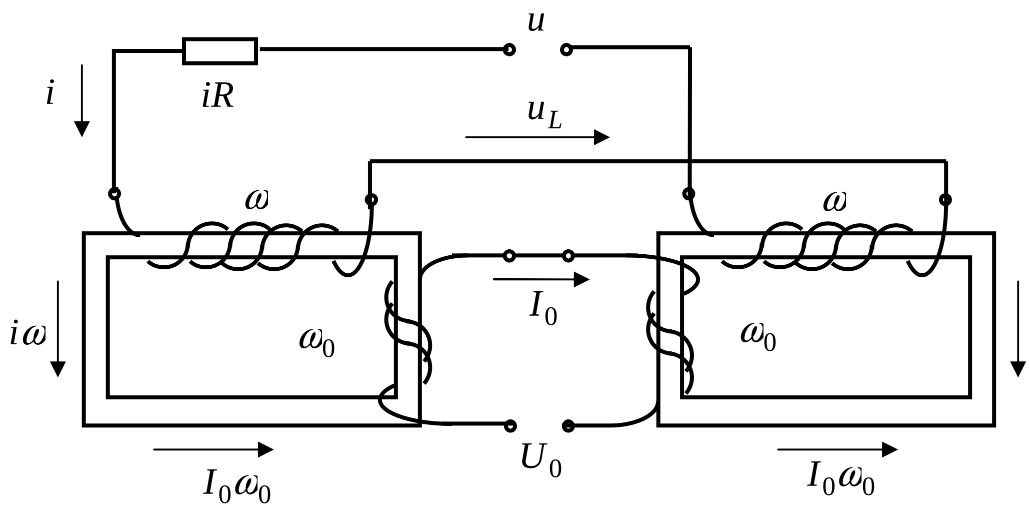

Fig. 7.41.

So-called ferromagnetic power amplifiers with nonlinear controlled ferromagnetic elements are rather widespread. Fig. 7.41 shows a circuit of a ferromagnetic amplifier.

The amplifier consists of two identical ferromagnetic cores upon each of which the two windings having number of turns *w* and w_0 are superimposed. Windings with number of turns *w* are connected in series with a load having a resistance *R*. This circuit is powered by an AC voltage *U* with frequency *f*. The windings with number of turns w_0 is the controlling circuit. Suppose that certain DC i_o is flowing through the controlling circuit. The larger value i_o, the stronger the magnetization of core coils by this current, and the smaller the inductive reactance coils for AC from the side of windings *w*, and accordingly the smaller the drop voltage U_L across the coils at the given current *I*. Fig. 7.42 shows a family of

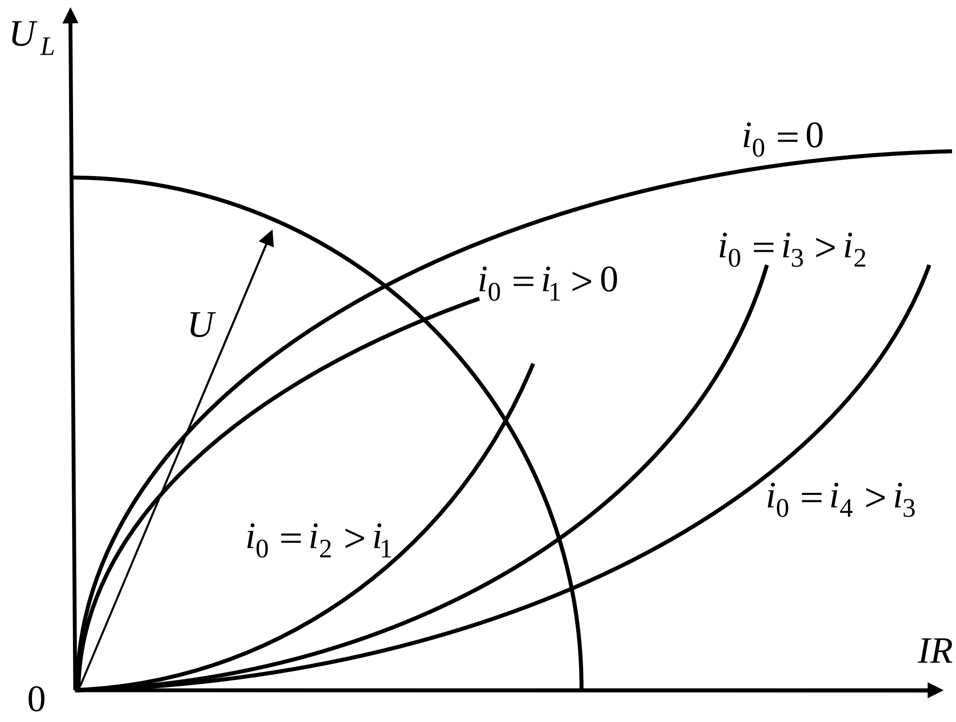

Fig. 7.42.

characteristics $U_L=f(IR)$ for different values of the magnetization current i_o. For the convenience of further discussion, the voltage drop at the load, being proportional to current *I* when $r= const$, is marked on the x-axis instead of the current *I*.

By introducing the equivalent sinusoids and neglecting losses in cores and winding of coils, one can consider that voltages *Ir* and U_L are shifted in phase by an angle π/2. In this case, we can write

$$U_L^2 + (Ir)^2 = U^2.$$

When $U = const$, this equation defines a circle of radius *U* with center at the origin (see Fig. 7.42).

The intersection points of circle with the characteristics constructed for different values of i_0 make it possible to find the dependence of *I* on the i_0. If the current i_0 through the control winding changes with the frequency is significantly lower than the frequency *f*, then it will cause a corresponding change in the rms value of current *I* at the load.

Provided $i^2_0 r_0 << I^2 r$, we get the possibility to control a large power at the load under low power of a control circuit, i.e. power amplifier is obtained.

In the amplifier in Fig. 7.41 the two identical cores are taken and windings are coiled in such a direction that even harmonics resulting from the core magnetization by current i_0 are mutually compensated in the load circuit. Herewith the EMF with frequency *f* caused by current *I* is likewise compensated in the controlling windings.

7.3.5.2. SEPARATION OF HIGHER HARMONICS IN NONLINEAR CIRCUITS OF THE FREQUENCY CONVERTERS

Till now, using the method of equivalent sine waves, in fact, we neglected the higher harmonics in curves of current and voltage. In the previous section we saw that for analytically determine the first harmonic it is necessary to take into account the presence of the higher harmonics; and the wider the range of harmonics taken into account, the more accurate the result.

In certain cases it is important to determine higher harmonics. In such cases, it is necessary to consider the real non-sinusoidal curves of current and voltage. This problem arises, for example, if we want to take advantage of the higher harmonics presence in nonlinear circuits for the purpose of frequency multiplication. The presence of any nonlinear inertialess element in an electric circuit causes the currents and voltages in the circuit to have non-sinusoidal shapes even under a sinusoidal voltage applied to the input terminals of the circuit.

Singling out one or another harmonic at output circuit, we can receive a substantial frequency multiplication. To obtain a sufficiently high level of efficiency in such a transformation of frequency it is advisable to use non-linear elements, in which the energy losses are small. These may be, for example, non-linear inductive and capacitive elements.

For the conversion of frequency, devices with controllable nonlinear elements such as electronic lamps and semiconductor triode are also widely used.

In symmetrical multiphase systems, the harmonics whose order is equal to or a multiple of the number of phase's *m* form the symmetrical system of zero phase-sequence. The remaining harmonics form a symmetrical system of positive or negative phase-sequences. We take *m* identical coils with ferromagnetic cores and we connect these coils in wye circuit without neutral wire. When these windings are connected to the source of sinusoidal symmetrical *m*-phase voltage with the positive phase-sequences, higher harmonics appear due of nonlinear coils characteristics in the current curves.

However, the harmonics whose order equals to or is a multiple of *m* cannot exist in the current curve since they form a system of zero phase-sequences that can only be closed through the neutral wire which is missing here. In such case, these harmonics appear in the curves of the magnetic core fluxes and, respectively, in the curves of the phase voltages across the windings of cores.

The initial condition of the absence of such harmonics in the line voltage is satisfied, since the line voltage is the difference between the phase voltages. If now identical secondary windings are set on all cores and connected in series, then EMF harmonics whose order is equal to *m* or a multiple of *m* are summed arithmetically, and the main EMF harmonics into secondary windings yield zero. Thus, voltage frequency at secondary terminals will be *m* times greater than the frequency in primary circuit voltage, i.e. we obtain the frequency multiplication by the factor of *m*.

It is essential to note that the *m*-phase system is converted into a single-phase, i.e., decreases the number of phases by the factor of *m*.

Frequency tripler and doubler are based upon this idea. The circuit of frequency tripler is given in Fig. 7.43, and the circuit of frequency doubler is shown in Fig. 7.44.

For a frequency tripler, primary supply of the circuit is performed by the symmetrical three-phase sinusoidal voltage source. The voltage of triple

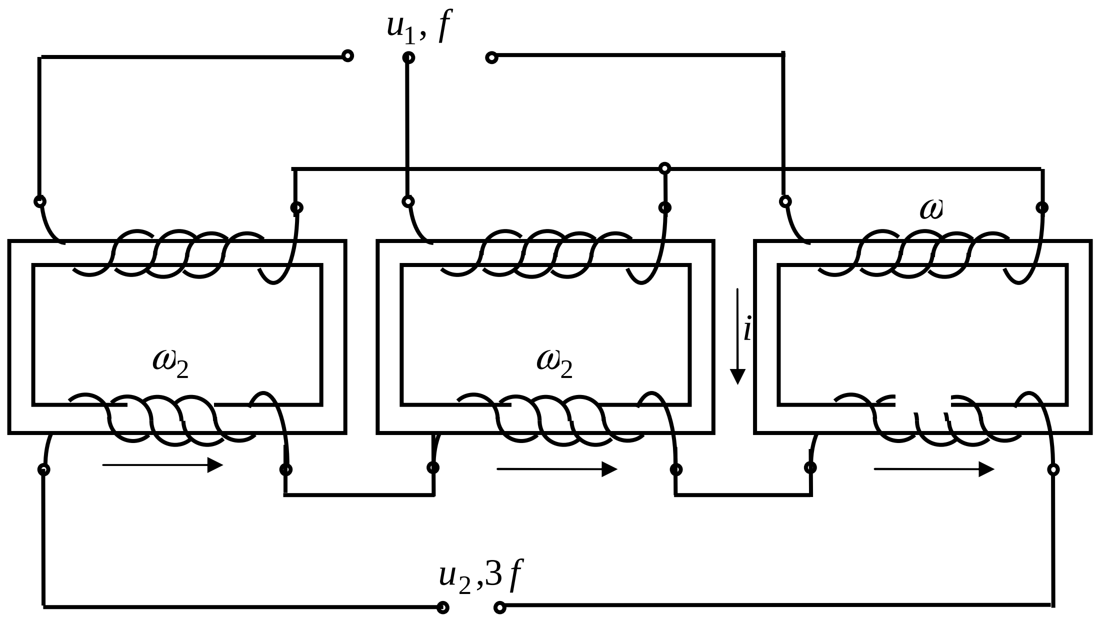

Fig. 7.43

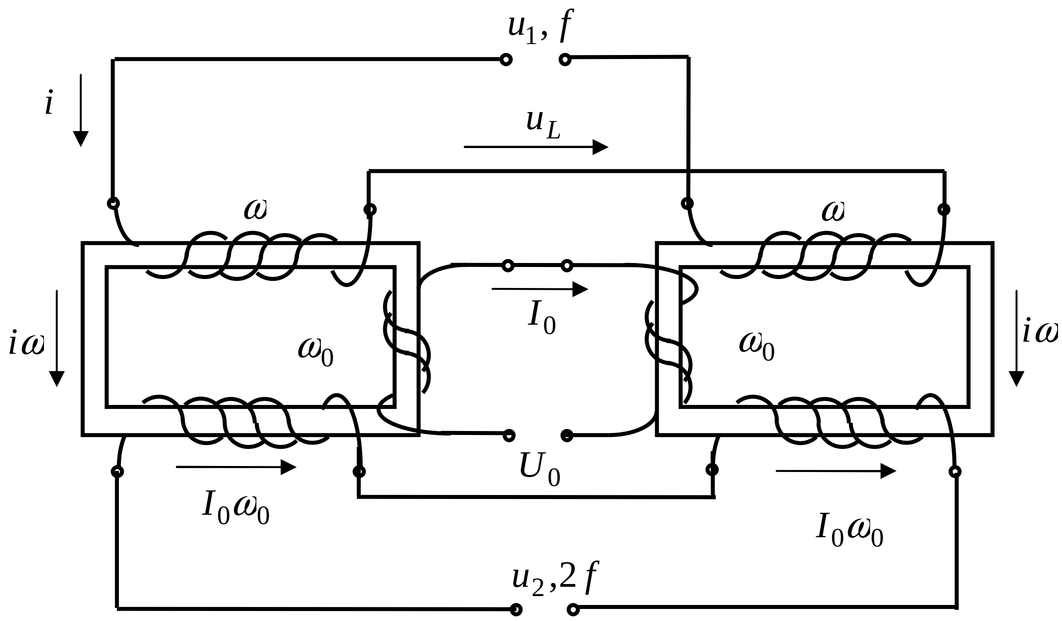

Fig. 7.44.

frequency at output terminals is obtained as a result of separating out the third harmonic. The output voltage will also contain all the odd harmonics whose order is a multiple of three (*9th*, *15th*, *21th*, and etc.). Even harmonics are absent due to the symmetry of the magnetization curve of the cores.

The supply for primary circuit of frequency doubler is carried out from source of sinusoidal single-phase voltage U_{12}. To obtain the voltages U_{01} and U_{02}, the nonsymmetrical bi-phase system with a phase shift by π is formed between neutral point *0* and terminals *1* and *2*. According to the above, in the secondary windings connected in series, harmonics of order $m = 2$ and of order multiple of two can be separated, i.e. all even harmonics.

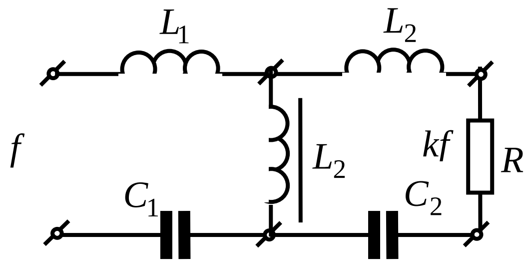

Fig. 7.45.

However when the magnetization curve has symmetry, then the even harmonics cannot exist. To create asymmetry in the magnetization curve, there is a third winding with constant magnetizing current i_0.

Besides frequency multipliers based on the principles outlined above, there may be multipliers with resonance selection of *k-th* harmonic. Fig. 7.45 shows a circuit of such multiplier. Coil *L* with ferromagnetic core is powered from a source with frequency *f*. A capacitor C_1 and an inductance coil L_1 are used to tune the entire primary loop at frequency *f*. Here the current through the coil L_1 is close to a sine wave, and the voltage across it has distinct peak-shape. Secondary loop L_2, C_2 tunes in resonance with the *k-th* harmonic of non-sinusoidal voltage appearing at the terminals of the coil *L*. Thus, frequency *kf* is allocated at the load having resistance *R*, and hence we obtain the frequency multiplication by the factor of *k*.

In practice, for frequency multiplication, more complex circuits having better working characteristics are applied.

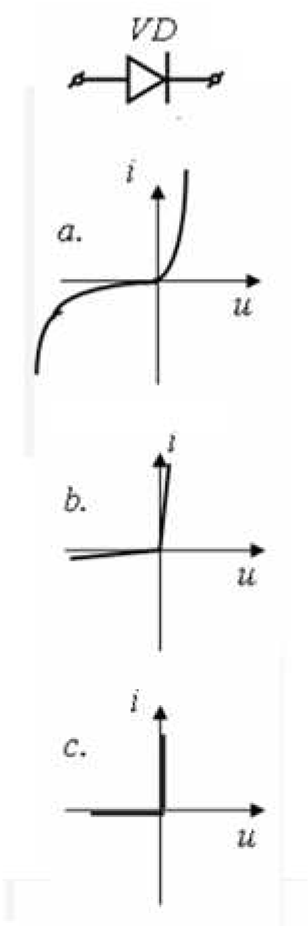

Fig. 7.46.

7.3.6. SPECIFICS OF CALCULATION OF NON-LINEAR CIRCUITS WITH SEMICONDUCTOR DIODES. AC RECTIFICATION

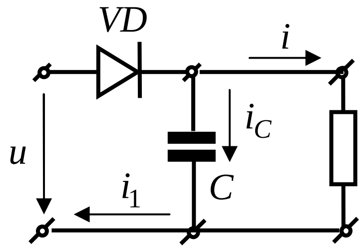

Fig.7.47.

The characteristics *u(i)* of semiconductor rectifier diode and a piecewise linear approximation of this characteristics are shown in Fig.7.46, *a*, *b*. If we neglect the drop voltage across the valve in the flowing of direct and reverse current, the characteristics of the ideal valve takes the form shown in Fig. 7.46.*c*.

As an example of applying the method of intervals conjugation, we consider a simple circuit of current

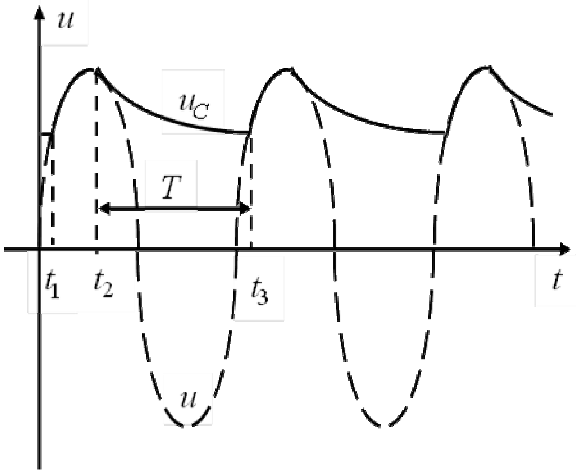

Fig.7.48.

rectification, shown in Fig.8.47 assuming that the diode has the perfect characteristics Fig. 7.46.*c*.

When the diode is opened, the voltage drop at it is zero, and when it is closed, the current through it is zero. Let the applied voltage be changed according to the law $u = U_m \sin\omega t$.

Within the time interval $t_1 < t < t_2$ (Fig. 7.48) the diode *VD* is open and the capacitor *C* is being charged. In this interval, we have the equation:

$u_c=u=U_m\sin\omega t;\ i=u/R= U_m \sin\omega t/R;$
$i_c = C du_c/dt=\omega C U_m\cos\omega t;$
$i_1=i_c+i= \omega C U_m\cos\omega t+ U_m \sin\omega t/ R.$

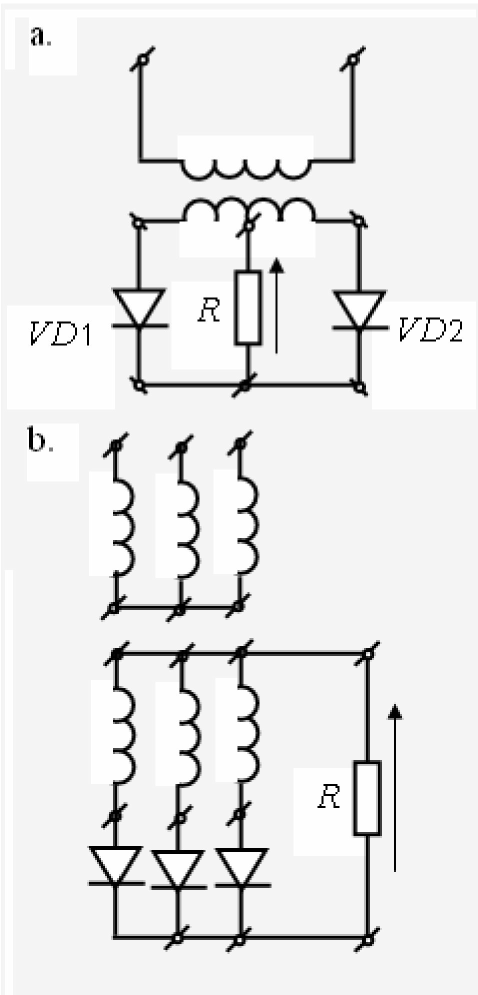

Fig. 7.49.

The diode *VD* closes at instant $t= t_2$, when the urrent i_1 by changing reaches the cusp point of the haracteristic (point *0* in Fig. 7.46,*c*), in given case, vhen the current i_1 drops to zero. Hence, to determine he time t_2 we obtain the equation

$$0= \omega C U_m\cos\omega t_2+ U_m \sin\omega t_2/R$$
$$\text{or}\ \ \omega t_2=-arctg(\omega CR).$$

Within the interval $t_2 < t < t_3$ the semiconductor liode *VD* is closed and the capacitor *C* is being lischarged through the resistance *R*. We have

$$u_C = iR =– i_C R =– RCdu_c/dt$$
$$\text{or}\ RCdu_c/dt+ u_C = 0,$$

rom which $u_C = Ae^{-(t-t2)/CR}$, where *A* is the initial value of u_C at the instant $t = t_2$ for this interval. A constant *A* s determined from the condition of conjugation processes in the considered adjacent intervals, namely t the instant t_2, bearing in mind that the voltage across he capacitor at this point cannot be changed step-wise. Equating values for u_C at $t = t_2$, taken from the expressions for the first and second intervals, we obtain

$$u_c (t_2) = U_m \sin (\omega t_2) = A.$$

The last thing is to determine the time t_1 of the diode opening. It is found from the condition that the recurrence interval of the process in this case is the period T of the applied voltage. Consequently, the voltage u_C in the beginning of the first interval at instant t_1 is equal to the voltage u_C in the end of the second interval at the instant $t=t_1+T$:

$$U_m \sin(\omega t_1)= U_m \sin(\omega t_2)\, e^{-(t1+T-t2)/CR}$$

or

$$e^{-(t1+T)/CR} \sin(\omega t_1)= e^{-t2/CR} \sin(\omega t_2).$$

From this equation t_1 is calculated.

Even this simple example shows that to calculate the desired values it is necessary to solve the transcendental equation. In more complicated circuits we will have to solve a set of such equations, which is the main difficulty of the method. However, this method makes it possible to define the actual shapes of the current and voltage very accurately in those cases where the characteristics of nonlinear elements are close to the piecewise–linear ones.

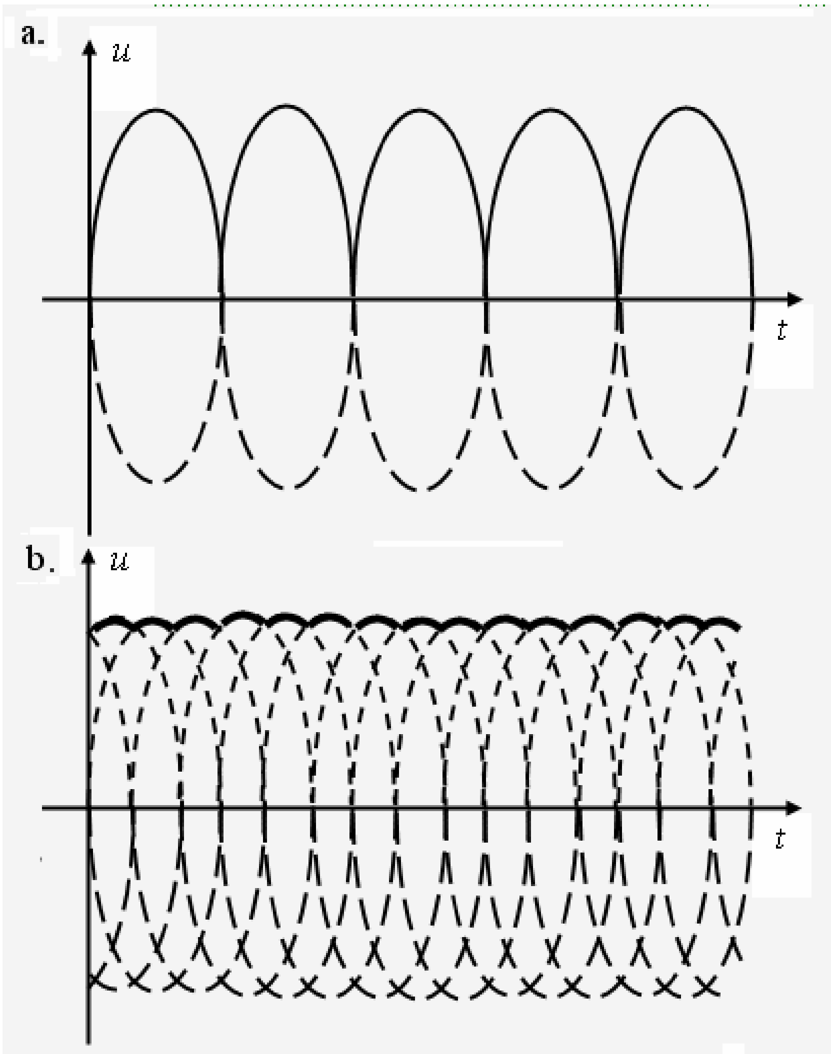

Fig. 7.50.

The value of the capacitor C in the considered circuit is easily seen from Fig. 7.48. The larger the capacitance C in a given R, the greater the time constant $\tau = RC$ of capacitor discharge, and the less the voltage will be different at the load R at instants t_2 and t_3, respectively, and the smaller the pulsation of the rectified voltage.

Usually more complicated rectifier circuits are applied. In Fig. 7.49,*a*, *b* are depicted the two-phase and three-phase rectifier circuits with neutral point *0* in the secondary windings of the transformer. If we neglect the inductance of the AC circuit, then the current through the secondary circuit flows at each instant only through one diode coupled to the transformer winding, the voltage at the terminals of the winding in the given instant is the highest.

If we also neglect the voltage drop at the diodes, then the voltage at the load will have a form shown in Fig. 7.50,*a*, *b* by bold lines. The Fig. 7.50,*a* relates to a two-phase rectifier circuit (Fig. 7.49,*a*), and Fig. 7.50,*b* – to three-phase rectifier circuit (Fig. 7.49,b). The larger the number of phases, the smaller the pulsation of the rectified voltage. The current through the load flows in one direction all the time indicated in Fig. 7.49,*a, b* by an arrow.

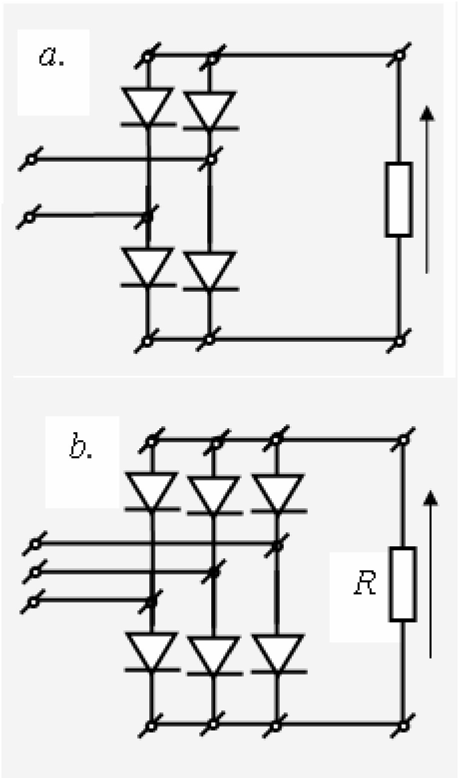

Fig. 7.51.

For current rectification, bridge circuits are also used. In Fig. 7.51,*a* is shown a single-phase circuit, and in Fig. 7.51,*b* three-phase bridge one. The curve of the rectified voltage for the first circuit has the form shown in Fig. 8.36,*a*, and for the second – the form shown in Fig. 8.36,*b*.

If in the circuits shown in Fig. 7.49 and 7.51, we take in consideration circuit inductance, and if these circuits have capacitors, the calculation processes in them should be carried out by the method of the conjugate intervals as described above. Herewith, the recurrence interval in the circuits Fig. 7.49,*a* and 7.51,*a* is equal to *T/2*, in the circuit of Fig. 7.49,*b* it is equal to *T/3* and in the circuit in Fig. 7.51,*b* it equals *T/6*.

7.4. CONCLUSIONS

I. NONLINEAR CIRCUITS UNDER DC.

1. Electric circuit that contains at least one non-linear element as a whole is non-linear. Non-linear element is such an element in which current-voltage characteristic is not presented by a straight line. The current-voltage characteristics of nonlinear elements can be given analytically, graphically or in a tabular form.

2. Non-linear elements have static and dynamic resistances. Static resistance is always positive. Dynamic resistances for the increasing current-voltage characteristics are positive, and for decreasing characteristics are negative.
3. The static and dynamic resistances in the linear elements coincide at all points of the current-voltage characteristics.
4. Methods for calculating the nonlinear DC circuits are based on Kirchhoff's laws, which connect the currents in the nodes, electromotive forces and the voltage drop across independent circuits.
5. For the calculation of nonlinear circuits the principle of superposition are not applied, so all the methods of circuit calculation based on the principle of superposition are not applied to non-linear DC and AC circuits.
6. If the working section of the nonlinear element may be approximated by a straight line, in this case, the calculation of circuit parameters can be linearized by replacing non-linear element by an equivalent dynamic resistance with a power source or by static resistance. Calculation of linearized equivalent circuit in the vicinity of the working point is made analytically.
7. If in a circuit there are two nodes then calculation is rational to perform graphically by using nodal analysis. For this, current-voltage characteristics of each branch are being constructed, which are displaced with respect to the origin of coordinates by value of the EMF included in the given branch. Then resulting current-voltage characteristic of the whole circuit is plotted, which allows you to find the voltage across the parallel branches. Knowing the voltage across the parallel branches we find the parameters of all the branches.
8. If an electric circuit comprises a single non-linear element, the calculations are efficiently produced by the method of equivalent generator. The circuit is divided into passive and active one-port network. The passive one-port network includes a non-linear element, and the active one-port network contains all the rest of the linear circuit. Active one-port network is replaced by an equivalent generator. After determining the parameters of the linear equivalent generator the problem is reduced to finding the parameters of a

nonlinear circuit with single-loop having EMF source or with dual-loop having current source. A working point on the non-linear voltage-current characteristic is determined, which allows a non-linear current-voltage characteristic to be linearized by dynamic resistance with an additional source of energy or static resistance. After linearization, calculus of the rest of parameters in the initial circuit is made like in the linear circuit.

9. If a circuit has two non-linear elements, calculation is rational to perform by using two-port network theory. Calculated circuit is presented in the form of the active two-port network, whose leads are connected to the two non-linear elements. Active two-port network is presented by T-shape equivalent circuit of passive two-port network with two open circuit EMF sources. As a result, the calculation is reduced to finding the parameters of the dual-loop circuit with three linear, two nonlinear resistors and two sources of EMF.

II. NONLINEAR CIRCUITS UNDER AC.

1. AC non-linear elements by its inherent properties are being subdivided into symmetrical, non-symmetrical, inertial, non-inertial, resistive, reactive, with positive and negative dynamic resistances.
2. The properties of symmetrical non-linear elements are not dependent on direction of current flowing through it or applied voltage.
3. The inertial properties of non-linear elements under AC are conditioned by the presence of heat inertia which results in linear dependence between instantaneous value of current and voltage, as well as in non-linear dependence between effective values of current and voltage.
4. We distinguish between non-linear electric circuits with homogeneous energy sources, when the sources of equal frequency are acting in circuits, and non-linear electric circuits with nonhomogeneous energy sources having the sources with different frequencies.
5. The circuits with electric valve (diode) elements are studied in assumption that diode has an ideal volt-ampere characteristic: in conducting direction (conducting state) its resistance tends to zero, and in

contrary (lock state) direction its resistance tends to infinity.

6. Diode is in conducting state when the anode potential is higher than the cathode potential.

7. The calculation of complicated AC electric circuits is performed on the grounds of Kirchhoff's laws. This calculation is performed analytically using given analytic expressions of volt-ampere characteristics, as well as graphically when the given characteristics have the form of graphs or tables.

8. The successive approximations method is efficient and allows making calculations of non-linear electric and magnetic circuits of any complexity, assuming that the characteristics have piecewise-linear approximation.

9. If it is known that the non-linear element operates in the section of the current-voltage characteristic, which can be approximately assumed linear, then the problem is linearized by replacing the working section of current-voltage characteristic of a straight line and determining the equivalent parameters of the equivalent circuit.

10. When the coils with ferromagnetic cores are studied then an efficient method of circuit parameters calculation is the equivalent sinusoids method, at which real sinusoidal values are replaced by equivalent sinusoidal ones. The conditions of equivalence are: the equal effective values and losses in the circuit from the initial non-sinusoidal curve and replaceable sinusoidal.